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A GEOMETRIC BASIS FOR PHYSICAL AND ORGANIC PHENOMENA

THE following notes refer to certain ideas which the writer has had in mind for many years, but in the form now submitted they are immediately suggested by a recent casual examination of D'Arcy W. Thompson's "Growth and Form" (Cambridge University Press, 1917) and particularly by reading certain paragraphs in this book relating to the various possible divisions of space by systems of surfaces or material films and membranes, in connection with a discussion of the internal structure of organic bodies.

A number of years ago the present writer submitted a brief paper to the American Physical Society under the title "A simple geometrical principle and its possible significance in connection with a general physical theory." The principle was stated as follows: "In any aggregation of an indefinite number of equal spherical bodies an arrangement giving minimum total volume occupied and perfect symmetry throughout is impossible." The quotations are from memory. An abstract of the paper was published at the time in the *Physical Review.*

Of course this principle might be dealt with by geometrical construction and mathematical analysis, but it can be demonstrated experimentally and in a simple and practical way by means of a number of balls of equal diameter like the hollow celluloid "ping-pong" balls, or the rubber balls sold as children's playthings. Thick mucilage, varnish, collodion, sealing wax or any other available adhesive substance may be used for sticking the balls together. Perhaps what follows may seem at first too elementary to be regarded as something of real scientific interest, but it is a matter of some surprise to find how many erroneous and confused ideas on so simple a

subject have been entertained and expressed even by scientific writers.

Place one of the balls on the table and arrange four others around and touching it with equal intervals between them in the form of a right-angled cross. Then place one ball directly on top of the central one, and finally one directly beneath it. This forms a group of seven balls which suggests one of the "jackstones" (generally made of iron or lead) that children play with. The group has a perfectly symmetrical arrangement which admits of indefinite extension on the same system in all directions by the addition of balls. In such a system any one ball, except of course those on the outer boundaries of the assemblage, is symmetrically surrounded by six others all touching the one that is central for this individual group of seven, but no two of these surrounding balls touch each other. The planes mutually tangent to each pair of balls at their common point of contact will obviously form by their intersections a system of equal cubes with common interfaces, each cube circumscribed about a ball. It is plain that these cubes "stack" together so as to make a solid volume, or in other words there will be no voids between—no waste of space. It will be clear that exactly the same arrangement results from placing on the table a number of balls in contact and in a single layer in "square" order, or with the balls in rows both ways at right angles like the squares on a checker board, and then placing another layer in the same formation with each ball directly over a ball of the first layer, and so on. The balls will have to be stuck together or very carefully placed or they will not retain this formation but they will fall down or spread and the pile will collapse under the influence of the gravitation of the earth.

It soon becomes apparent that this cubical arrangement is not the most compact possible or not the one which permits placing the greatest number of the balls in a given volume. For example, after placing the first layer in square formation greater concentration is attained by placing each ball of the second layer over an interval or space among the balls of each group of four in the first layer rather than directly over another ball, and so continuing the succeeding layers.

Now undertake to effect the most compact arrangement possible beginning with one ball, and place about a central ball on the table as many others of the same size as there is room for in one layer with all touching the central ball. There will of course be six side balls, all tangent to each other throughout as well as to the central ball, in hexagonal order. Then three more balls can be placed above touching the central one—and only three, though there are six intervals among the balls of the foundation layer—and likewise three others can be placed below, making twelve surrounding balls or a group of thirteen, all in mutual contact throughout, so that the position of each ball in the group is definitely fixed relative to its neighbors. This arrangement may be extended without limit and it is the most compact possible for an indefinite number of balls, but it is not perfectly symmetrical throughout. The mutually tangent planes at the points of tangency between the balls make a system of rhombic dodecahedrons, each one surrounding a ball. Equal rhombic dodecahedron will stack together without voids when similarly oriented but they do not form a completely symmetrical division of space, since the rhombic dodehedron is not one of the regular polyhedrons, or not a solid with all equal regular polygons for faces. All of the diedral angles of this solid are 120 degrees, but its twelve faces are equilateral oblique angled parallelograms or rhombs and the plane angles meeting at the vertices or solid angles are not therefore all equal.

It should be noted that the formation resulting from starting with a layer in square order and placing the balls of the next layer over the intervals in the first one and so on, is also this same rhombic dodecahedronal arrangement, only differently disposed with respect to the table or the horizontal plane. It is what we so often see in a pile of oranges in the groceries and on the fruit stands. In all horizontal layers of such a pile the balls are in square order, but there are other sys-

tems or series of layers in the pile, inclined to each other and to the horizontal, in which the balls are all in the hexagonal order, which is the closest assemblage possible in any one layer or plane.

We have thus developed one arrangement—the cubical—that gives universal symmetry with the balls in contact throughout, but not maximum concentration; and another one—the rhombic dodecahedral—that gives maximum concentration and density, but not universal symmetry. Now try for a formation that will give both.

The sphere is itself a shape of the most perfect symmetry and it has the very maximum quantity of contained volume or space for a given area of enclosing surface. It seems at first axiomatic and a foregone conclusion that an assemblage of equal spheres *must* admit of an arrangement or grouping that will give to the aggregate collection characteristics exactly similar to those of the individual sphere, with complete internal symmetry and equilibrium.

Recalling that in the second experiment the twelve side balls were placed about the central one all in mutual contact throughout so that the position of each ball in the group was definitely fixed and with no room for relative movement, it will perhaps be somewhat surprising to find that another arrangement for the twelve surounding balls is possible which gives a disposition perfectly symmetrical with respect to the central ball while the balls are nowhere in contact with each other at all, but each is equally and symmetrically spaced from all of its side neighbors. There is room to spare among the side balls but not enough for another ball. In this arangement the common tangent planes between the central and the surrounding balls form a regular polyhedron—the regular or pentagonal dodecahedron—about the central ball. This is a volume with twelve equal pentagons for faces, and of course having all its diedral angles as well as its vertices or polyhedral angles equal respectively. Each diedral angle of this solid, or the angle between any two adjacent faces, is 116° —34′ —54″; that is more than 90 degrees and somewhat less than 120 degrees, or between one quarter and one third of the complete angular space about one edge. Equal volumes or solids of this form may be assembled, face matching face, about a central one of the same size in a group of thirteen, but there must be a wedge shaped void, with a diedral angle at the edge of over ten degrees, between each two adjacent side members of the group where three edges of the solids coincide, and therefore the system can not be extended in the same formation by adding other equal solids of the same size and shape. From this it is apparent that a grouping of spheres inscribed in the equal regular dodecahedrons does not admit of this symmetrical arrangement beyond the group of thirteen.

(A compact grouping of eight equal spheres which is symmetrical with respect to a central point, not within any one of them, may also be arranged as follows: Place three of the spheres in contact on the table, with a fourth over the interval, making a triangular pyramid group or a regular tetrahedral grouping. Then place a sphere over each of the four spaces or openings that will be found over the outer surfaces of the group, each opening surrounded by three tangent spheres. The limit in number for this grouping is eight spheres—there is no available space for any more placed symmetrically—and here is a suggestion of possibly some relation to the "periodic law" of physical chemistry).

To summarize: The only possible arrangement or grouping of equal spheres in contact that gives perfect symmetry as a fixed condition throughout for a group of an indefinite number is the cubical system, and this does not give maximum density: while the only possible arrangement that gives maximum density as a fixed condition throughout such a group is the rhombic dodecahedral, but this does not give universal symmetry. *There is no arrangement possible giving both maximum density and universal symmetry.*

It is scarcely necessary to add that these relations in no manner depend on absolute dimensions—they are true for spheres of the

minutest diameter conceivable as well as for those of the most colossal size we can imagine, and for all intermediate sizes.

Now regard the spheres as equal masses of homogeneous matter endowed with the property of mutual attraction or gravity. They will tend to collect together in a group if free to move relatively, and to remain so. The cubical arrangement would be entirely consistent with complete equilibrium of the attracting forces, but this can not be permanent since it is not a formation of maximum density or concentration. It does not fully satisfy the collecting tendency under the forces of mutual attraction, and the equilibrium of the cubical formation must be unstable. If arranged on the cubical system the group will collapse on the slightest disturbance and the members will seek another arrangement permitting greater concentration. The rhombic dodecahedral grouping affords maximum concentration but it too fails to give complete stability, for it is not perfectly symmetrical and the forces of attraction can not be permanently balanced or in complete equilibrium throughout. Any one group of thirteen of the balls or spheres would be "satisfied" as to concentration and balance or equilibrium by the regular dodecahedral arrangement, but as above set forth this could not possibly obtain as a fixed condition throughout a group of more than thirteen of the spherical bodies. For a group of an indefinite number of the equal spheres greater than thirteen, *there is no stable and permanent arangement possible.*

We can now in imagination expand the diameter of the balls to any extent limited only by infinity—which means without limit—and likewise their size may be reduced to any dimensions short of zero, while their number may be multiplied also without restriction. The above relations are true for the smallest units of matter that can exist as well as for the most gigantic bodies. Furthermore the truth of these principles is not dependent on the complete *occupation by matter* of each of the individual spherical spaces or volumes considered. These spherical spaces may be only the respective "fields" or space loci of one or more separate portions of matter in a state of motion respecting neighboring portions in other similar spaces or fields—all having motions of revolution, of vibration, of oscillation, or of pulsation, with limitless combinations and variations as to size and number of the individual portions, their velocity, direction and amplitude of movement, etc.

Every assemblage or group of matter tends to assume the form with an external bounding surface of spherical shape under the mutual attractions of its parts, but however large or small such an assemblage or whatever may be the number of its individual members, its internal structure is governed by the principles above outlined. This indicates an explanation of the paradox involved in the first assumptions or impressions above referred to. The conceit that a perfectly symmetrical grouping of equal spheres with maximum concentration can be made, at first seemingly entirely simple and even axiomatic, turns out to be inconsistent with elementary facts of geometry and therefore impossible.

A direct corollary of this proposition is that a *plenum* of matter in any form, or any material "continuous medium," is impossible and non-existent. All material substances affected by gravity—which is equivalent to saying all real matter whatever—*must* be atomic or "granular" in its structure and in its behavior, and this does not depend upon an assumption of "intermolecular repulsion" or of "kinematical energy," nor indeed even upon the theory of energy as a separate entity, nor on any other extraordinary force or attribute. Plain gravitational attraction with the resultant unrelenting stress and struggle for a status which is *geometrically unattainable* is all-sufficient. This may even be made to account for apparent repulsion.

The reason for the conviction and belief that these principles have an intimate and fundamental relation to the universal and eternal unrest of matter and to all physical phenomena of whatever nature will now be apparent and we have at least an interesting and suggestive side light on Boltzmann's demonstration of "the indispensability of atom-

istics in natural philosophy," as recently referred to by Professor Bumstead.[1]

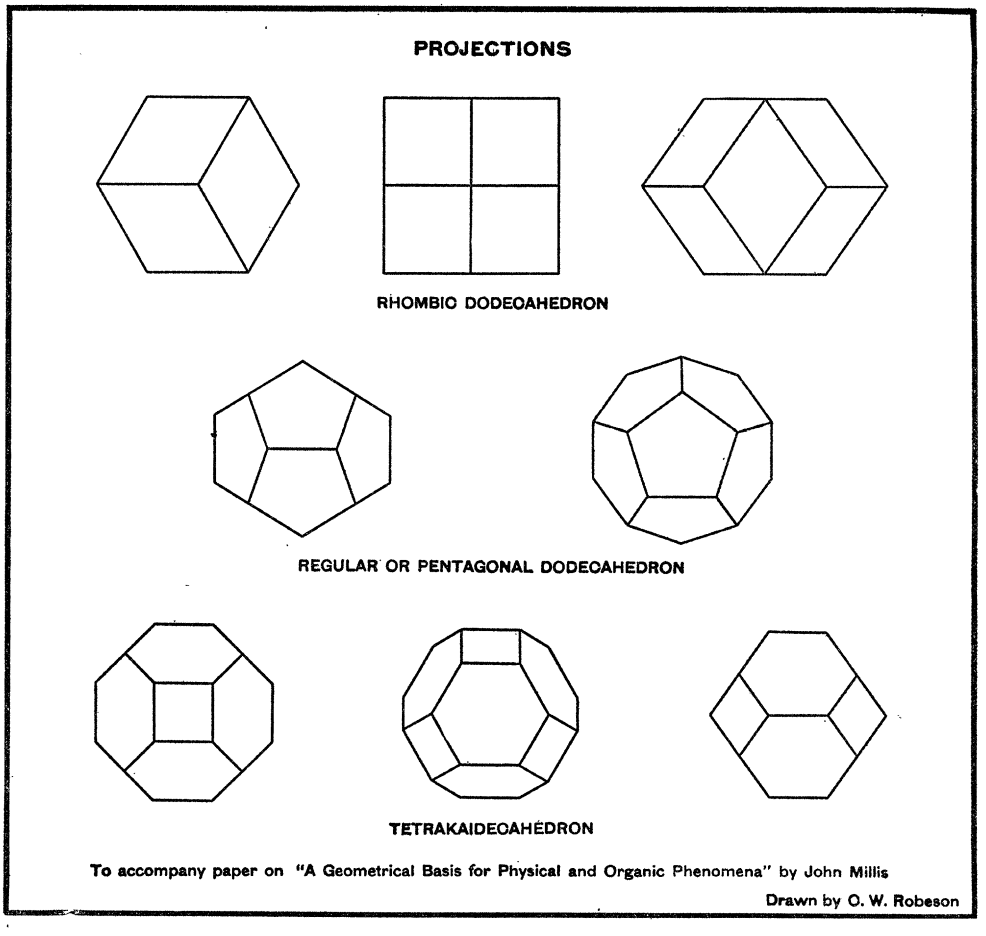
Going back to D'Arcy W. Thompson's book on "Growth and Form," there are found some exceedingly interesting discussions and references pertaining to the various possible divisions of space by plane surfaces that have a direct bearing on this subject. We can assume our equal spheres to be soap bubbles. The shape of a single bubble by itself is determined by the tendency of the enclosing film to contract due to its "tension," or the mutual attraction among its own particles, and the opposition to this contraction tendency presented by the enclosed air. By the same principles that have been explained above it can be shown that in a group of such bubbles the tendency is to assume an arrangement that will give complete symmetry and a minimum total partitioning area, that these conditions can not both obtain as a fixed and simultaneous status for the whole group, and that there can not be a condition of equilibrium and stability throughout such a group. The same will be true of any similar group of compartments or cells enclosing a fluid and with walls or partitions composed of substance that is of a fluid nature. Thompson seems to have fallen into some errors in his discussion, as where he calls the rhombic dodecahedron a "regular solid"[2] and where he understands that by means of an assemblage of equal and similar "tetrakiadecahedrons" space may be homogeneously partitioned into similar and similarly situated cells "with an economy of surface in relation to area (volume ?) even greater than in an assemblage of rhombic dodecahedra" (p. 338). The "regular" tetrakiadecahedron is a semi-regular polyhedron, a fourteen-sided volume with six equal square faces and eight that are regular and equal hexagons, the sides of these squares and hexagons all being equal. Such a solid may be formed by cutting off the corners of a cube, also by cutting off the corners of a regular octahedron. Space can not be divided into equal volumes of this shape without surplus, or in other words these volumes can not be stacked together without leaving voids. This may readily be determined by a study of the diedral angles, or practically by constructing a number of the tetrakiadecahedrons and trying it. However, these errors are not material to the present purpose, and it was Thompson's book that first suggested to the writer's mind a still broader generalization of the principles herein referred to.

It may be stated that all processes and phenomena of life are associated directly with some form of fluid substance. This includes not only gaseous matter and liquids but many forms of matter that are not solid in the ordinary sense nor yet liquid, but which have a certain degree of mobility among the constituent particles. The essential primary element of all organisms is the cell. *All* material substances whatever, whether affected by influences of life or whether only dead matter, are alike governed by the physical and mathematical laws here outlined. Is there not therefore a remarkable and intimate relation between the "simple geometrical principle" above explained and all organic existence and processes;—All life, growth, repair, decay, and dissolution:—Even all mind, intelligence, emotion, and all reasoning and thought. The speculative philosopher might indeed go so far as to add all health, satisfaction, and pleasure; all sickness, distress and pain; all relations and struggles among humans, all endeavors of man, all events of history, *everything*:—And the psychologist may here note an analogy to the unending strife between good and evil which figures in so many of man's superstitions and religious beliefs, primitive and otherwise.

A further conception of the profound significance of these elementary geometrical relations in connection with all activities and phenomena of the material universe may be formed by imaging a region or space, apart from any known real one, where it *is* possible for equal spheres to be so grouped that the arrangement will have at the same time maximum concentration and universal symmetry—an imaginary space where the regular dodeca-

[1] See SCIENCE for January 18, 1918.

[2] "Growth and Form," p. 328.



hedron has each of its diedral angles exactly 120 degrees instead of 116° —33′ —54″ (or, if preferred, where the full circle is made up of about 349—2/3 real degrees instead of 360), and where equal solids of this form will therefore stack together without voids. The curious and interesting speculations and deductions that follow from imagining a space with more dimensions than real space possesses (as four dimension or n dimension space), or a space having special properties like "curvature," etc., are quite well known. All matter in the hypothetical space permitting the special arrangement of equal spheres as above will, if we assume gravity to remain normal, tend to concentration as in real space but this tendency will not be checked or modified or counteracted by a departure from equilibrium resulting from an approach or approximation to the rhombic dodecahedral grouping of spherical elements, since the tendency will be to assume the regular dodecahedral grouping throughout. All matter under these conditions must eventually become a stagnant and dead plenum, an amorphous and non granular mass, in which no physical activity or life could possibly have being.

Another curious paradox, not altogether devoid of usefulness, is found in the self contradictory conception, which is at least semi logical, that there could only be a "real" continuous *ether* in a space with the imaginary properties above described! There are as yet unsurmounted (and unsurmountable?) diffi-

culties in the way of attempts at such a conception for *real* space, but even in the hypothetical space a continuous ether would find obstacles to the exercise of its principal function which is "to undulate." Moreover there would be no particular object in undulating—nothing to incite undulation—and so from all angles the ether idea has a rather hard struggle for a real existence.

We deduce properties of the circle by assuming that it is possible to divide the continuously curved circumference into parts so small that each one will be a straight line, and the rigid accuracy of the results so obtained is in no degree vitiated by the fact that the assumption can not possibly be true. Likewise we may deal with physical phenomena *as though there were* a "medium," an all pervading plenum of substance, itself devoid of gravity and of most other properties of real matter saving only the capacity to undulate. We can deduce, explain and predict with entire success—with consistent results and even astonishing confirmations—notwithstanding our medium or ether may be entirely hypothetical, its assumed properties may be contradictory in themselves, and it may not be possible for such an imaginary medium to have a real objective existence.

To locate and corner the remaining major difficulty in the way of a full comprehension of things of nature will at least contribute to our plans and measures for mobilization of forces and will indicate the main objective and the methods of attack, even if the adversary shall prove forever invincible.

Body B is separated from body A by an intervening distance. It is not possible for an "impulse" of mutual influence which requires time for the passage to be in transit between A and B, to be disconnected from both for the moment—suspended between them in other words—*without any intervening medium except space?* What a flood of light and clarity would be shed on and through the accumulated mass of physical facts and data, as well as the tangled maze of speculative perplexities, if an affirmative answer to this conundrum could only be given.

No confession of individual faith can be claimed to be of itself a useful contribution to our knowledge of material things, and this is distinctly true so far as concerns the ideas of the undersigned, but as has recently been said by a distinguished physicist it is well sometimes to declare ourselves in this respect "for this naturally has its influence upon all that is said and done and to the end of making the point of view of the writer clearly understood." (Crehore) It will be advantageous for the reader if he can feel that the writer is setting forth ideas with a certain degree of self felt confidence and "intellectual rest." I will therefore add the following, which is stated partly in the first person since it is a sort of individually conceived framework or background on which to pin ideas and new facts; frequently with a satisfactory fit in place, sometimes for the moment quite detached from the general pattern, but as a rule with a constant tendency towards accordance with what may ultimately turn out to be a complete and consistent picture.

There is only one kind of real space, the kind of everyday experience.

There is no material substance that does not have the common attribute of gravity.

There is no "force" except gravity, and all physical phenomena are resolvable into this conception.

Gravity is an inherent, essential, and universal attribute of matter. It is and ever will remain inexplainable. What is more (and this is another paradox) if an "explanation" were possible this would actually be a retrograde step in the progress of knowledge, since we would then have at least one remaining mystery on our hands, almost certainly still more troublesome than is that of gravity.

There is no such real thing as a continuous medium or ether. This however in no manner or degree disparages the vast majority of the facts, results and predictions that have been accumulated and accomplished on the "as though there were" assumption regarding an ether, nor is it inconsistent with the confident belief that there will be very many additional real and useful developments and advances in

our knowledge of natural things and phenomena on the same assumption.

It will be recognized that whatever there may be of novelty in the above first principles is found in the combination rather than in any one element.

Finally I concede with the eminent philosophers of long ago that an idea of the real nature of "action at a distance" without any intervening medium is inconceivable to the human mind (*my* human mind—they no doubt likewise meant theirs) and especially so is the suggestion that an impulse which requires time for transmission from one body to another may have left the one and be on the way to the other—in a state of detachment between—with nothing but empty space along the road. (It is probable that the "velocity of light" as a physical constant is the same as the velocity of transmission of a gravitational impulse or change from one body of matter to another, or at least that there is some very direct relation beween the two.)

Here however is the parting of the ways. I have faith that it will some day be accepted that this inconceivableness is attributable, *not* to the fact that the suggestion is incompatible with the real workings of nature, but to the limitations in the powers of human comprehension.

If it can be accepted that "philosophy" is only a shorter term for peace of mind arrived at or approximated to after long pondering, then the above may be set down as a sort of personal philosophy of the writer's.

And the path of future progress? We are apt to regard the human intellect of our period as already in a stage of its development which may be called maturity, but this is not at all certain. If something like a curve is plotted to indicate the mental status of man at different periods or "ages"—the primitive state, the stone age, the bronze age, the age of iron, etc., its general shape will indicate whether the present is the age of finality in this respect. There was just as much reason for regarding any one of the previous ages as a culmination as there is for assuming that we are now on an ultimate crest of the curve of human powers of understanding. In fact if we consider the varying rate of change in direction of such a curve, or the rate of its departure from a base line of zero intelligence, there is less ground for thinking our present mental capacity is at a maximum than there was for such a belief at any previous age or period.

Let us therefore "play" that there *is* an ether, with all its seemingly necessary though improbable attributes, and go ahead with our observations, experiments, studies and researches until the mind of man, now possibly only in the juvenile or youthful stage of its growth, may have so far advanced towards maturity as to be able to put aside this elementary conception and to substitute something more grown up. Meanwhile let us not lose sight of this all-important coordinate part of the program for advancing—the development of the human mind in capacity for comprehension so it can assimilate and interpret the facts as they accumulate and keep pace with the general progress. The super intelligence capable of fully comprehending all nature will doubtless always remain a limiting ideal—something to be eternally striven for, to be approached all the while more nearly, but forever unattainable.

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SCIENTIFIC EVENTS

MANGANESE ORE IN GEORGIA

As manganese is urgently needed in the war several geologists of the United States Geological Survey, Department of the Interior, have been making systematic examinations of areas that are believed to contain deposits of manganese and manganiferous ores in the United States and the West Indies, in order to appraise our available resources of manganese and to assist in stimulating its production and use.

Manganese is a metal resembling iron. It is used principally in the manufacture of steel, to which it is added in the form of alloys with iron, such as ferromanganese and spiegeleisen. It is used also in glassmaking, in many chem-