sent the table, as in the expression "Thogh Argus (that is, al-Khowarizmi) the noble covnter Sete to rekene in hys counter." The story is connected with Fitz-Neal's "*Dialogus de Scaccario*" of 1178 and the court of the exchequer, with backgammon, and with divers other ramifications. The counter went by various names, such as Rechenpfennig, Zahlpfennig, and Raitpfennig in Germany; projectiles and abaculi as well as calculi in Latin; jetons, gectz, getoirs, giets, and the like in France; Leggelt and Werpgeld in Holland; and jettons and Venetian money as well as counters in England.

It is the field of counters that Professor Barnard has made his own in the monumental and sumptuous work under review. For many years he has been collecting specimens of counters of the various European countries. He has examined upwards of 40,000 specimens and has in his own cabinet some 7,000, most of those described being in this collection. With great care he has selected typical specimens, the choice being determined by their historical importance, artistic merit and general value. Fifty-nine Early English jettons are described, nineteen Italian, three hundred seventy-two French, ninety Low Country, one hundred twenty-two German, and four Portuguese. Each specimen is illustrated photographically and each is described with all the care of a trained numismatist. Nothing could be more satisfactory to the collector or the student, and it would be difficult to suggest a single particular in which the descriptive material could be improved.

To the historian of mathematics one of the items of greatest value is the set of photographs of reckoning tables at Basle and Nürnberg, of reckoning cloths at Munich, of illustrations from manuscripts and early printed books, and of Dutch jetton cylinders, together with a description of each. There is also a valuable list of one hundred fifty-nine extracts from English inventories with references to counting boards, thus showing that these devices were common from the fourteenth to the sixteenth century at least. The first two of these extracts, of date 1321 and 1337, respectively, make mention of "Camera quædam cum mensa quadrata ad calculandum" and of "Unum computatorium," while other interesting items refer to "the cheker," "counting-bord," "unum scaccarium," "cowntynge borde or table," "accomptyng borde," "unam mensam vocatam a counter," and "the counterborde in the Hall."

Perhaps the most important part of the work from the standpoint of the historian is that on "the methods of casting with jettons" (pages 254–319). Here Professor Barnard has given a very satisfactory summary of the more important European works on the subject, such as those by Recorde, Awdeley, Reisch, Cusanus, Siliceus, Köbel, de Moya, and Trenchant. There should also be mentioned as of great value to students the bibliography of upwards of six hundred titles.

Taken as a whole the work may be safely characterized in superlatives. Such an elaborate treatise on any special field of the history of mathematics has never before appeared, nor are we likely soon to see another. The infinite pains taken by the author in his research, the munificence shown in the publication itself, and the fact that a mass of technical material is presented in a style that makes every page readable, all combine to render the work unique in its way. No library of reference can afford to be without the book, and students of the history of mathematics should add it to their personal libraries as soon as they can arrange to do so. It will be the classic upon the subject for generations to come.

David Eugene Smith

The Nature of Solution. By Harry C. Jones. New York, D. Van Nostrand Co. 1917. 23 x 15 cm.; pp. xxiv + 380.

The present work is not a text-book, but a general discussion of some of the more important properties of solutions, true and colloidal. It is therefore written in a non-mathematical, indeed, largely in a semi-popular style. It is hoped that this work may interest students of the various branches of science to go on into the real physical chemistry of solutions, and from this into physical chemistry in its broadest sense.

As far back as 1893 Jones did some work with Arrhenius on the hydrates of sulphuric acid. In 1900 he suggested that part of the abnormal changes of the freezing-point with concentrated solutions is due to the solute forming a compound or solvate with the solvent, so that the real concentration is or may be much greater than the apparent concentration. From this time on Jones devoted practically all his time to a study of solvates in solution. The results are remarkable in quantity and in range. Jones points out, p. 346, that "the following sixteen lines of evidence bearing on the solvate theory of solution have been established:

"1. Relation between lowering of the freezing-point of water and water of crystallization of the dissolved substance.

"2. Approximate composition of the hydrates formed by various substances in solution.

"3. Relation between the minima in the freezing-point curves and the minima in the boiling-point curves.

"4. Relation between water of crystallization and temperature of crystallization.

"5. Hydrate theory in aqueous solutions becomes the solvate theory in general.

"6. Temperature coefficients of conductivity and hydration.

"7. Relation between hydration of the ions and their volumes.

"8. Hydration of the ions and the velocities which they move.

"9. Dissociation as measured by the freezing-point method and by the conductivity method.

"10. Effect of one salt with hydrating power on the hydrates formed by a second salt in the same solution.

"11. Investigations in mixed solvents.

"12. Spectroscopic evidence bearing on the solvate theory of solution; work of Jones and Uhler.

"13. Work of Jones and Anderson on absorption spectra, in which the presence of 'solvate' bands was first detected. This showed that the solvate had an effect on the absorption of light, and this could be explained only as due to a combination between the solvent and the resonator, or something containing the resonator.

"14. The work of Jones and Strong on absorption spectra established the existence of a large number of 'solvent' bands. They showed that these were formed by many salts and in many solvents. They could even distinguish between the bands of a salt in a given alcohol and in its isomer. This was regarded as very important. The temperature work of Jones and Strong was strong evidence for the solvate theory.

"15. The work of Jones and Guy on the effect of high temperature on the absorption spectra of aqueous solutions, and also on the effect of dilution, led to results which were also in keeping with the solvate theory. The most important work of Jones and Guy which bears on the solvate theory of solution is that in which the radiomicrometer was used. It was here shown that solutions of certain strongly hydrated non-absorbing salts are more transparent than pure water having a depth equal to that of the water in the solution. In the case of non-hydrated salts the solution was the more opaque. This shows that water in combination with the dissolved substance—water of hydration—has less absorption than pure water. This is regarded as striking evidence that some of the water in the presence of salts which are shown by other methods to hydrate is different from pure, free, uncombined water; and the simplest explanation seems to be that this is the combined water or water of hydration.

"16. The work of Jones and Guy was repeated and extended by Jones, Shaeffer and Pauls. They obtained results of the same general character as those found by Jones and Guy. Solutions of hydrated salts were in general more transparent than pure water, especially at the centers of the absorption bands. Solutions of non-hydrated or only slightly hydrated salts are more opaque than pure water, especially at the centers of the bands."

The quantitative conclusions are based on the two assumptions that the van't Hoff formula holds absolutely when the true concentrations are used, and that the percentage dissociation can be calculated from the conductivity. Unfortunately, neither of these assumptions is true. There are many who think very highly of Jones' work and there are those who are more critical. All will agree, however, with the words of Professor Reid in the biographical sketch, that Jones was an advocate rather than a judge. WILDER D. BANCROFT

SPECIAL ARTICLES

COMPARATIVE PERMEABILITY OF FERTILIZED AND UNFERTILIZED EGGS TO WATER

IN general the rate of the osmotic entrance or exit of water, in any living cell, after transfer from its normal medium to another non-injurious medium (*i. e.*, one not impairing semi-permeability) of lower or higher osmotic pressure, varies directly (1) with the gradient of osmotic pressure between the interior and the exterior of the cell, (2) with the area of the semi-permeable membrane enclosing the cell, and (3) with the permeability of this membrane to water. It is to be expected that this permeability to water will vary in different physiological states of the cell; and that the same species of cell, placed in the same medium, will show variations in the rate of the osmotic transfer of water, *i. e.*, in its rate of swelling or shrinkage, according to its physiological condition or state of functional activity at the time of the transfer. There is in fact a definite and constant difference between the fertilized and the unfertilized eggs of the sea-urchin *Arbacia* in this respect, fertilization being followed regularly by a marked increase in the permeability of the egg-surface to water—as may readily be shown by bringing the eggs into either dilute or concentrated sea-water; in the former medium they swell, in the latter they shrink, but in both cases the rate of the process is much greater in the fertilized than in the unfertilized eggs. In the unfertilized eggs both swelling and shrinkage are surprisingly slow, so that in a medium whose osmotic pressure differs from the normal by so much as ten or twelve atmospheres these eggs show little apparent alteration of size at a time (*e. g.*, two minutes after placing in this medium) when the fertilized eggs are conspicuously swollen or shrunken. This difference of behavior relates entirely to the *rate* at which water either enters or leaves the egg; the *degree* of swelling or shrinkage when osmotic equilibrium is reached is approximately the same in both cases. It is clear therefore that this difference has nothing to do with any possible change in the osmotic pressure of the egg-protoplasm, resulting from fertilization, but is determined simply by the greater readiness with which water enters or leaves the fertilized as compared with the unfertilized egg. According to former measurements made on the rate of swelling of fertilized and unfertilized eggs in dilute sea-water, the resistance to the passage of water across the plasma-membrane is decreased approximately four times as a result of fertilization.[1]

The most striking and convenient method of showing this difference is to place a mixture of equal numbers of unfertilized and fertilized uncleaved eggs (the latter fertilized at least 15 minutes previously) in a somewhat strongly hypertonic sea-water (*e. g.*, 1 volume of van't Hoff's artificial sea-water of 2.5 m concentration *plus* 4 volumes normal sea-water). The fertilized eggs at once shrink rapidly and undergo crenation, and within less than one minute exhibit a collapsed, shrunken, and angular appearance; at this time the unfertilized eggs show little change, so that a striking contrast is presented. Shrinkage continues slowly in the unfertilized eggs, and becomes well marked in the course of five or six minutes, but a curious

[1] *Amer. Jour. Physiol.*, 1916, Vol. 40, p. 249.