In the detailed application of this principle, there is one point where confusion might arise, though it can readily be avoided. It is the anomalous behavior of the unit, radian, which appears as a perfectly respectable unit when an angular velocity is converted from $\frac{\text{rev}}{\text{min}}$ to $\frac{\text{sec}}{\text{rad}}$ but does not appear when the same angular velocity is found from $\frac{v}{r}$. This anomaly is the only one of its kind, and is not nature's fault, but our own. If we define angle as degree of opening, to be measured in units of the same kind, the substitution method outlined above is the most natural method of converting say $\frac{\text{rev}}{\text{min}}$ to $\frac{\text{rad}}{\text{sec}}$. If, on the other hand, we define angle as a mere ratio of arc to radius it is necessarily a pure number (like a sine or a tangent). If we swap horses in midstream, we shall either miss this unit later or else see it floating up where we do not expect it. This means we must insert or rub out the unit radian whenever it is convenient to do so. Fortunately angle is the only quantity treated in such a way.

For the sake of such mathematical purists as may not approve of the above on philosophical grounds, a few words should be inserted here on the meaning of the term "multiplication." In elementary arithmetic it means merely repeated addition, but with the introduction of irrational numbers the term is extended by mathematicians to an operation that is not strictly repeated addition. The plan here advocated extends the notion of multiplication still further, to cover a physical combination of concrete quantities. In general the definition of multiplication in each individual case amounts to translating into algebra the ordinary verbal definition of the compound quantity involved (area, velocity, work, etc.). This extension is made practicable by the fact that the operation thus defined obeys the same logical postulates as the corresponding algebraic operation on pure numbers. In other words, the machinery of mathematics can be applied not merely to numbers, but to any group of concepts and operations satisfying the same postulates. This fact is accepted intuitively by most students; and incidentally the emphasis it puts on the definitions prevents most of the well-known confusion between acceleration and velocity, power and work, and so on.

To sum up, it seems to me after several years' experience with this system, that it has the following important advantages: (1) It treats equations as neat shorthand statements about real physical things and emphasizes the esthetic side of mathematics in general; (2) It provides an enlarged principle of dimensions by which equations may be checked during computation; and (3) It removes completely all restrictions on the units to be used and enables the student to concentrate his attention on the facts of nature without the disturbing influence of arbitrary rules.

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SPECIAL ARTICLES

ON THE SWELLING AND "SOLUTION" OF PROTEIN IN POLYBASIC ACIDS AND THEIR SALTS

THERE are available only scattered observations on the absorption of water by proteins in the presence of various polybasic acids and their salts. In order to obtain further experimental data in this field, we undertook a rather detailed study of this problem during the past year. As examples of proteins, dried gelatin discs and powdered fibrin were used. For the polybasic acids we chose phosphoric, citric and carbonic. In connection with the swelling of gelatin, we studied also its "solution." The general results of our experiments may be summed up as follows.

I

The amounts of water absorbed by gelatin from equimolar solutions of monosodium, disodium and trisodium phosphate depend not only upon which of these salts are present, but upon their concentration. Gelatin absorbs but little more water in a solution of monosodium phosphate than it does in pure water.

In low concentrations of disodium phosphate, gelatin swells decidedly more than in pure water, but as these lower concentrations give way to higher ones, the gelatin swells less and less until, when sufficiently high concentrations are attained, the gelatin swells decidedly less than in pure water.

These same general truths may be stated for trisodium phosphate, except that the absolute amounts of water absorbed in solutions of this salt are, at the same molar concentration, decidedly higher than in the case of the disodium salt. Low concentrations of trisodium phosphate bring about much greater swelling than higher ones. With progressive increase in the concentration of the trisodium salt, there is a progressive decrease in the amount of swelling until a concentration is finally reached in which the swelling is decidedly less than in pure water.

Having studied in this fashion the relation of swelling to type of salt and its concentration, we investigated next the amount of water absorbed by gelatin in phosphate mixtures of compositions varying from the extreme of pure phosphoric acid on the one hand through mono-, di- and trisodium phosphate to pure sodium hydroxid on the other. These mixtures were made in different ways. Beginning with pure phosphoric acid, we added successively greater quantities of sodium hydroxid, or beginning with sodium hydroxid, we added successively greater amounts of acid until the theoretical neutralization had been accomplished; or we began with pure acid and replaced this with more and more of the mono- di-, or trisodium phosphate until the opposite extreme of a pure alkali was reached; or we began with a definite concentration of any one of the phosphates and added progressively greater amounts of either acid or alkali. The results in all these expriments were practically the same. In 24 to 48 hours the gelatin attained its maximal swelling (practically). When the amount of swelling is plotted on the vertical and the changes in the composition of the solutions from acid through the mixtures of the mono-, di- and trisodium salts to pure alkali on the horizontal, a curve, roughly V-shaped, is obtained. Greatest swelling is observed in the pure acid solution and least in a solution consisting essentially of monosodium phosphate. From this point on, there is a gradual increase in the swelling of the gelatin until the disodium salt is passed, when there occurs a more abrupt rise until the trisodium salt is reached, beyond which the curve rises still more steeply until the sodium hydroxid end of the series is attained.

The swelling of gelatin in monosodium, disodium and trisodium citrate follows the same general laws as its swelling in the corresponding salts of phosphoric acid. Monosodium citrate in all concentrations increases somewhat the swelling of gelatin over the amount of swelling in pure water. The same is true of low concentrations of disodium citrate. But the higher concentrations of this salt depress the swelling to below that attained in pure water. These statements also hold for the trisodium salt. As we succeed in getting more base into the citrate, there appears a distinctly greater tendency to depress the amount of water absorption.

In studying the amounts of water absorbed in citrate mixtures varying between the extreme, on the one hand, of pure citric acid, through mono-, di- and trisodium citrate to pure sodium hydroxid, we observed that the results (when amount of swelling is plotted on the vertical and progressive change in composition of solution on the horizontal) yield a U-shaped curve. Greatest swelling is obtained in the pure acid, the amount of this swelling decreasing progressively as we approach the monosodium salt. From the monosodium to the disodium salt the curve falls more gently, until a minimal point is reached in a mixture of about equal parts of monosodium citrate and disodium citrate. From here on, the curve rises gradually to the trisodium salt, after which it ascends steeply as we pass toward the extreme of the pure alkali.

We have also studied in this fashion the effects of carbonate mixtures. As the sodium bicarbonate in a pure solution of this salt is gradually displaced by a molecularly equivalent amount of sodium carbonate, and this in its turn by an equivalent of sodium hydroxid, the amount of water absorbed gradually increases in the form of the right arm of the letter U. Swelling is least in the pure sodium bicarbonate, increases slowly in the sodium carbonate and then more rapidly as this is replaced by sodium hydroxid. The swelling of gelatin in pure sodium bicarbonate is slightly higher, in the concentration employed by us, than in pure water.

II

Practically the same findings as have been detailed for gelatin in the paragraphs given above were encountered when the swelling of fibrin was studied in different concentrations of the pure salts or in mixtures of these, varying between the extremes of acid on the one hand and alkali on the other.

III

It has been pointed out in previous papers[1] that the swelling of a protein and its liquefaction or "solution" are totally different processes. The "solution" of gelatin is, in other words, not merely the extreme or a continuation of the swelling of a protein. We were able to verify these results in studying, in parallel with the swelling of gelatin in polybasic acids and their salts, its "solution" under the same circumstances.

When gelatin containing a unit amount of water, and solid at ordinary room temperature, has mixed with it phosphoric acid, phosphate mixtures or sodium hydroxid in the concentrations already discussed above, it is found that the "solution" or liquefaction of the gelatin parallels its swelling. In other words, gelatin remains solid in phosphate mixtures of various kinds, but tends to lose in viscosity, to liquefy and to remain fluid as we pass from the phosphates in the direction either toward acid or toward alkali.

IV

We hold these experiments to be corroborative of, and to bear upon notions previously expressed regarding the importance of acids, of alkalies, of various salts and of these in mixture in determining the amount of water absorbed by protoplasm under physiological and pathological conditions. The well-established qualitative and quantitative analogy between the absorption of water by various hydrophilic colloids (like the proteins) and isolated cells, organs or organisms, whether of animal or vegetable origin, shows that protoplasmic water absorption is essentially a colloid-chemical phenomenon. These studies with polybasic acids and their salts therefore bring further proof of the importance of an abnormal production or accumulation of acids within such colloid systems for increasing the amount of water thus held, and so of explaining the mechanism by which the abnormally high hydrations of living cells are brought about as observed in edema, excessive turgor and plasmoptysis, or in those various "diseases" which are in essence only edemas of the involved organs like nephritis, glaucoma and "uremia." These experiments also show how coincident with, but not synonymous with the increased swelling there also occur a "softening"[2] and an increased "solution" of the colloids of the involved tissues, thus explaining further the "softening" of organs after an initial swelling together with the appearance of increased amounts of colloid (like protein) in the fluids bathing or expressed from the involved edematous tissues (albuminuria, excessive protein content of spinal fluid in edemas of the central nervous system, increased protein content of serous accumulations, etc.).

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[1] Martin H. Fischer, SCIENCE, N. S., Vol. XLII., p. 223 (1915); *Kolloid Zeitschr.*, Vol. XVII., p. 1 (1915).

[2] For a discussion of tissue softening as due to the breaking of an emulsion see Martin H. Fischer and Marian O. Hooker, SCIENCE, N. S., Vol. XLIII, p. 468 (1916); "Fats and Fatty Degeneration," 76, New York, 1917.