specific acidity, developed his method so that it would yield just this degree of precision. These points are mentioned specially because Olsen, in the Danish paper, criticizes the writer's method severely on the basis of "inaccuracy." But if a soil, the acidity of which varies in general by a factor of 3, is sampled at an arbitrary depth and then altered by long soaking and filtration, there is certainly nothing to be gained by making highly precise acidity determinations on the resulting extract. Indeed, both Arrhenius and Olsen, upon assembling the results obtained on given species or associations of plants, also find that there is always a range of at least 0.5 in p_H (a factor of 3 in specific acidity). The fact that all three come to recognize the same range indicates that it is of fundamental significance.

All three investigators find that the soils of native plants in general extend from a specific acidity of a few thousand to a specific alkalinity of about 10. All find that the greatest number of species as well as of individuals occur in soils lying just to the acid side of the neutral point. And, most remarkable of all, it turns out that many individual species of plants have essentially the same soil acidity preferences in Europe as in America, indicating that this is not a question of location, climate, or surroundings, but a physiological feature of the species. For illustration: the lily-of-the-valley, *Convallaria majalis,* grows in Denmark in soils of specific acidity 1000 to 400. Isolated colonies of this plant in the southern Appalachian Mountains have been studied by the writer and found to have specific acidity 500 to 300, practically the same range. Hepatica (*Hepatica triloba* or *Anemone Hepatica*) shows in Denmark preference for soils ranging from neutral (specific alkalinity 1) to specific alkalinity 8. In America a near relative of the European plant thrives best in black leafmold with an average specific alkalinity of 3.

How soil acidity affects plants is a subject requiring further investigation. Olsen's data led him to infer that the action may be direct, but others have found that it is usually indirect. There is evidence both for and against the view that the acidity affects primarily symbiotic organisms, and only indirectly through them the higher plants. Recent American work has indicated that the effect of acidity is produced largely through the agency of aluminium or iron salts, although Olsen is unable to find evidence of their toxicity. But in view of the general agreement of the results of the three independent investigators as above outlined, it can no longer be questioned that soil acidity is of fundamental importance in controlling the distribution of native plants.

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SPECIAL ARTICLES

THE EINSTEIN EQUATIONS FOR THE SOLAR FIELD FROM THE NEWTONIAN POINT OF VIEW

1. About a year ago I determined the law of attraction from the Newtonian point of view of action at a distance which gives the equations of planetary motion obtained in the Einstein theory. Two months ago Professor Birkhoff, of Harvard, told me that he had obtained similar results in his class this year, and suggested that I publish my results. In doing so I am not advocating the rejection of the Einstein point of view which seems to me the correct one, but I am merely indicating a modification in the Newtonian law which will account for the motion of the perihelion of Mercury and the deflection of light rays. It may be also that by means of this formulation of the law it will be possible to solve, with sufficient accuracy, problems which are not readily handled by means of the equations of general relativity.

2. The Schwarzschild form of the linear element of the Einstein field of gravitation of a mass m at rest with respect to the space-time frame of reference is

$$(1)\quad ds^2 = \left(1-\frac{2m}{r}\right)dt^2 - \frac{1}{1-\frac{2m}{r}}dr^2 - r^2(d\theta^2+\sin^2\theta d\varphi^2)$$

where r, θ and φ are the space coordinates as measured by astronomers, and t is the coordinate of time.

If we write

(2) $I =$

$$\sqrt{\left(1-\frac{2m}{r}\right)\dot{t}^2 - \frac{r}{r-2m}\dot{r} - r^2(\dot{\theta}^2 + \sin^2\theta\dot{\varphi}^2)}$$

where dots indicate derivatives with respect to s, the world-lines of particles in the gravitational field are the curves in the 4-space for which the integral

(3) $$\int I ds$$

is stationary. The conditions that (3) be stationary are four differential equations of the second order. From one of them it follows that the path is plane; the coordinates may be chosen so that the equation of the plane is $\theta = 0$. Two of the other equations admit as first integrals

(4) $$\frac{dt}{ds} = \frac{A}{1-\frac{2m}{r}},$$

(5) $$r^2\frac{d\varphi}{ds} = h,$$

where A and h are constants. It is readily shown that $I = k$, a constant, is a first integral of the four equations. When k is not zero, s can be chosen so that $k = 1$. Then we have $\theta = 0$, (4), (5), and

(6) $$\left(\frac{dr}{ds}\right)^2 + r^2\left(\frac{d\varphi}{ds}\right)^2 = (A^2 - 1) + \frac{2m}{r} + 2m\frac{h^2}{r^3}$$

for the equations of a world-line of a particle in the gravitational field.

When $I = 0$, the integral (3) is stationary and the corresponding world-lines are those of light in accordance with the Einstein theory. Their equations are (4), (5) and

(7) $$\left(\frac{dr}{ds}\right)^2 + r^2\left(\frac{d\varphi}{ds}\right)^2 = A^2 + 2m\frac{h^2}{r^3}$$

Some writers have obtained these equations by solving $I = 0$ for dt and expressing the condition that $\int dt$ be stationary, in accordance with the Fermat principle. The above method was given by Professor Veblen in his lectures, and appears also in Laue, *Die Relativitätstheorie,* Vol. 2, p. 225. Putting $I = 0$ in (2), we see that the units are such that the velocity of light is unity for $r = \infty$, and that it diminishes as the light approaches the sun. If the unit of length is taken as a kilometer, then the unit of time is 1/300,000 of a second.

3. In classical mechanics for a central force of attraction $f(r)$ the equations are

(8) $$r^2\frac{d\varphi}{dt} = h$$

and

(9) $$\left(\frac{dr}{dt}\right)^2 + r^2\left(\frac{d\varphi}{dt}\right)^2 + 2\int_{\infty}^{r} f(r)dr = E,$$

where h and E are constants. For planetary motion about the sun, whose mass in gravitational units is denoted by m, equation (9) assumes the form

(10) $$\left(\frac{dr}{dt}\right)^2 + r^2\left(\frac{d\varphi}{dt}\right)^2 = -\frac{m}{a} + \frac{2m}{r},$$

where a is the semi-major axis. For the solar system m/a and m/r are of the order of 10^{-8}, for the units previously defined. Thus if we identify $A^2 - 1$ in (6) with $-m/a$ in (10), $A - 1 = \frac{1}{2}10^{-8}$ approximately. Then from (4), $\frac{dt}{ds} = 1 + 3/2\ 10^{-8}$ approximately, which shows the order of discrepancy so far as the solar system is concerned in interpreting ds and dt as the same in (5), (6), (8) and (9) (*cf.* Eddington, Report, p. 50).

It is well-known that it is the term $2mh^2/r^3$ in (6) which accounts for the motion of the perihelion of Mercury. Comparing (6) and (9), we see that from the point of view of action at a distance this is accounted for if we take

(11) $$f(r) = m\left(\frac{1}{r^2} + \frac{3h^2}{r^4}\right).$$

From the preceding remarks it follows that if we put

(12) $$\omega = \frac{d\varphi}{ds}$$

then ω may be interpreted as the angular velocity of the planet about the sun. Then from (5), (11) and (12) we have that

(13) The attraction $=$

$$m\left(\frac{1}{r^2} + 3\omega^2\right) = \frac{m}{r^2}(1 + 3v^2),$$

where v is the component of the velocity perpendicular to the radius vector.

We have remarked in the preceding that the velocity of light at ∞ is equal to 1 in the units chosen. If we denote it by c in any system of units, we may formulate the law as follows:

Two bodies attract one another inversely as the square of their distance and directly as the product of their masses and $(1 + 3v^2/c^2)$, *where v is the component of their relative velocity perpendicular to the line joining the bodies.*

The form (1) is obtained from the Einstein theory on the hypothesis that the planet is small in comparison with the sun. It may be that the above law applies only to this case. However, it may be that the law would work if the bodies were approximately of the same mass. As formulated the law enables one to set up the differential equations of n bodies in a manner analogous to the classical theory. It would be interesting to know whether known discrepancies in the motion of the moon would be overcome by the use of this law.

Although the term $3v^2/c^2$ produces an observable effect only in the case of Mercury, it may produce a significant effect in molecular motion.

4. When in like manner equation (7) is compared with (9) we find that for a ray of light the attraction is

$$3m\omega^2 \tag{14}$$

where ω may be interpreted as the angular velocity of the light about the sun. Thus it is the term $3m\omega^2$ in (13) which accounts for the deflection of light, and the term m/r^2 does not enter. Einstein and his followers have calculated the deviation of light by noting that the velocity changes in a manner analogous to that of a refracting medium, and by applying Huygen's principle. Since the same term appears in the attraction of a planet, it may very well be that the sun affects the medium through which both the light and planets pass, and that the difference between Newton's law and (13) is due to this situation. From this point of view one would expect that the law would not be accurate for two or more bodies of relatively the same mass, but it may lead to a sufficiently close approximation.[1]

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[1] I have just found that A. V. Bäcklund in the *Arkiv för Matematik, Astronomioch Fysik,* Vols. 14 and 15 (just received) has made an extensive study of the relation between classical dynamics and the Einstein theory of gravitation. In the course of his three articles he obtains equation (11) and one similar to (13).

THE AMERICAN ASSOCIATION FOR THE ADVANCEMENT OF SCIENCE

SECTION F—ZOOLOGICAL SCIENCES AND ASSOCIATED SOCIETIES

AT the Toronto meeting of the American Association for the Advancement of Science, December 27-31, 1921, Section F (Zoology) offered no separate program, but met jointly with the American Society of Zoologists. The program was arranged by the latter society.

Six joint sessions were held, the program including 101 titles distributed by subject as follows: embryology, 4; cytology, 8; comparative anatomy, 7; evolution and genetics, 24; ecology and zoogeography (with the Ecological Society of America), 13; general zoology, 2; protozoology, 2; parasitology, 22; comparative and general physiology, 17; unclassified, 2.

The session of Friday afternoon, December 30, was devoted to a symposium on orthogenesis. A biologists' smoker was held Wednesday evening, December 28, and the zoologists' dinner Friday evening, December 30.

The business meeting of Section F took place at the morning session on December 29, with Vice-president Kofoid acting as chairman. M. M. Metcalf is vice-president for Section F for 1922. J. A. Detlefsen was elected a member of the section committee for four years in place of the retiring member, A. M. Reese.

F. R. Lillie presented the following resolutions drawn up by a conference of representatives of the biological societies in regard to a proposed federation of biological societies:

RESOLVED: 1. That it is the sense of this conference that an inter-society conference should be called to study and report upon the feasibility of federation of the biological societies and to develop plans for the said federation.

2. That for the purpose of effecting such an organization, each society, and Sections F and G of the American Association for the Advancement of Science, be requested to designate its president and secretary as members of an inter-