

From the point of view of the future historian this would serve as a large labor-saving device, especially in view of the fact that human knowledge is ever becoming more specialized.

It might be well to call attention to the fact that a bibliography of relativity has also been in progress in England,² namely, the International Catalogue of Scientific Literature, under the direction of Dr. H. Forster Morley. Dr. Morley has made a selected chronological bibliography of relativity and related problems from 1886 to the end of 1920.

The recent visit of Dr. Albert Einstein has not alone stimulated interest among scientific men, but he has strengthened his theory by his own clear presentation of relativity.

Of course the theory has yet to receive its final verification, before the whole can be accepted, and Dr. Einstein has expressed confidence in the final answer.

Not since the doctrine of evolution was promulgated, has any advance of intellectual progress, either of philosophic or scientific importance, caused such profound interest, popular or scientific, as the theory of relativity. And like all epoch-making ideas, the synthetic character of the theory of relativity will mark off a period of great importance in the history of science. Hence the value of a bibliography of a subject in relation to the history of science is in direct proportion to the importance of the subject itself.

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SPECIAL ARTICLES

EINSTEIN'S COSMOLOGICAL EQUATIONS

In two earlier notes published in *SCIENCE* (Vol. 52, p. 413, Vol. 53, p. 238) I gave certain geometrical theorems connected with Einstein's original (1914) equations of gravitation, $G_{ik}=0$ (in space free from matter). I shall now extend some of the results so as to apply to the modified equations employed in Ein-

²Dr. H. Forster Morley, *Nature*, 106, 811-13, Feb. 17, 1921.

stein's cosmological speculations. These he first wrote (1917) in the form, $G_{ik} - \lambda g_{ik} = 0$; but more recently (1919) he has employed the form $G_{ik} - \frac{1}{4} g_{ik} G = 0$, which includes the previous form and which, when the energy impulse tensor T_{ik} is introduced in the right hand member, has the advantage of being possibly applicable to the microcosm (atoms and electrons) as well as to the macrocosm (the stellar universe). Here G_{ik} is the contracted curvature tensor and G is the scalar curvature.

For brevity we shall term any four dimensional manifold which obeys the last equations, a cosmological solution.

I. The only cosmological solutions which have the same light rays as the euclidean or Minkowski world are those which have constant curvature in the sense of Riemann. In other words, if a cosmological world is to admit conformal representation on a euclidean world, it must be of spherical (or pseudo-spherical) character. This result is analogous to the earlier result for $G_{ik}=0$, that the only manifolds having the Minkowski light equation are flat (zero curvature). Both results are obviously valid also for geodesic representation (same equation of orbits).

II. Here we discuss four-dimensional curved manifolds which can be regarded as imbedded in a flat space of five dimensions. Our result is that for the cosmological equations, there are two distinct possibilities.

(a) In the first case at every point of the manifold the four principal curvatures are equal, that is $K_1=K_2=K_3=K_4$, so that every point is umbilical. The manifold is then simply a hypersphere.

(b) In the second case $K_1=K_2=-K_3=-K_4$, that is, the four principal curvatures are numerically equal, but two are positive and two are negative. Such manifolds may be regarded as a generalization of ordinary minimal surfaces (where $K_1=-K_2$), and may be described as hyperminimal spreads. (It would be interesting to find an actual example in finite form of such a spread.)

It will be recalled that for our previous discussion of $G_{ik}=0$, no solution in five dimen-

sions existed, the simple case of the solar field being actually six dimensional,¹ as are also certain other physical solutions obtained by Weyl.

III. The author has found all solutions of $G_{ik}=0$ of the orthogonal form $\lambda_1 dx_1^2 + \lambda_2 dx_2^2 + \lambda_3 dx_3^2 + \lambda_4 dx_4^2$ in which the four coefficients are functions of one variable say x_1 . An example of such a field is

$$x_1^{-2} dx_1^2 - x_1^4 (dx_2^2 + dx_3^2 + dx_4^2).$$

All cosmological solutions which satisfy the same hypotheses are determined and can be expressed by elementary, algebraic and transcendental functions.

The principal solution is

$$ds^2 = \frac{4dx_1^2}{c^2(1+x_1^2)^2} + \left(\frac{2x_1}{1+x_1^2}\right)^{\frac{2}{3}} \left[x_1^{2\alpha_2} dx_2^2 + x_1^{2\alpha_3} dx_3^2 + x_1^{2\alpha_4} dx_4^2 \right],$$

where c is arbitrary and $\alpha_2, \alpha_3, \alpha_4$ obey the relations.

$$\alpha_2 + \alpha_3 + \alpha_4 = 0, \quad \alpha_2\alpha_3 + \alpha_3\alpha_4 + \alpha_4\alpha_2 = -\frac{1}{3}.$$

These fields can all be represented in flat space of seven dimensions. A paper on this subject has been sent to the *Mathematischen Annalen*.

IV. If we require the quaternary form ds^2 to be the sum of two binary forms, that is the sum of the squared elements of two surfaces, then the only cosmological solution (neglecting the trivial euclidean form) is

$$ds^2 = x_1^{-2} (dx_1^2 + dx_2^2) + x_3^{-2} (dx_3^2 + dx_4^2).$$

This represents a quartic manifold of four dimensions imbedded in a 6-flat. The finite equations are

$$X_1^2 + X_2^2 + X_3^2 = 1, X_4^2 + X_5^2 + X_6^2 = 1.$$

This is apparently the simplest solution of Einstein's equations which has thus far been found, and the first one (beyond the obvious flat and spherical spaces) which in its finite form is algebraic.

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¹ See *American Journal of Mathematics*, Volume 43 (1921), pp. 126-133.

THE PRODUCTION OF ENHANCED LINE SPECTRA BY A NEW METHOD

THE ordinary spark spectrum differs from the arc spectrum in that certain lines are weakened, others are enhanced and new lines appear. In general the more violent the stimulus of the source the more intense are the new enhanced lines as compared to the weakened lines. It is customary to refer to the lines which are the more prominent in the spectrum produced by an arc as arc lines, while those which are enhanced by the spark are known as spark lines and constitute the pure spark spectrum.

Lorenser and Fowler, as well as Sommerfeld and Kossel, have shown that modern theories of atomic structure and radiation leave little doubt that the enhanced lines in the spectrum are due to radiation from atoms that have lost an electron, *i.e.*, ionized atoms; and that arc lines are due to radiation from the un-ionized or neutral atom. The varying facility of producing the enhanced lines of different elements depends, then, on the intensity of the forces which bind the electron to its nucleus and on the energy used in tearing the electron off. For example, no enhanced lines of lithium have ever been produced while the enhanced doublet of calcium, H and K, is strong even in the flame spectrum.

In a study of the enhanced lines of the calcium spectrum begun by examining the spectrum of calcium wires exploded by the Anderson method¹ it was found that as the size of the wires used was decreased, while the energy of the stimulus remained the same, the intensity of the enhanced lines increased. This increase in intensity indicated a more complete ionization of the calcium atoms. In seeking a way by which the amount of calcium in the source could be still further reduced a new source of light was developed.

A fine asbestos fiber about three centimeters long was saturated with an aqueous solution of some salt of calcium. The saturated fiber was fastened in place as the fine wires had previously been fastened and the charge of the high tension condensers thrown across it,

¹ *Astro. J.*, 51, 37, 1920.