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PLANS FOR A HISTORY OF MATHEMATICS IN THE NINETEENTH CENTURY[1]

MY distinguished predecessor in the presidency of this association at one time ingeniously concocted a plan of procedure which if adopted, would have enabled him and succeeding presidents to escape the ordeal of preparing a presidential address. As a member of the council, I greatly enjoyed cooperating with others in nipping the president's scheme in the bud. Little did I know at that time that I was working against my own best interests and against the pleasure and comfort of the association on the present occasion.

An address being expected, it is my intention briefly to discuss plans of an organized movement for the writing of the history of mathematics of the nineteenth century on a scale commensurate with what has been achieved for previous periods. Taking for granted that such a history is desirable, in order that the present age may apprehend itself by comprehending its origin and growth, three important questions present themselves for consideration: Is it possible so early in the present century to write a satisfactory history of the preceding century? What will be the magnitude of the task, as compared with the labor involved in writing the earlier history? What should be the aim and nature of such a history?

As regards the first question, the material for the writing of modern scientific history is quite easily accessible. In this respect the writer of the history of science enjoys a great advantage over the writer of the history of recent diplomacy or war. Nor are the feelings and prejudices as intense on matters of

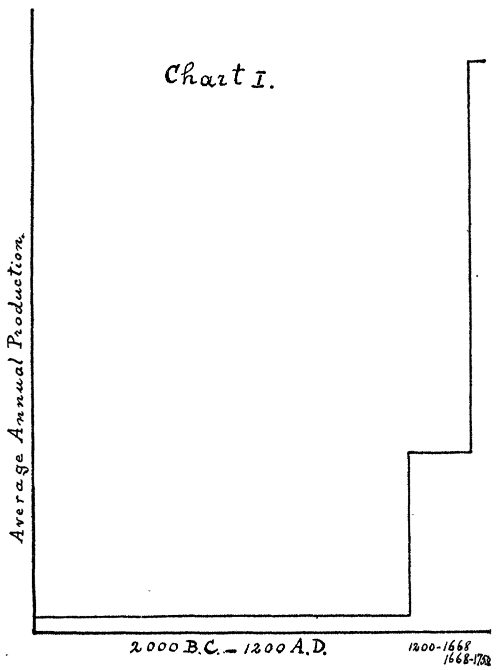
[1] Address delivered by Professor Florian Cajori as retiring president of the Mathematical Association of America, at Dartmouth College, on Saturday, September 7, 1918.

science as they are in religion, or in wars which involve the life and death of individuals and of nations. But, even if we have the material at hand, is it possible to write the mathematical history of the nineteenth century at this early date? Do we not now lack the proper perspective? In reply, we admit at once that we can not now write a history which will satisfy mathematicians seventy-five or one hundred years hence. All we can hope to do is to render a service to the present and the next generation of mathematical students and investigators. Nor can the history of the nineteenth century be written seventy-five or a hundred years from now in a manner that will be fully acceptable to all posterity. The general proposition holds true that no decade can write history which does not have to be rewritten later; no decade can write history for all future decades. There is an inevitable relativity of historical narrative. The reason for this relativity is obvious. The point of view changes; the attention of mathematicians will be directed to new concepts. Part of the history of mathematics will have to be rewritten in order to give proper emphasis to these new concepts. If it were possible, after the lapse of a few centuries, to impart finality to a history, then assuredly the history of Greek mathematics should have been rigidly determined and fixed long ago. But the facts disclose no such finality. In recent years the history of Greek mathematics has been partly recast. Zeno of Elea, whose arguments on motion formerly received little or no attention from mathematicians and were completely ignored by Montucla, the great eighteenth century historian of mathematics—this Zeno who was berated by philosophical writers as an insincere dialectician or as the progenitor of modern pettifogging lawyers—has been interpreted by Paul Tannery and other recent historians as having dealt sincerely and ably with questions of infinity now playing a leading rôle in modern mathematics. Geometric ideas of the last fifty years have brought into prominence the postulate of Eudoxus and Archimedes which the older historians of mathematics passed over in silence. The advent of the non-Euclidean geometry has thrown Euclid's parallel postulate into a wholly different light. Euclid's once criticized definition of equal ratios as contained in the fifth book of his Elements acquires a fresh interest when seen in the light of Dedekind's theory of the irrational. Many other illustrations might be cited to prove that historical narrative is relative, that history can not be written by a historian of one age, however keen, to satisfy all succeeding ages. Since this is so, should historians in despair drop their pens, remain idle and permit mathematicians to labor without the stimulus and light which a history of the modern developments of their science can give? Assuredly, no. A history of nineteenth century mathematics can be written acceptably to workers of to-day and to morrow, but will probably be in need of revision on the day after to-morrow. To wait for the moment when a history of mathematics can be written that will answer the demands of all future time is to postpone the crossing of a great river until all the water has flowed by.

An important step is the ascertainment of the magnitude of the task, the determination of the volume of mathematical literature to be penetrated. For purposes of comparison we have found it convenient to consider the mathematical productiveness during seven periods. Moritz Cantor's four volumes of lectures on the history of mathematics furnish the data for the first four periods. Statistics which I gathered from the first three volumes of Poggendorff's "Biographisches Handwörterbuch" were used to characterize the next two periods, while Professor H. S. White's "Forty Years' Fluctuations in Mathematical Research"[2] that took place since 1870 furnished the figures for the last period. Cantor's first volume gives the history of about 32 centuries down to 1200 A.D.; the second volume covers 468 years, to 1668; the third volume gives the history during 90 years, to 1758; while the fourth volume is limited to 41 years and carries the history down to 1799. We shall assume that the sizes of the volumes are ap-

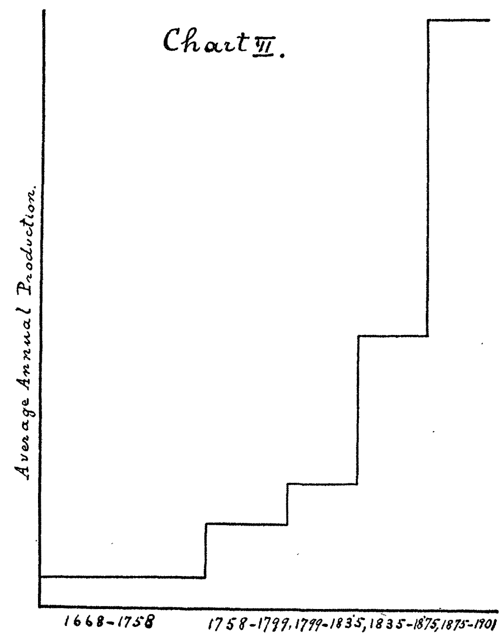
[2] SCIENCE, N. S., Vol. 42, 1915, pp. 105–113.

proximately proportional to the mathematical output of the periods covered. In Chart I., letting a short ordinate represent the average annual mathematical production during the first period extending from antiquity to 1200



A.D., we find that the average yearly production for the second period of 468 years is 6.9 times greater than for the first period, and for the third period of 90 years, 35.3 times greater. It will be convenient to take the average annual output during the first period as a unit measure of mathematical production. Accordingly, the figures for the first three periods are per annum, respectively, 1, 6.9, 35.3.

That we may graph more conveniently the expansion of mathematical literature since 1758, the third period is represented in Chart II. by a short ordinate. The fourth period of 41 years shows that the annual publications are 2.7 times more voluminous than those of the third period, indicating a yearly production of 95.8. To carry the comparison down to later periods I counted the number of mathematical writers given in the first two volumes of Poggendorff's "Handwörterbuch" for the eleven years 1790–1800. I found this number to be 244. For the period 1830–1840 I obtained 341 names. This indicates an increase in the ratio 1.398. I also counted the number of lines taken up in the enumeration of titles of books and articles of 134 mathematical writers for 1790–1800 and of 134 mathematical writers for 1830–1840. I began at random with the name of Jolly and in succession took the names of mathematical authors who wrote during these intervals. By this test the average productiveness of the individuals of these two groups proved, to my surprise, to be nearly the same; it was in the ratio of 1 : 1.04 in favor of the later group.



Combining the ratios of 1.398 and 1.04 I get 1.45 as the rate of increase in volume of mathematical literature of the period 1830–1840 over that of 1790–1800. The gradient fixed by the middle points of the third and fourth periods in Chart II. indicates for 1799 an approximate production of 114.4. Multiplying 114.4 by 1.45 gives 165.9 as the production for the year 1835. The average of 114.4 and 165.9, or 140.1 is the average annual production during the fifth period.

For the eleven years 1870–1880 I found in the third volume of Poggendorff 888 names of mathematical writers; this number is 2.6 times greater than that for 1830–1840, indicating an annual production of 2.6 times 165.9 or 431.3. The average annual production for the sixth period 1835–1875, is the average of 165.9 and 431.3, or 298.6.

From Professor White's statistics it appears that the annual output was in 1905 about 2.1 times greater than in 1875, or 905.8. From these data it follows that during the seventh period, 1875–1905, the average yearly quantity of mathematical literature was 668.8. Our results, put in tabular form, are as follows:

	Average Annual Production	Production during Period
I. Period, 2000 B.C.–1200 A.D.	1	3,200
II. Period, 1200–1668	6.9	3,149
III. Period, 1668–1758	35.3	3,177
IV. Period, 1758–1799	95.8	3,928
V. Period, 1799–1835	140.1	5,044
VI. Period, 1835–1875	298.6	11,944
VII. Period, 1875–1905	668.8	20,064

The tremendous increase is shown strikingly also by the two charts. They give emphasis to the biblical declaration, "Of making many books there is no end."

The total literary production during the last three periods is 37,052 and for the interval 1799–1901 it is 34,377. How long would it take to survey this field and write the history of mathematics of the nineteenth century? Moritz Cantor published his first historical paper in 1856 and completed the third volume of his well-known history in 1898. Perhaps it would not be fair to claim that all of these 42 years were given to the three volumes. Three smaller historical books from his pen preceded the three volumes in question. The early years were years of necessary preparation. The first volume appeared in 1880, the second in 1892, or 12 years later. If we assume that the preparation of the first volume required the same time as that of the second, then 30 years is the time Cantor devoted to the three volumes. In the tabular view given above the first three volumes are the history covering mathematical material amounting to 9,526. At this rate it would take a man 108.3 years, or 20 men 5.4 years, to write the history of mathematics of the nineteenth century, provided that these men gave, as did Cantor, a liberal amount of time to research and confined their efforts to this particular project. If only one third of their research time were given to it, then 65 men would be needed for 5 years.

Let us estimate the number of men from a different set of data. Volume IV. of Cantor's history was prepared by 10 men, each working about one year and a half. Some of these prepared short chapters and probably completed their parts in less time. On the basis of $1\frac{1}{2}$ years and on the supposition that each worker will give one third of his research time to the enterprise, it will take 79 men 5 years to write the mathematical history of the nineteenth century. If this history is written with the elaborateness of Cantor's "Vorlesungen," it will embrace 14 or 15 volumes of the size of the Cantor volumes; it will contain $5\frac{1}{2}$ million words. Probably it would be wise to plan a considerably smaller number of volumes.

Considering the magnitude and difficulty of the undertaking, it is quite evident that American mathematicians alone would find the task excessive. If the services of English, French and Italian mathematicians could be enlisted, the enterprise would be comparatively easy. In past years it has been Europe that has initiated organized efforts among scientists. In proof of this I need only mention the "Encyklopädie der mathematischen Wissenschaften," the "Encyclopédie des sciences mathématiques," the "Royal Society Catalogue of Scientific Papers," as also the publications of Euler's complete works, of the *Fortschritte der Mathematik*, of the *Revue semestrielle*. It would seem to be our turn to take the initiative. The history of mathematics of the nineteenth century might very well be planned and financed by America. The magnitude and difficulty of the task would exert a healthful stimulus upon our 550 colleges and universities. Such an ambitious scheme would require the exercise of energies now latent.

Let us hope that the places where mathematical research is carried on will cease to be limited to a small number of localities on the great map of our country—the seats of our stronger universities. The spirit of research should penetrate many other institutions. Young men of ability, now doomed to pass their lives with broken wings through the crushing weight of excessive hours of teaching, should find relief in the future and be encouraged to accomplish the highest that is in them to do. Professor Bjerknes, of Stockholm, when lecturing at Columbia University, ventured the statement that had his teaching schedule at home been as onerous as is that of American professors, then he would never have been invited to lecture in America in a difficult branch of science.

To carry the enterprise we are considering into successful operation calls for the united efforts of three groups of men: The mathematician, eager to enter a rich but difficult field of research, the administrator, willing to provide the mathematician with the necessary leisure, and the philanthropist, ready to supply the funds necessary for the publication of the results of research. Has America the ideals and the genius of organization for the creation of such a triumvirate?

In recent years a new ideal for the history of mathematics has arisen. The new movement calls for a much more careful and comprehensive scrutiny of historical material; it demands higher standards of historical accuracy. This ideal has been championed in the *Bibliotheca Mathematica,* a journal edited by Gustav Eneström, of Stockholm. According to the standards set by Eneström, the labors of Moritz Cantor, of Heidelberg, especially the third volume of his well-known history, hardly reach the high mark of excellence demanded. Cantor worked diligently and persistently for many years until finally failing eyesight lessened his powers and, more recently, compelled him to enter the darkened universe of the blind. If his third volume is inferior to his first two, an additional cause thereof is found in the fact that in the seventeenth and eighteenth centuries so many new mathematical subjects were introduced that the task of setting forth their history almost transcended the powers of a single individual. Necessity compelled the fourth volume to be prepared on the cooperative plan, by a group of men.

The present danger is that, in the effort to attain extreme accuracy in historical detail, we shall lose sight of literary quality. The publications of Huxley and Tyndall have demonstrated that scientific writing admits of combination with literary finish. The two ideals are not incompatible. Good literary form challenges attention and provokes admiration. Macaulay, the great master of words, made history enjoyable to thousands who otherwise would have shunned historical reading. Arago's biographies of scientific men have a charm their own.

In order to make the history of mathematics interesting, it is desirable to consider not only literary finish, but also the amplitude of topics selected. In writing the mathematical history of the ninetenth century, it does not seem to me sufficient merely to endeavor to trace the line of scientific progress, the evolution of new concepts. That much is done by the "Encyklopädie" and the "Encyclopédie," which we hope may be carried to completion soon after the termination of the great war. The mathematical reader does not subsist on logic alone. He is eager for color. He desires to know the personality of great mathematicians, the environment in which they worked, their idiosyncracies, their struggles with scientific difficulties, the circuitous route by which they reached their results, the influence which mathematicians exerted, one upon another. The heroism of some scientific men makes a tremendous appeal to the reader. These topics create not only interest but enthusiasm. In fact, if we consult our experience, as well as that of others, we are forced to admit that the best that we have from history is the enhusiasm which it generates. The mathematician, like the warrior, is a hero worshipper. In the case of Archimedes, the reader expects the historian not only to state accurately the connection of the writings of Archimedes with those of his predecessors, the advances that he made and the relations that his writings bear to the modern mathematics, but the reader expects also a presentation in vivid style of the imagination and enthusiasm that Archimedes exhibited in the saying "Give me a fulcrum on which to rest, and I will move the earth;" the reader likes to be entertained by the narrative of his behavior when the true solution of the problem of the crown flashed on his mind—how he jumped out of his bath and shouted "I have found it"; the reader likes to be able to cite as marks of high patriotism his services to his sovereign in the construction of war engines and his alleged use of reflecting mirrors to set the Roman ships afire; the reader wishes to be reminded of the tragic death of Archimedes at the hands of a Roman soldier whom he had requested not to spoil his circles drawn in the sand, of his desire that the figure for his theorem on the sphere and circumscribed cylinder be inscribed upon his tomb, and how, over a century later, Cicero found the tomb of this the greatest mathematician of antiquity almost hidden amongst briars near one of the gates of Syracuse and forgotten by the people of the city.

Details of this sort do not strictly belong to a history of scientific ideas, but they add color to the narrative. Like all men, the mathematical reader is largely dominated by feeling. A modern poet has said of Horace,

> It is a curse
> To understand, not feel thy lyric flow,
> To comprehend, but never love thy verse.

Where is the science which appeals to the intellect alone, never to the heart? Such a science, if it exists at all, can not be found in the camp of the mathematicians. Certainly, then, the history of mathematics should appeal to the heart as well as to the head. Such a history should create respect and love for mathematics; it should excite admiration for this science; it should make the mathematician feel stronger than ever that he is contributing his bit toward the true grandeur of nations.

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THE BRITISH NATIONAL PHYSICAL LABORATORY

The National Physical Laboratory is now conducted as a government institution under the financial control of the Committee for Scientific and Industrial Research, though its scientific work and researches remain under the direction of the Royal Society as before, and its annual report, of which the London *Times* gives an abstract, is for the first time published by the Stationery Office (2s. 6d. n.). As has been the case since 1914, a large part of the laboratory's work last year was in connection with the requirements of the war, and therefore can not be described in detail, even if referred to at all, but there is a considerable body of investigations to which this limitation does not apply.

In the electrical department additional magnetograph 24-hour records were taken and coincidences found between the running of electric trains on the London and South-Western Railway and magnetic disturbances at the laboratory. The question as to the magnitude and cause of certain of the disturbances was submitted to the arbitration of Mr. A. J. Walter, K.C., in November and December, and he adjudged that the use of electrical power by the railway has caused an increase in the horizontal magnetic field at the laboratory to an extent exceeding that stipulated in the South-Western Railway Act of 1913.

Among general electrical measurements a large number of the small mica condensers used in magnetos were tested for capacity and power factor; much work was done in connection with the composition and treatment of steel used for compass needles, and some special search coils proved valuable in testing permanent horse-shoe magnets. Some investigation was also made of the properties of non-magnetic steels. Sensitive vibration galvanometers for frequencies of the order of 10 cycles were constructed, and some preparations made for a research on effective resistance at radio frequencies. A new 400-volt secondary battery is being installed for use with large valves to generate the high-frequency current.

In the division of electrotechnics, in view of