SPECIAL ARTICLES

NOTE ON THE EQUATIONS FOR TORTUOSITY IN THOMSON AND TAIT'S NATURAL PHILOSOPHY

THE occurrence of two very different expressions for the same thing is always puzzling. It may not, therefore, be superfluous to consider them.

1. If $\alpha, \beta, \gamma, \lambda, \mu, \nu$ are the direction cosines of the radius of curvature R, and of the corresponding normal N to the osculating plane, respectively, the first expression in the well-known classic (section 9, edition 1879), is $t^2 = \lambda'^2 + \mu'^2 + \gamma'^2$, where the dashes denote d/ds and s is the independent variable. This is equivalent to the vector tortuosity, $T = d\,(RN)/ds = i\lambda' + j\,\mu' + k\,\nu'$, and is the obvious increment of the unit normal, RN, per cm. along *s, in full,* and in the direction $RN' + R'N$. But as both N and N′ are vector products each containing r′, this T has no projection on the tangent; and, therefore, for counter-clockwise rotation seen in the direction of the tangent, points in the direction of the radius of curvature.

2. The second expression reads:

$$t^2 = \left(\mu \frac{d\nu}{ds} - \nu \frac{d\mu}{ds}\right) + \text{etc.}$$

and is thus vectorially equivalent to

$T' = R\,N \times d\,(R\,N)\,/\,ds = R^2\,N \times N' + 0;$

and if we insert the vector equations for N and N′

$$\begin{aligned}
-T' &= R^2\,(r' \times r'') \times (r' \times r''') \\
&= r'\,R^2\,(r' \times r''')\,.\,r'' \\
&= r'\,R^2\,(i\,\lambda' + j\,\mu' + k\,\nu')\,/\,R - NR'/R)\,.\,r'' \\
&= r'\,T\,.\,R_1 = r'\,(\alpha\,\lambda' + \beta\,\mu' + \gamma\,\nu'),
\end{aligned}$$

the subscript denoting a unit vector. Thus, in value, T′ is again the projection of T on the radius of curvature R. The former (T′) is laid off, however, along the tangent, since T and R are opposed in direction, and is thus a very different thing from T, which is not an immediate tortuosity. The case is analogous to the occurrence of two expressions for the radius of curvature: $(r'')^{-1}$ in the osculating plane and pointing towards the center; and $(r' \times r'')^{-1}$, laid off along the corresponding normal to that plane, inward.

If the normal N had been defined (preferably here, I think) as $r'' \times r'$, rather than as above, the clockwise rotation of the osculating plane seen in the direction of the tangent would correspond to a vector T in the direction of radius of curvature. T′ would at once be positive in the direction of the tangent r′, without compromise; but opposition of sign is none the less inherent in the two projected tortuosities or angles, $d\,R_1/ds\,.\,N_1$ and $d\,N_1/ds\,.\,R_1$.

CARL BARUS

BROWN UNIVERSITY,
PROVIDENCE, R. I.

VERY SOFT X-RAYS—THE M — SERIES FOR IRON

DURING the past two years several investigators have reported attempts to study the portion of the spectrum lying between the Millikan region and the X-ray region by means of the photoelectric action of the radiation. In every case the curves obtained by plotting the photoelectric current against the accelerating voltage applied to the electrons which generated the radiation showed one or two discontinuities which were interpreted as indicating the excitation of one of the X-ray series in the region under investigation. Now in the ordinary X-ray region the characteristic absorption limits have been found to possess a fine structure which has been interpreted by Kossel as corresponding to the ejection of an electron from one of the inner shells of the atom to the various optical orbits in the outer portion of the atom. If such an explanation is correct then a similar phenomenon should be observed in the soft X-ray region, and therefore we should expect to find not one break in the photoelectric current but several for each X-ray absorption limit. This has been observed by the writer in his research on the soft X-rays, and as the work is still in progress it seems advisable to make a preliminary report at this time.

The apparatus used is in principle the same as that employed by Hughes. The radiation was excited by bombarding a solid target with electrons from a hot tungsten cathode. The radiation fell on a platinum plate producing a photoelectric current which was measured by means of a Compton electrometer using the constant deflection method. Positive ions and electrons were kept from reaching the photoelectric plate by two gauzes, one of which was