Among the comparative recent developments in geodesy that are especially well stated in the book are (1) the importance of determining the relative strength of different proposed chains of triangulation as fixed by the geometrical relations, and the methods for quickly doing so; (2) the relation between the average length of the lines in a triangulation and the rapidity, economy, and accuracy of that triangulation and its convenience to the user; (3) the advantages of the light and rapidly built towers such as are now used in the Coast and Geodetic Survey; (4) the advantages of the transit micrometer on portable instruments for determining time accurately; (5) the application of the interferometer to determination of the flexure of the support of a pendulum used to determine the relative values of gravity at different points. These things are stated forcefully and with good judgment as to their relation to older ideas and methods.

Though he has looked carefully for errors of omission, the reviewer, who has a background of experience which naturally tends to make him keenly critical, finds only three that are, in his opinion, important.

1. On its best direction theodolites the Coast and Geodetic Survey uses two sets of double parallel lines in the micrometer microscopes with which the horizontal circle is read, the two sets being so placed that the observer moves the micrometer screw only one turn between a forward and the corresponding backward reading, instead of five turns. This is a time-saving convenience which also increases the accuracy, and surely should have been mentioned in the book.

2. The necessity of tracing back the adopted field length of a base measuring tape to the standard meter and the methods of doing so are inadequately treated in the book. The developments of the past twenty years have made it clear that one must concentrate much more keenly on this part of the work than the book indicates.

3. The area method of computing the figure of the earth from geodetic and astronomic observations is barely referred to on page 204 without explanation. In view of the fact that this method gives a much higher degree of accuracy from the same observations than the traditional arc method, it certainly deserves a page of general exposition in the book, even if it is possibly too difficult for the student to grasp in full. The student and the engineer should know that the more accurate method exists, should know its general character, and in a general way why it is more accurate than the arc method.

The author of the book has shown such ability to see with the eye of an expert, and to exercise the judgment of a practicing geodetic engineer, that one may confidently expect that even these three omissions will not occur in a second edition.

JOHN F. HAYFORD

SPECIAL ARTICLES

CONCERNING APPLICATION OF THE PROBABLE ERROR IN CASES OF EXTREMELY ASYMMETRICAL FREQUENCY CURVES

IN a study of the fecal pollution of shellfish, Dr. James Johnstone[1] raises an important question: that of determining the most probable value of a measure from a series whose frequency distribution is highly asymmetrical. In such instances it is evident, although prevailing practise contradicts the statement, that it is illegitimate to apply the probable error in the usual manner. For such application presupposes a symmetrical (Gaussian) distribution, and, since a wide range of biological measurements is characterized by an asymmetrical distribution, the matter merits consideration.

Dr. Johnstone lists the following counts of colonies of bacteria growing on twenty plates, each having been incubated a standard length of time after being inoculated with 1 c.c. of an emulsion, in 250 c.c. of water, of five muscles collected at random from the polluted area: 7, 24, 40, 15, 22, 20, 17, 9, 16, 29, 7, 9, 10, 26, 15, 11, 21, 17, 10, and 41. Dr. Johnstone assumes each count to be an estimate of the number of bacteria per c.c. of the emul-

[1] "The Probable Error of a Bacteriological Analysis," Rept. Lanc. Sea-Fish. Lab., 1919, No. XXVII., p. 64–85.

sion, the variation between the counts being attributed to errors in sampling. He then raises the question as to the most probable number of bacteria present, and, after pointing out that, according to custom, the arithmetic mean of the counts (18.3) would be regarded as the most probable number, proves this to be untrue by showing the frequency distribution to be highly asymmetrical, as follows:

Counts	Frequency
6–10	6
11–15	3
16–20	4
21–25	3
26–30	2
31–35	0
36–40	1
41–45	1

Although Dr. Johnstone discusses this distribution, and, by employing Galton's graphical method, determines certain constants, he fails to answer the question he raises.

In cases of this kind it seems as though the simplest procedure is to find some function of the measurements whose frequency distribution is Gaussian, and apply the probable error to that function. The reason is that an asymmetrical distribution implies that some influence other than "chance" is operative, and substitution of a function whose distribution is Gaussian enables their separation. In the particular case at hand, and it is typical of many within the province of biology, this function is the logarithm. This is easily demonstrated by grouping the logarithms of the counts with respect to a deviation of ± 0.1 from their mean ($=1.2046$) as follows:

Logarithm	Frequency
0.505–0.704	0
0.705–0.904	2
0.905–1.104	5
1.105–1.304	6
1.305–1.504	5
1.505–1.704	2
1.705–1.904	0

The arithmetic mean of the logarithms (1.2046) is the logarithm of the geometric mean of the counts ($=16.02$), the geometric mean, by definition, being the twentieth root of the product of the twenty counts. Accordingly, the Gaussian distribution of the logarithms shows that the counts cluster in approximately constant ratio about their geometric mean, or, to express it otherwise, that variations in the count are compensatory in the geometric mean. This signifies that variation in the count is not primarily attributable to errors in sampling and that each count is not an estimate of the number of bacteria present per c.c. in a homogeneous emulsion, but rather that conditions favoring the propagation of bacteria fluctuated in an "accidental" way either during the period in which the twenty samples were removed from the emulsion, or from place to place within the emulsion, or both. Whether or not this interpretation be correct, the logarithmic frequency distribution demonstrates that something of like nature occurred. In any case the most probable number of bacteria per c.c. corresponding to the most typical condition of the emulsion is the geometric mean of the counts (16.02); and, in the same sense, $250 \times 16.02 = 4{,}005$ is, of course, the most probable number of bacteria in the whole emulsion.

The reliability of this estimate may be approximated by applying the probable error to the logarithms. The standard deviation of the logarithms, σ, is 0.224, the probable error, or, better, the "probable departure" from the logarithm of a single count is $0.6745\ \sigma = \pm 0.1511$ and the probable departure from the logarithmic mean is $0.1511/\sqrt{20} = \pm 0.0337$. It follows from tabulated values of the probability integral that, had the entire 250 c.c. been examined, it is as likely that the logarithmic mean would have been within 1.2046 ± 0.0337 as that it would have been outside these limits, while the odds are about 4.6 to 1 that it would have been within $1.2046 \pm 2(0.0337)$, about 22 to 1 that it would have been within $1.2046 \pm 3(0.0337)$, and nearly 142 to 1 that it would have been within $1.2046 \pm 4(0.0337)$. The numbers corresponding to these logarithms are the limit-

ing values of the estimated number of bacteria per c.c.; that is, the odds are even that this number lay between 14.82 and 17.31, about 4.6 to 1 that it lay between 13.72 and 18.72, about 22 to 1 that it lay between 12.69 and 20.22, and nearly 142 to 1 that it lay between 11.74 and 21.86.

This, I believe, answers Dr. Johnstone's question in so far as the small series of counts permit. The problem is typical of many that have not received due consideration by either biologist or statistician; and conclusions departing widely from the truth are continually being reached through failure to apply any criterion of reliability on the one hand, and through an erroneous application of the probable error on the other hand. It is hoped this brief presentation will stimulate discussion.

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THE AMERICAN MATHEMATICAL SOCIETY

THE twenty-sixth annual meeting of the society was held at Columbia University on Tuesday and Wednesday, December 30–31, with the usual morning and afternoon sessions on each day. The attendance included 96 members. President Frank Morley occupied the chair, being relieved at the last session by Professor J. L. Coolidge. The following new members were elected: Dr. H. E. Bray, Rice Institute; Professor I. L. Miller, Carthage College; Dr. Helen B. Owens, Cornell University; Professor E. W. Pehrson, University of Utah. Ten applications for membership were received.

At the annual election the following officers and other members of the council were chosen: vice-presidents, C. N. Haskins and R. G. D. Richardson; secretary, F. N. Cole; treasurer, J. H. Tanner; librarian, D. E. Smith; committee on publication, F. N. Cole, Virgil Snyder, and J. W. Young; members of the council to serve until December, 1922, T. H. Hildebrandt, Edward Kasner, W. A. Manning, H. H. Mitchell.

The total membership of the society is now 733, including 80 life members. The total attendance of members at all meetings, including sectional meetings, during the past year was 393; the number of papers read was 187. The number of members attending at least one meeting during the year was 252. At the annual election 156 votes were cast. The treasurer's report shows a balance of $10,692.23, including the life membership fund of $7,168.87. Sales of the society's publications during the year amounted to $1,811.52. The library now contains 5,690 volumes, excluding some 500 unbound dissertations.

It was decided to proceed with the incorporation of the society under the general law of the state of New York. A committee was appointed to consider plans for the organization and administration of the society after the retirement of the present secretary and librarian from their offices at the close of the present year. A committee was also appointed to consider the formation of an international union of mathematicians. The committee on mathematical requirements presented a report, which was laid over for consideration at the February meeting.

The following resolutions, introduced by Professor R. C. Archibald as chairman of the committee on bibliography, were adopted by the council:

The council regards the preparation and publication, in America, of a dictionary of mathematical terms as not only most desirable but also entirely feasible, provided that financial aid for the preparation of the manuscript can be secured.

Impressed with possibilities for the more extensive development of pure and applied mathematics in America, and with the importance of such development to the nation, the Council records its conviction that there are undertakings whose active consideration would be highly desirable if adequate financial assistance might be regarded as available. Among such undertakings are: 1. The preparation and publication by societies or individuals of surveys, introductory monographs, translations, memoirs, and treatises, in important fields, including the history of mathematics. 2. The organization of research fellowships. 3. The preparation and publication of an encyclopædia of mathematics in English. 4. The preparation and publication of an annual critical survey, in English, of the mathematical literature of the world. 5. The preparation and publication of a biographical and bibliographical dictionary of mathematicians.

The meeting of the society immediately preceded that of the Mathematical Association of America on January 1–2. A very pleasant occasion was the joint dinner of the two organizations on New Year's eve with an attendance of 114 members and friends.

The following papers were read at the annual meeting:

The sum of the face angles of a polyhedron in space of n dimensions: H. F. MACNEISH.