

tin or glass receptacle quickly dissipates it; once lost, it can not be restored. Observers have been able to detect the sound from a New England beach sand over 400 feet away, when a small bagful is suddenly shaken.

While the analogy to the snow crystals may account for part of the phenomenon in some cases, it can not account for the singing of limestone, coral or other non-crystalline sands. Moreover, when one walks barefooted on musical sands, or runs the hand through them, there is felt a distinct tingling sensation. To some, this has suggested an electrical property. The latest and most plausible theory is that upon clean, dry sands, atmospheric gases condense, just as gases will adhere to particles of some metallic minerals and not others, and that the sounds and the sensations described are due to the disturbance of these air cushions. At any rate, the sensation experienced when walking barefoot through a patch of musical sand is very similar to that felt when the hand is immersed in a solution in which nascent oxygen is being generated.

By the way, I wonder if it has ever occurred to any archeologist that a possible explanation of the "Vocal Memnon" which Strabo and other travelers attested some two thousand years ago, might be the presence near the colossi, of musical sands, long since buried by the drift from the Libyan Desert.

ALBERT R. LEDOUX

MODERN INTERPRETATIONS OF DIFFERENTIALS

TO THE EDITOR OF SCIENCE: Professor E. V. Huntington, in an article entitled "Modern Interpretation of Differentials" (SCIENCE, March 26), states with reference to the definition $\lim \Delta y = 0$, $\lim N\Delta y = dy$, that, "The inevitable consequence of such a definition is that $dy = 0$, which is futile." Every school boy in the theory of limits knows that this is not true when N varies.

To take his figure of a graph of a function $y = f(x)$, it is logically correct to denote a point on the graph by $P(x, y)$ without subscripts, and $P'(x + \Delta x, y + \Delta y)$ is any other point on the graph, where $PQ = \Delta x$, $QP' = \Delta y$.

Produce PQ to $PR' = N\Delta x = \Delta'x$, and draw $R'S' = N\Delta y = \Delta'y$, parallel to OY . Then $S'(x + \Delta'x, y + \Delta'y)$ is any point on the produced chord PP' (i. e., variation in the same ratio is along the chord).

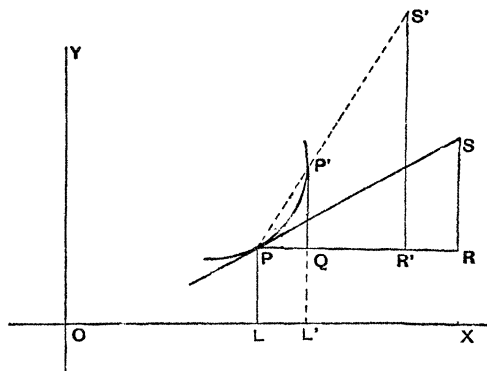


FIG. 1.

Professor Huntington asserts that $S'(x + \Delta'x, y + \Delta'y)$ inevitably approaches coincidence with $P(x, y)$ when $\Delta x, \Delta y$, approach zero, although it is obvious that it may, if N increase appropriately, approach any chosen point $S(x + dx, y + dy)$ on the tangent at $P(x, y)$, so that $\lim \Delta'x = dx$, $\lim \Delta'y = dy$. Variation in the first ratio is therefore upon the tangent.

Professor Huntington should also have investigated the historical questions involved before venturing to assert that the above theory of differentials "would prove highly misleading to the modern student." It is a sad commentary on the present state of the calculus in respect to its fundamental ideas, when we note the variety of explanations of these ideas by authors with little historical knowledge, all of whom, no doubt, would term their productions "modern," though most explanations will be found to date back several centuries, if they be anything more than vaporizing.

Sir William Rowan Hamilton in his Elements of Quaternions (Bk. III., p. 392) states that ordinary definitions by derivative methods do not apply in quaternions, and that after a careful examination of the Principia, he would formulate and adopt Newton's definition as follows:

Simultaneous Differentials (or *Corresponding Fluxions*) are *Limits of Equimultiples of Simultaneous and Decreasing Differences*.

As we have seen, Newton also made this definition in "Quadrature of Curves," essentially as Hamilton gathered it from the "Principia." Many better mathematicians than myself, or than Professor Huntington, have, in fact, examined this definition carefully, and have found it to be rigorous, simple, and of great generality.

The infinitesimal method of Leibniz is to be found essentially in Newton's first tract "De analysi per aequationen . . .," which Newton himself later rejected as illogical. A third method of explanation is that of Lagrange, which consists in assuming (for independent variables), $dx = \Delta x$, $dy = \Delta y$, and for a dependent variable z $dz = \text{principle part of } \Delta z$, which Lagrange proposed to determine as *the terms of first degree in the expansion of $z + \Delta z$ in ascending powers of Δx , Δy* . Newton's dz is the same, if we put $dx = \Delta x$, $dy = \Delta y$. The adoption of the derivative method, led to devices to obtain the same significance of dz by derivatives, without assuming expansions in series. These involve various logical difficulties, especially when there are several independent variables. Also the differentials appear to change their values by changing the independent variables, whereas, Newton's method shows that for every equation between the variables, there exists (if differentiation be possible) a definite corresponding equation between their differentials, irrespective of the choice of independent variables.

Unquestionably, there has been a long continued propaganda, fostered at bottom to protect the claims of Leibniz, and aided by the inertia of established usage, to keep the methods of Newton in abeyance. Imagine, if the nationalities of these men had been reversed, the number of pamphlets that would have exploited the matter, and the number of textbooks in that method which would years ago have been published.

ARTHUR S. HATHAWAY

ROSE POLYTECHNIC INSTITUTE

CARBON DIOXIDE AND INCREASED CROP PRODUCTION

TO THE EDITOR OF SCIENCE: In 1912, at the International Congress of Chemists held in New York, Professor Ciamician, of the University of Bologna, presented a paper on the "Photochemistry of the Future," in which, among other things, the suggestion was made that crop production might be increased by increasing the concentration of carbon dioxide in the air. Of course, the idea underlying such a suggestion is that since the carbon dioxide of the air is a necessary constituent in the synthesis of carbohydrate by the plant, and since, furthermore, the percentage of the gas in the air is comparatively small, any increase in the amount of carbon dioxide may tend to increase the amount of carbohydrate produced.

That such is actually the case has been found by a number of German chemists, according to the Berlin correspondent of the *N. Y. Tribune* (April 4). Working in greenhouses attached to one of the large iron companies in Essen, and utilizing the carbon dioxide (freed from impurities) obtained from the blast furnaces, the yield of tomatoes was increased 175 per cent. and cucumbers 70 per cent. Further experiments in the open air, on plots around which, punctured tubes were laid, and through the latter of which the carbon dioxide was sent, gave increases of 150 per cent. in the yield of spinach, 140 per cent. with tomatoes and 100 per cent. with barley.

BENJAMIN HARROW

STRUCTURAL BLUE IN SNOW

TO THE EDITOR OF SCIENCE: The recent blizzard began here with a heavy downpour of rain on the evening of March 5, which later turned into a glistening snow that was shattered by the furious wind and formed a crystalline-looking glittering coherent mass whose structure was maintained by the low temperature (about 20° F.).

When the sun finally came out on Saturday afternoon, I noticed that the shadows of the trees and the shadow masses of the distant snow, appeared unusually *blue*, and that the