Kassowitz is obliged to admit, however, that "in warm-blooded animals which are in a position to maintain their own body temperature under the most diverse conditions, one can claim the appearance of some justification that their living parts produce heat in order to protect the body against loss by radiation, etc."[9]

Whether this is a real justification or only the appearance of one will not trouble the practical physiologist so long as the generalization that human beings of different size produce heat in proportion to surface rather than weight, and therefore, require food energy in this proportion, helps him to understand his feeding problems; and there is no doubt that the law of surface area has been immensely useful in this connection. It explains the much higher basal metabolism per unit of weight of the small individual in comparison with the large better than the so-called causal explanation cited by Kassowitz. It explains also much better the need for conservation of heat in the infant, and the role which subcutaneous fat plays in this connection.

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ON THE SIGNIFICNACE OF AN EXPERIMENTAL DIFFERENCE, WITH A PROBABILITY TABLE FOR LARGE DEVIATIONS

The results of experiments from which scientific conclusions are drawn always constitute a sample, limited in number, of a potentially unlimited universe. The argument is always from the limited number to the infinite number, and assumes that the sample is representative of the universe. This is a priori not necessarily true, which is proven in the fact that two sets of measurements of supposedly the same quantity never agree in any absolute sense, that they may disagree widely, and that they therefore have to be qualified by a measure of their precision, which is derived from the magnitude of the mutual disagreement of the individual measurements of the same set.

This fact becomes of trying significance in many biological measurements. We may make two sets of measurements, A and B, under conditions alike except for one experimentally varied factor, and find that although their means show an apparently definite difference, many of the measurements A lie beyond the mean of B, and vice versa. It may be that a plot of the aggregate of the two distributions shows little or no bimodality corresponding to the difference in the respective conditions of A and B.

The usual mode of procedure in such a case is, first, to compute the measure of precision of the difference of the two means, according to the formula:

$$\sigma_\Delta = \sqrt{\frac{\sigma_1^2}{N_1} + \frac{\sigma_2^2}{N_2}},$$

where Δ is the difference between the arithmetic means $(M_1 - M_2)$, σ_Δ its standard deviation, σ_1 and σ_2 are the standard deviations[1] of the two distributions A and B, respectively, and N_1 and N_2 are the respective numbers of measurements.

Then the probability, P, of a deviation lying within the limits $\pm\Delta$, in a normal distribution of standard deviation σ_Δ, is found from the table.[2] The complement of this, $1-P$, is the probability of such a deviation lying outside the limits $\pm\Delta$.

The accompanying probability table was computed by the writer for deviations higher than those included within the range of most such tables extant, with a view to giving values of P much nearer to unity than usual. An approximate method of computation was used. While the computation of values of

$$\int_0^x e^{-x^2} dx$$

[9] Kassowitz, M., *ibid.*, p. 240.

[1] This assumes that

$$\sigma_1 = \sqrt{\frac{\Sigma\delta_1^2}{N_1 - 1}}.$$

Where N_1 is large the error due to the use of N_1 instead of $(N_1 - 1)$ tends to become negligible.

[2] Such as Table IV., pp. 119–125, Davenport, "Statistical Methods," third edition, New York; or Table 24 or Table 25, Smithsonian Physical Tables, Seventh Edition, Washington, 1920.

is laborious, excessively so where, as in the present case, many decimal places are required, it is possible to make a closely approximate integration of small segments of the function:

$$ydx = e^{-x^2}dx$$

such as

$$\int_{-z}^{z} e^{-(a+z)^2}dz,$$

where a is any abscissa and z is small.

Expanding the exponent, this becomes:

$$\int_{-z}^{z} e^{-a^2} \cdot e^{-2az} \cdot e^{-z^2},$$

which by putting

$$e^{-z^2} = 1 - z^2,$$

becomes

$$e^{-a^2}\int_{-z}^{z}(1 - z^2)e^{-2az}$$

$$= e^{-a^2}\left[e^{-2az}\left(\frac{z^2-1}{2a} + \frac{z}{2a^2} + \frac{1}{4a^3}\right) - e^{2az}\left(\frac{z^2-1}{2a} - \frac{z}{2a^2} + \frac{1}{4a^3}\right)\right].$$

Reducing and substituting for z the value 1/10 gives:

$$\int_{-1}^{1/10}{}_{0} e^{-(a+z)^2} = \frac{e^{-a^2}}{4a^3}\left[(e^{a/5} + e^{-a/5})a/5 + (e^{a/5} - e^{-a/5})\,(1.98a^2 - 1)\right].$$

Thus, by assigning values to a, progressing by 0.2, the areas of the segments of the integral for the abscissal intervals $a \pm 1/10$ could be closely approximated and summated, the values in the table being finally:

$$\log\left(2\int_x^\infty \frac{h}{\sqrt{\pi}} e^{-h^2x^2}dx\right):$$

or log $(1 - P)$, according to the usual symbology. It was found that it was only necessary in the extreme value given ($hx = 7.0$) to carry the computation a few steps farther, in order that the sum of the subsequent segments to infinity should be a vanishing quantity with respect to the degree of precision decided upon. The table is not to be looked upon as more than supplementary to the tables in general use, and upon examination, it will appear that the error introduced by assuming that $e^{-z^2} = 1 - z^2$ is negligible since, for $z = 1/10$ this error at its maximum is only as 0.99 — 0.99005 to 0.99, or 5 parts in 99,000 *with respect to* $1 - P$, and on the whole, even less than this; and it is the values of $1 - P$, smaller than those obtainable from the usual tables, in which we are here interested. The values of this table check with those in the usual tables, as far as the latter go, and also (in the extreme cases, especially where $hx = 5.0$, 5.5 and 6.0) with the values given in the original work of Burgess.[3]

EXPLANATION OF TABLE

Common logarithms of the values of the integral:

$$2\frac{h}{\sqrt{\pi}}\int_x^\infty e^{-h^2x^2}dx \ (= 1 - P)$$

for various values of hx.

$$hx = \frac{0.4769x}{E} = \frac{0.7071x}{\sigma},$$

where E is the probable error and σ the quadratic mean error.

Interpolations will be fairly accurate to the fourth place if proper account be taken of the second difference.

hx	log (1 — P)	hx	log (1 — P)
0.0	0.0000	3.5	3.8710–10
0.1	9.9482–10	3.6	3.5513
0.2	9.8906	3.7	3.2231
0.3	9.8270	3.8	2.8865
0.4	9.7571	3.9	2.5415
0.5	9.6808	4.0	2.1880
0.6	9.5978	4.1	1.8261
0.7	9.5081	4.2	1.4557
0.8	9.4115	4.3	1.0768
0.9	9.3077	4.4	0.6895
1.0	9.1967	4.5	0.2936–10
1.1	9.0784	4.6	9.8893–20
1.2	8.9527	4.7	9.4764
1.3	8.8195	4.8	9.0551
1.4	8.6787	4.9	8.6252
1.5	8.5301	5.0	8.1868
1.6	8.3739	5.1	7.7399

[3] Burgess, *Trans. Roy. Soc. Edinb.*, XXXIX., p. 257 ff. "On the Definite Integral $(2/\pi)\int_0^t e^{-t^2}dt$ with Extended Tables of Values."

1.7....8.2098	5.2....7.2844
1.8....8.0378	5.3....6.8204
1.9....7.8579	5.4....6.3479
2.0....7.6700	5.5....5.8668
2.1....7.4741	5.6....5.3771
2.2....7.2702	5.7....4.8789
2.3....7.0581	5.8....4.3721
2.4....6.8379	5.9....3.8567
2.5....6.6095	6.0....3.3328
2.6....6.3730	6.1....2.8003
2.7....6.1282	6.2....2.2593
2.8....5.8751	6.3....1.7096
2.9....5.6138	6.4....1.1514
3.0....5.3442	6.5....0.5846
3.1....5.0663	6.6....0.0092–20
3.2....4.7800	6.7....9.4252–30
3.3....4.4854	6.8....8.8326
3.4....4.1824	6.9....8.2314
3.5....3.8710–10	7.0....7.6216–30

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POLARIZATION OF SOUND

The term polarization, applied to a wave motion, is generally associated only with transverse waves, more especially with light-waves, as referring to a state in which certain qualities are different in certain directions at right angles to one another and to the direction of propagation. By its origin, however, the term may be used with the same justification for longitudinal waves exhibiting qualities that are different in different directions, irrespective of the nature of such qualities and the relation of the various directions to each other.

It is thus proper to speak of a polarization of sound when conditions prevail under which a quality like its pitch is of opposite character to opposite sides of a fixed plane or axis.

Such conditions may be brought about by putting the source, which for the sake of simplicity is supposed to produce a sustained sound of uniform pitch, through certain movements. It is well known that when such a source is in motion the pitch of the sound emitted into space will be a function both of the direction of the movement and its speed.

This is due to the relative displacement of the individual wave rings by the motion, and is readily observed by anyone standing close to a railroad track while a locomotive blowing its whistle is passing. At the instant of passage there is a sudden fall in the pitch of the blast, the fall being approximately proportional to the speed of the locomotive.

The pitch observed at any point may be expressed by the formula:

$$p = q\frac{v}{v-u},$$

p denoting the pitch observed, q the pitch produced, v the velocity of sound, and u the speed component of the movement in the direction of the observer, with due consideration of its sign.

If the source, instead of being moved at uniform speed in one direction, is made to perform a harmonic oscillatory movement at right angles to a plane P, and symmetrical to it, then the resulting sound will be of uniform pitch only at points located in this plane, assuming the extent of the movement to be small as compared with the distance to the point of observation. To either side of the plane the pitch will be undulating, the undulations reaching their maximum amplitude at points directly in line with the movement.

While the undulations will be of the same amplitude at any two points symmetrically located with respect to the plane, they will be opposite in phase and, therefore, of opposite character. Accordingly, if the source is made to emit sound while to one side of the plane only, *i.e.*, during alternate half oscillations, then, by the above formula, the resulting sounds will be of descending pitch to that side of the plane, while to the opposite side of it the same sounds will be of ascending pitch.

The sound may thus be said to have been polarized with respect to the plane P.

If the oscillatory movement of the source is substituted by a rotation at uniform speed about an axis A, results of a similar nature are obtained. In this instance, however, the resulting sound will be of uniform pitch only