from teaching fellow to associate in medical entomology.

DR. LOUIS J. GILLESPIE, professor of physical chemistry at Syracuse University, who was formerly with the Department of Agriculture, Washington, D. C., has resigned to go to the Massachusetts Institute of Technology as assistant professor of physico-chemical research.

DR. ARTHUR F. BUDDINGTON, Ph.D. (Princeton, '16), and Dr. Benjamin F. Howell, Ph.D. (Princeton, '20), have been appointed assistant professors of geology at Princeton University.

DISCUSSION AND CORRESPONDENCE

MODERN INTERPRETATION OF DIFFERENTIALS

IN an advance copy of a note to SCIENCE, which Professor Huntington has kindly sent to me, he says that "*some indication as to the manner in which N is to vary*" is necessary to define $dy = \lim N\Delta y$. This is not true. Of course, there must be some relation between N and Δy, in order that, for example, $\lim N\Delta y = 5$, but the number of such relations is infinite, and it is only necessary to know that they exist. For example, if $\Delta y = (5/N) + (8/N^2)$, then $N\Delta y = 5 + (8/N)$, and for $\lim N = \infty$, $\lim \Delta y = 0$, $\lim N\Delta y = 5$. It was stated in my note which Professor Huntington is criticizing[1] that N varies from zero to infinity. We are not concerned with the method of approach, but only with the possible value of the limit. The preceding illustration shows that if y be an independent variable, such limit dy exists, and in any value we please to name. It is different if y be dependent, and my note in SCIENCE of May 7, contained a demonstration that $df(x)$ exists when the graph of $f(x)$ has a tangent, and determines its construction, corresponding to any value of dx, including in particular, $dx = \Delta x$, which is, of course, not always true.

The problem of differentiation is larger than that of a single value, since it determines an infinite number of corresponding values. We have the analogy of the infinite number of corresponding values of the derivative variable and its argument x. We justify this variable as a limit on the ground that it is a true limit for each numerical value of x. The example having been set, its extension to differentials can not be denied.

The infinite number of corresponding differentials (dx, dy, dz) pertain to the one set of corresponding variables (x, y, z), just as the increments (Δx, Δy, Δz) pertain to it, and are *corresponding increments of the instantaneous state of the variables,* also, *increments in the first ratio* (Newton's "prime" ratio), etc. This is not a vague idea but one which, in numerical cases, determines numerical values. The source of this terminology is the physical idea that *equimultiples of very small simultaneous increments* are *approximately* increments of the instantaneous state. The differential analysis of Newton, which carries this idea to its logical conclusion, is therefore the mathematical foundation for such physical idea.

It is easy to make statements appear vague by separating them from the facts on which they are based, and such facts appear in the article from which Professor Huntington quotes, with a figure showing the *finite equimultiples* which are becoming exact differentials—differentials which his "modern" method can not represent, since they pertain to a system of two independent variables, and of which the derivative calculus can give no adequate idea. although they are of great practical importance.

Such so-called modern method is crude in its limitation $dx = \Delta x$, narrow in its application *only to plane curves in rectangular coordinates.* A natural extension to space is impossible, but Newtonian differentials are coordinates of tangent planes, from their points of contact as origin. By Newton's method, all kinds of continuously variable quantity, in plane or space, lines, areas, volumes, forces, may have corresponding differentials represented in finite quantities of the same kind, and by the limits of finite and visible values.

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[1] SCIENCE, February 13.