SCIENCE

FRIDAY, OCTOBER 26, 1917

CONTENTS

MSS. intended for publication and books, etc., intended for review should be sent to The Editor of Science, Garrison-on-Hudson, N. Y.

THE SIGNIFICANCE OF MATHEMATICS[1]

SEVERAL circumstances combine to render peculiarly fitting a consideration at this time of the significance of mathematics. Of late we have heard much from real or alleged educators, tending to show a lack of appreciation on their part, if not on the part of the public, of the vital part which mathematics plays in the affairs of humanity. These attacks were beginning to receive some hearing in the educational world, on account of their reiteration and their vehemence, if not through intrinsic merit.

A counter influence of tremendous public force, whose import is as yet seen only by those most nearly interested, has now arisen through the existence of war and the necessities of war. To the layman, lately told by pedagogical orators that mathematics lacks useful application, the evident need of mathematical training on every hand now comes as a distinct surprise.

The attacks on mathematics, and the lay conception of the entire subject, centers naturally around elementary and secondary instruction. We ourselves, college teachers of mathematics, have commonly talked of current practise and of reforms, largely with respect to secondary education. The third influence which contributes toward the present situation and which may strongly affect its future development is the formation and the existence of this great association, which affords for the first time in the history of America an adequate

[1] Retiring Presidential Address, Mathematical Association of America, summer meeting, Cleveland, September 6, 1917.

forum for the discussion of the problems of collegiate instruction in mathematics.

As retiring president of the association, I know of no more fitting topic than that I have chosen. It vitally concerns us; it is bound up with the functions of this association; and the times in which we live seem to point forcibly toward its consideration. I shall attempt to outline to you my own views on the true significance of mathematics, and to sketch what I for one would be glad to see this association promote.

In speaking of the significance of mathematics, I understand that we mean not at all the baser material advantage to the individual student, not at all a narrow utilitarianism, but rather a comprehensive grasp of the usefulness of mathematics to society as a whole, to science, to engineering, to the nation. Any narrower view would be unworthy of us; any narrower demand by educators means a degraded view of the purposes of education in a democracy.

Especially under the stress of war, public attention may be secured for the real claim of mathematics as a public necessity, not only to be employed by a few specialists, but also to influence and to determine the conduct and the efficiency of thousands.

Thus a knowledge of trigonometry and of the trigonometric theorems of geometry is a prime requisite for the successful and efficient conduct of our armies, not only by a few engineers who are to make maps and to train artillery, but also for all officers to whom the lives of men are entrusted. Any one of these officers, cut off with his force, without a superior engineer at hand, may lose his position and the lives of his men if he is ignorant of the significance of these propositions. Ignorance at such a crisis would be next to treason; it would be incompetence.

Do we, in trigonometry, so bring out the significance of the fundamental ideas on right triangles that the officer who faces such a test will sense the possibility of finding a range, or estimating a distance, without help and without instruments or tables? Frankly I do not believe that we have been doing this, even in such a practical subject as trigonometry. We have been too often content, and too often solely seeking, even here, the knowledge of intricate formalisms, of formulas and rules and theorems, of operations done mechanically. Too often we have omitted, even here, to give insight into the rather obvious significance of these rules and formulas.

On the whole, however, trigonometry is the one subject in which some small measure of insight has usually been secured.

If I now turn to other topics of our curriculum, may I not name scores of equally vital topic, usually studied by our students, in which insight is rarely gained? Let me mention some such instances:

In algebra, as taught in colleges, among the topics always considered are fractional exponents, logarithms and arithmetic and geometric progression. To many, fractional exponents remain a pure formalism, learned by rote and unappreciated, connected neither with the other topics just mentioned nor with any realities of life. That fractional exponents occur in expressions for air-resistance (as in airplanes), in water resistance (as in measuring stream-flow), in electricity (as in induction), would surprise most students who pass our courses. That these exponents are determinable and are determined by logarithms would surprise students and some teachers, even if the essential equivalence of exponents and logarithms is adequately emphasized. The idea of a compound interest law, namely, that one quantity may proceed in arithmetic progression as another

related quantity proceeds in geometric progression, is ordinarily not brought out, nor is the fact that this same situation leads to a logarithmic law.

The omission of these and similar vital connections, both of mathematics to the exterior world and of one topic in mathematics to another, is directly responsible for the failure of algebra to reach the hearts of our students, and for the failure of the students to gain real insight into the significance of the subjects they so dully learn.

I shall not dwell long on any one topic, for I desire to emphasize the existence of significance for life and society in the entire range of mathematical courses, and I desire to call your attention to the failure—shall I not say *our* failure?—to bring to light that significance.

Let me turn to analytic geometry for another instance of our traditional blindness, if it be that—our sin, if it is not blindness. Here, as before, applications abound. Most of the results of scientific experiment to-day are known and are recorded not by algebraic formulas of traditional form, but solely by curves traced in our traditional style, showing graphically the functional relations between two or more interdependent variables. Laws of physics, of chemistry, of every quantitative science, expressed by such means abound. The effort of science may well be said to be to deduce from such graphical functions the corresponding laws in algebraic or formalistic form.

Yet to most students of analytic geometry, precisely the reverse view seems to be our aim. The significance of analytic geometry as a piece of scientific machinery is totally lost, and the subject sinks to the level of dubious value in the minds of our students and of half-informed educators. In the present emergency, popular conviction of the real significance of analytic geometry for society is being attained, and may be fostered, through the occurrence of just such graphical laws in the dynamics of airplanes, in artillery performance (ballistics) and in wireless telegraphy. Here as in general in science, most of our information on functions is now in graphical form, and the desire to express the function in equation form illustrates the fundamental demand of science, and the fundamental significance of analytic geometry.

That the calculus is regarded as dry and uninteresting by many students, and that its value is occasionally doubted, is the strongest proof possible that its significance is not grasped. Here the connection with realities is so easy and so abundant that it is actually a skillful feat to conceal the fact. Yet it is done. I know personally of courses in the calculus (and so may you) in which the pressure to obtain and to enforce memory of formal algebraic rules has resulted in absolute neglect of the idea that a derivative represents a rate of change! I know students whose whole conception of integration is the formalistic solution of integrals of set expression by devices whose complexity you well know. That an integral is indeed the limit of a summation, and that results of science may be reached through such summation is often nearly ignored and not at all appreciated. That the ideas of the calculus should fall so low as to consist mainly in formal differentiations and integrations of set expressions must indeed astound any one to whom the wonderful significance of the subject is at all known. Moreover, it must convince any liberally minded educator who takes our own courses as a true representation of mathematical values that even the calculus is of no importance for real life or for society.

I might proceed to other courses—differential equations as given by Forsyth, the theory of equations as by Burnside and Panton or as by even the most recent

writers, the theory of function (without any hint of its manifold connections with physics), the calculus of variations (denatured, without a hint of its vast importance in mechanics and elsewhere), projective geometry (with no mention of descriptive geometry nor the representation of space forms).

In all these, tradition has been leading us as far astray as it has in those more elementary courses of the secondary school, which we are wont to criticize. Shall we not search our own house? Shall we not ask if our own collegiate and graduate courses in mathematics demonstrate to students the real significance of the theory they cover? Have we denatured each subject until insight is eliminated and only formalism and logical tricks remain?

So long as this blight remains, we must expect and we shall deserve public disdain and sincere doubt of our value to humanity.

It should be unnecessary for me to explain my own deep interest in the logical and cultural side of mathematics. Certainly I would be the last to belittle its great spiritual values. But this is for the specialist rather than for the usual student. Values to the world at large must be stated in terms of more concrete realities. Shall we hide the fact of the immense service of mathematics to society? To emphasize beauty and pleasure to the entire exclusion of the more convincing argument of benefit to mankind is as quixotic and short-sighted as is the corresponding formalization of our courses of instruction. To ignore the significance of our great subject is to spurn our birthright.

Let me then, in retiring from office in the association, leave with you the sincere hope that a part of the work of this association be to impress upon the public the great value of mathematics in its direct effects upon life and upon human society. To accomplish this end, a most effective means, and one ready to hand, is to bring out to our own students, not halfheartedly, but with vigor, not a few but all available facts that shed light on the real meaning of what we teach. Let this association be a focus from which such doctrine may emanate, a forum in which such views may be emphasized and detailed. Thus I to-day have mentioned to you a few samples of our neglect, in haste and by name only. Shall we not discuss among ourselves these and other means toward the end, other topics whose significance is commonly lost or neglected, other points of view that will increase insight, even if it be at the expense of a few formulas or theorems that we traditionally treasure.

To the same end, may I now emphasize what seems to me a great if not the greatest function of this association? In America, up to recent years, the beauty and interest centering in pure mathematics has so absorbed all mathematical talent that we have almost if not quite neglected that other phase of mathematics in which the significance of all we do is so self-evident: applied mathematics. This association has, through its journal and through its meetings, already demonstrated its willingness and its ability to foster mathematics of this type. On this side of mathematics, not only discussion of the mathematics taught or to be taught, but even research papers of high grade have had in the past no adequate means of exposition. The wonderful work of Gibbs was for this reason long buried in the obscure Connecticut Academy and mathematical advancement along the important lines that he laid down was delayed or wholly prevented. The great work of G. W. Hill, which included profound investigations on infinite determinants, was for the same reason unknown and unappreciated by many mathematicians in this country until near his death, and work by others along the lines he mapped out was

discouraged and delayed. Thus American mathematics has suffered not only in reputation, through the suppression of what are perhaps the greatest American achievements in mathematics, but also in that encouragement necessary to the establishment of a strong school. The same may be said of the essentially mathematical researches of other men still living, whom I hesitate to name,[2] whose work is scattered through journals on general science, journals on astronomy, journals on life insurance, journals on engineering, and so forth.

Already there have been published by the *Monthly* articles of research on topics in insurance, on mathematical history, on mechanics, and on other applied branches of mathematics. In the first annual meeting, Professors Wilson and Webster presented their own studies on the mathematical theory of the dynamics of the air. At the last summer meeting Professors Huntington and Hoskins presented studies on the foundations of mechanics. Such work, though deserving of high praise, has long had no suitable center for exposition and for encouragement. This association has afforded a means for exposition, as within the field of mathematics in its broader meaning, of papers in applied branches of mathematics. I trust that we shall continue this policy, and that we shall no longer rule out of our circle in mathematics, those who find the problems of applied mathematics peculiarly attractive. It should be our aim to encourage them and their students; to hear their work and to print it; to listen to their counsel on the needs of our traditional mathematical courses, to learn from them ourselves to appreciate more keenly the significance of mathematics as a whole.

In both these ways—by reorganizing our own instruction under the auspices of this association, and by the recognition and encouragement of workers in the various fields of applied mathematics, we may, and I think we should, increase the appreciation of the significance of mathematics among our students, among the public and even among ourselves. Incidentally we shall have done a service, not only to the public, in the increased emphasis upon phases of mathematics of real public service, but also to the advancement of mathematics itself, in that a better insight into the significance of mathematics will prevent or nullify mistaken attacks on the subject as one of little public worth.

Such to my mind should be one function, if not the chief function of this association; the regeneration of a significant mathematics, the encouragement of workers in applied mathematics, and the effort to obtain recognition of the true public worth of mathematics in every phase.

E. R. HEDRICK

UNIVERSITY OF MISSOURI

[2] One of the men I have in mind is in attendance at these meetings.

AN INSTITUTE FOR THE HISTORY OF SCIENCE AND CIVILIZATION

TO THE EDITOR OF SCIENCE: The appeal concerning "an Institute for the history of science and civilization" published in SCIENCE, March 23—ill-timed as it was—has met with the most encouraging response. Two communications relating to it have been published in SCIENCE, June 22 and July 6,[1] and a great many more have been privately addressed to me. Most of them, however, lay so much stress on some special feature of our plan that I feel it necessary to state again, briefly, the fundamental idea that underlies it, lest the real purpose of the institute be lost sight of.

But let me say first of all that there is at least one point upon which an unanimous agreement seems to have been reached. The whole budget of letters which I have received

[1] *Cf.* also F. S. Marvin in the *Positivist Review*, London, June, 1917.