very encouraging, there being some two score British fliers, besides foreign aviators, notably the Dutch airman, M. Fokker. A large number of short flights and some quite long flights were made; yet on the whole the results of the meeting were not of a sensational nature until the last day of the meeting. The general conclusion is that British aviators do not fall behind those of Germany, and that it is possible to find suitable arenas in this country for the practice and display of motorless flight. The most notable achievement of the first day of the contest was a thirty-seven-minute glide by M. Fokker, but this was surpassed by a fine flight executed by Mr. F. P. Raynham. This aviator had already taken a place in the front rank of British pilots in the recent air-race round England: he added to his laurels by remaining in the air in a motorless machine for one hour and fifty-three minutes, thus putting himself in the same category as the German record-makers, Martens and Hentzen. But on the last day, Saturday, two world-records were nevertheless established. J. R. Olley went up in a Fokker biplane, and remained in the air with a passenger for forty-nine minutes, while M. Maneyrolle, in a tandem monoplane glider, succeeded in remaining in the air for three hours twenty-two minutes, thus winning the *Daily Mail* prize and beating the previous record, that of Hentzen, by twelve minutes. During the last ninety minutes of his flight, M. Maneyrolle was accompanied by a monoplane glider flown by Squadron-Leader A. Gray, and it was night when the two machines landed within 100 yards of the point from which they started. These competitions on the South Downs will serve as an encouragement to motorless flight in this country, and will help in the accumulation of knowledge and experience on one of the most interesting developments in modern aeronautics.—*Nature.*

SCIENTIFIC BOOKS

The Theory of Functions of a Real Variable. By E. W. HOBSON, Sc.D., LL.D., F.R.S., Sadleirian Professor of Pure Mathematics and Fellow of Christ's College, in the University of Cambridge. Second edition, Vol. 1. Cambridge at the University Press, 1921. Pp. xvi + 671.

Because of the war and relativity, we are at present in a period of increasing scepticism towards so-called established principles and facts. Many still believe, however, that mathematical knowledge, at any rate, is beyond dispute. For these there will be great disappointment in the pages of Hobson. The book reads in places like unconvincing philosophy; and instead of statements made with full personal conviction, we find, at times, conflicting opinions of "authorities"—beings supposed, by some, to be unknown to mathematicians—and inconclusive attempts at mediation. And yet it is on the Theory of Functions of a Real Variable that rigor in Mathematical Analysis depends. The first edition appeared in the course of Zermelo's work on *Wohlordnung,* when mathematicians were just beginning to get their bearings on certain controversial matters; after a lapse of fourteen years, there is the same indecision. Other writers on Real Variables and Point Sets—for example, Hausdorff, Carathéodory and Hahn—adopt a single point of view and proceed joyfully without misgiving. Professor Hobson wants to give a comprehensive report—his book is the most voluminous treatment of the subject—to include historical matter, and to be as fair as possible, "no attempt has been made to give dogmatic decisions between opposed opinions." This wish to be fair makes Professor Hobson exchange, at times, the rôle of mathematician for that of reporter; it has the advantage, however, of making the reader independent of the author—a significant advantage when we observe that even among mathematicians instances are observable of opinions held not on their merits but on the ground of personal and nationalistic associations.

The rôle of the mediator, however, is apt to be a hard one, not only in industrial, but even in mathematical affairs. For fear of being one-sided, he may lose vivacity. Thus we read on page 238 [author's italics], *"In order that a transfinite aggregate . . . may be capable of being ordered, a principle of order must be explicitly or implicitly contained in the*

norm by which the aggregate is defined. Or again, page 239, " . . . an aggregate has a cardinal number only when it is one of a plurality of equivalent aggregates, distinct from one another." Again, what success can be expected from an attempt to clarify the notion of aggregate by the introduction of a new word, "norm," itself of debatable meaning? Professor Hobson intimates that something must be "universally accepted" to be admitted as mathematical knowledge; but he does not mention the attack made by Brouwer and Weyl upon some of the fundamental theorems in Analysis. Still again, it may be asked, if (page 6) "the justification [for a certain assumption] is to be found in the fact that no contradiction arises in the theory based on it," why is not the multiplicative axiom justified? Zermelo's *Grundlagen* have led to no contradiction.

The present edition is "revised throughout"; "the parts of the subject dealt with in the first five chapters of the first edition have been expanded into the eight chapters of the present volume." This expansion is due chiefly to the recent developments in the Theory of Integration. The theories of Hellinger, Young and Denjoy are also included. The mathematical world owes a debt of gratitude to Professor Hobson for presenting in a smooth, connected exposition a huge mass of research, a considerable part of which is of recent origin.

The style is, on the whole, very lucid, great pains being taken to prepare the reader's mind for the reception of new ideas. However, here and there we find a lack of compactness—witness the proofs that cover pages 97 and 98, and which may be compressed into one tenth of the space by the use of points with rational coordinates. The treatment retains, in part, something of the freshness of a memoir and will thus prove more stimulating, in one way, than treatises written with a constraining finish.

At this late date, strange to say, the definition of cardinal number as given by Professor Hobson is not without an objectionable feature. Russel is right. Professor Hobson's reference to the "degree of plurality" is like saying, "You know what I mean"; it does not make his definition mathematically acceptable.

On page 259 occurs the following statement: "No elaborate theory is required for functions which retain their complete generality, . . . since few deductions of importance can be made from that definition which will be valid for all functions." It may be of interest to remark that this view is rendered untenable in the light of the results to be announced soon in the *Proceedings of the National Academy of Sciences* in a paper by the reviewer, entitled "New Properties of All Real Functions."

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Analysis Situs. The Cambridge Colloquium Lectures, Part II, Vol. V. By OSWALD VEBLEN. Published by the American Mathematical Society, 501 West 116th Street, New York. 150 pp., octavo. 1922.

The Cambridge Colloquium lectures on Analysis Situs were delivered in 1916, but the publication having been postponed because of the war, the lectures were completely rewritten before publication, and the resulting book is a treatise on the elements of Analysis Situs. It is furthermore the only modern book on the subject. By a study of this book it is possible to acquire a knowledge of Analysis Situs without going through the many widely scattered memoirs as was formerly the case. Every one interested in Analysis Situs will welcome Professor Veblen's book as an important and useful contribution to the subject. Part I by G. C. Evans on Functionals and their Applications was published in 1918.

H. L. RIETZ

SPECIAL ARTICLES

ZOSTERA MARINA IN ITS RELATION TO TEMPERATURE[1]

IN connection with some work on the temperature control of the geographical distribution of the marine algæ, it seems to be demonstrated that the terms eurythermal and stenothermal apply only to the power of endurance of a wider or narrower range of temperature

[1] Preliminary communication.