"National Research Council." Membership of committees.

EDWIN BIDWELL WILSON
MASSACHUSETTS INSTITUTE OF TECHNOLOGY, CAMBRIDGE, MASS.

## THE DECIMAL POINT

STUDENTS of the history of science are continually impressed by the fact that we are truly in scientific achievements the heirs of all the ages with progenitors not limited by any bounds of time or place. The historian John Fiske in one of his essays[1] pertinently remarks:

> The thinker who elaborates a new system of philosophy, deeper and more comprehensive than any yet known to mankind, though he may work in solitude, nevertheless does not work alone. The very fact which makes his great scheme of thought a success, and not a failure, is the fact that it puts into definite and coherent shape the ideas which many people are more or less vaguely entertaining, and that it carries to a grand and triumphant conclusion processes of reasoning in which many persons have already begun taking the earlier steps.

The late and lamented Pierre Duhem in that wonderful work, "Le Système du Monde," with the fifth and concluding volume fortunately completed in manuscript before his recent death, opens his work with a statement to the effect that to trace the origin and genesis of great scientific ideas one is gradually led back to the point where history ceases.

The development of decimal fractions furnishes an excellent illustration of the process mentioned. This story was well told in the *Teachers College Bulletin* of 1910 by David Eugene Smith, in an article entitled, "The invention of the demical fraction." I propose in the present paper to discuss briefly one point of the development, namely the appearance of the decimal point itself; I am adding also an early approach to decimal fractions not known to writers on the subject, based upon a study of a Vienna manuscript of the fifteenth century.[2]

[1] "A century of science, and other essays," New York, 1899.

[2] Codex Vindobonensis 4770.

N. L. W. A. Gravelaar, in discussing Napier's works[3] ascribes to Napier priority in the use of the decimal point. In a further article, "De notatie der decimale breuken," Gravelaar[4] purports to show that Napier was not familiar in 1616 and 1617, with the editions of the "Trigonometry" of Pitiscus which appeared in 1608 and 1612, containing the first appearance in print of the decimal point after Stevin's systematic exposition of the subject of decimal fractions in 1585. To me the whole procedure of Gravelaar borders so closely on the absurd that it would not merit discussion, if it had not been accepted somewhat seriously by other writers.[5]

Eneström, the editor of the Bibliotheca Mathematica, refers[6] to Gravelaar's work, as follows: "Nach Herrn Gravelaar ist Neper der erste, bei dem das Komma (Rhabdologia, 1617) und das Pünktchen (Constructio, 1616) als wirkliche Dezimalzeichen vorkommen; Pitiscus hatte zwar schon ein Pünktchen angewendet, aber dies ist nur als ein Scheidezeichen auzusehen." That the point used with decimal significance in Pitiscus is used with full appreciation by Pitiscus should be evident simply from the fact that at this time the work of Stevin on decimal fractions was widely known; further Pitiscus who uses a bar in the text of his "Trigonometry" of 1608 and of 1612, 13/00024 where the point is used in the Tables, explicitly says of 13/00024, "fractione scilicet 24/100000 neglecto," meaning that in place of 13.00024 he uses 13 as an approximation. Among others von Braunmühl, in his "Vorlesungen über Geschichte der Trigonometrie,"[7] mentions the use of the point in decimal fractions by Pitiscus.

The further fact should be noted that the Constructio of 1616 is the English transla-

[3] *Verh. d. Koningl. Akad. v. Wetenschappen*, Amsterdam, 1899, Deel. VI., No. 6.

[4] *Nieuw Archif voor Wiskunde*, Amsterdam, 1900.

[5] Notably by Glaisher and others in the "Napier Tercentenary Volume."

[6] *Bibliotheca Mathematica*, Vol. VI., third series, pp. 108–109.

[7] Vol. I., p. 225.

tion by Wright of Napier's work, and while it did receive examination by Napier before publication, yet the probability is that the innovation of the decimal point was introduced by Wright. To prove even independent development of the decimal point one would have to show that neither Wright nor Napier had the 1608 or 1912 edition of Pitiscus or any knowledge of the work; the earlier edition of Pitiscus is cited by Napier.

Cantor does refer[8] to the use *in manuscript*, as he carefully mentions, of the decimal point by Bürgi "wahrscheinlich kurz nach . . . August 1592"; even of this use in MS. Cantor says[9] "scheint zuerst," whereas of the use by Pitiscus, he says "angeführte Thatsache, dass Pitiscus in Tabellenanhange seiner Trigonometrie von 1608 sowie von 1612 . . . das Decimalstellen abtrennende Pünktchen benutzt hat." My own feeling is that Cantor mentions Bürgi's use of the point in MS. here to strengthen Cantor's similar claim on MS. evidence for priority for Bürgi in the invention of logarithms.[10]

So far as the complete explanation of decimal fractions is concerned that appears in the work of Stevin, "La Disme," of 1585, which work in Flemish and in French evidently attained rapidly wide circulation. In 1603 Johann Beyer in his "Logistica decimalis" gave the explanation of operations with decimal fractions, using a period in combination with Stevin's method of designating the last order of the last place by a superimposed numeral; thus, $8.798^{\text{V}}$ for 8.00798. Even in 1616 Kepler in his "Auszug auss der uralten Messe-Kunst Archimedis" used a comma (reversed) quite as we do a decimal point, and gives a sufficiently complete exposition of the use of decimal fractions.

It seems to me to be distinctly unfortunate that Gravelaar should have given, without any ground as I believe I have shown, credit to Napier for a somewhat important contribution to the theory of notation of decimal fractions; certain writers have been misled into according to Napier honor in this field, where no credit is due.

Except for the fact that a question of nationality is introduced we might say that neither Pitiscus nor Napier is worthy of more than a passing note in the history of the development of decimal fractions. Decimal fractions and a more or less convenient notation for the same were historically inevitable, just as logarithms, the analytic geometry, the graphical representation of complex numbers, the calculus, and many other developments of mathematics were inevitable. The earlier steps leading to these processes can now be traced and it is increasingly evident that a succession of thinkers made possible these attainments. Similarly the preparatory contributions of many minds have made possible the simultaneous discovery, frequently, of apparently new mathematical theories.

In the pamphlet by Professor Smith, which I have cited, there appears the reproduction of a page in Christoff Rudolff's arithmetic of 1526 in which there is the computation of interest at 5 per cent., involving fundamentally decimal fractions, using a bar as a separatrix. Concerning this Enestrom says:[11]

In the year 1492 Francesco Pellizzati (or Pellos) published at Turin an arithmetic in which, in one example, division by 100 of 6976587 is given by 69765.87; the point is also used in dividing by 30, 400, 3000, and the like, pointing off one, two, or three places and afterwards writing the remainder as an ordinary fraction. Pellizzati can, however, not be credited with any real comprehension of decimal fractions. The page in question

[8] "Vorlesungen," Vol. II., second edition, p. 618.

[9] *Loc. cit.*, pp. 617, 619.

[10] See Cajori, "Napier Tercentenary Volume," pp. 101–102.

[11] *Bibliotheca Mathematica*, Vol. X., p. 243, in his article, "Über das angebliche Dezimalbruchzeichen einiger der ältesten gedruckten Rechenbücher."

Hier hat Rudolff wirklich mit Dezimalbrüchen gerechnet; wenn man ferner bemerkt, dass der Strich bei Rudolff gewissermassen ein Komma ist . . . so kann man sagen, dass sowohl die moderne Anwendung wie die moderne Bezeichnung bei Rudolff vorkommt . . . ohne dass Rudolff die Tragweite seines Verfahrens verstanden hat.

is reproduced by Smith in his article, abovementioned, and in the "Rara Arithmetica."

Some time ago in discussing before the American Mathematical Society the "Quadripartitum" of John of Meurs, written about A.D. 1325, I called attention to the fact that this writer also should be included among those who by use of an analogy with the sexagesimal fractions made a near approach to decimal fractions.[12] In the passage in question, which occurs in the twenty-second chapter of the second book, dealing with the extraction of square root, zeros in pairs are annexed to the number whose approximate root is desired. In extracting the square root of 2, six ciphers are annexed to the 2 and the square root of 2000000 is obtained as 1414. At first this is changed to sexagesimal fractions and by successive multiplication of remainders by 60 the root is obtained as 1° (gradus), 24 minutes, 50 seconds, and 24 thirds, or 1 24/60 50/3600 24/216000. The manuscript proceeds, as follows: "Et si vis posses dicere ab inicio cum habuisti hanc radicem 1414 quod unitas que est in quarto loco est sicut integrum et 4 que sunt in tercio loco sunt decima pars integri, et unitas que est in secundo loco est decima decium, et 4 que sunt in primo loco sunt decima decime decime sic 1·4·1·4. Et nota quod istam radicem preinuentam 1414 potes multum bene ponita unitate in loco integrum tres differentias remanentes multiplicare per 10, 20, 30, 40, 50, 60, 70, 80, 90 et semper a producto demptis figuris que excedunt numerum medietatis circulorum, residuas multiplica per illum articulum per quem alias extendisti et hoc facto tociens quod non remaneant nisi tres circuli qui sunt medietas circulorum. Habebis radicem secundum proporcionem integri ad articulum per quem fueris operatus." The statement here that in the 1414 (regarded as the square root of 2, evidently) that the first unit (fourth as counted from the right or ordinary unit's place) is to be regarded as an integer, the following 4 as tenth parts of an integer, the following unit as the tenth part of a tenth, and the final (first, counting in the ordinary way from the right) 4 as tenth parts of the tenth part of one tenth borders closely on the idea of decimal fractions. However, John of Meurs, like so many others who made initial steps in this direction, carried the idea no further.

LOUIS C. KARPINSKI
UNIVERSITY OF MICHIGAN

[12] I am using photographic copies of pages of the Vienna MS. 4770; the passage in question is found on folio 224 verso; the manuscript was probably written in the early part of the fifteenth century, containing also Robert of Chester's translation of the algebra of Al Khowarizmi, recently published in the University of Michigan Humanistic Series.

## THE SHORTAGE OF PLATINUM

ACCORDING to a bulletin issued by the U. S. Geological Survey most people are apt to think of platinum as preeminently adapted to settings for precious stones, but the metal is in fact indispensable to many essential industries. Platinum dishes and utensils are absolutely needed in all chemical laboratories, and upon these laboratories all great industries are dependent for guidance. Alloys have been devised for use in the ignition systems of internal-combustion engines, but no substitute for platinum has been found for certain delicate parts of these systems. Platinum and allied rare metals are widely employed in instruments of precision required for making physical tests of materials of all kinds. Probably platinum is now most valuable for its use in the contact process of making concentrated sulphuric acid, which is essential to a great number of industries that are vitally important at all times, and particularly in time of war.

The United States alone annually uses about 165,000 ounces of fine platinum and produces less than 1,000 ounces of crude platinum. Realizing the urgent necessity of increasing the country's production of the metals of the platinum group, the United States Geological Survey, Department of the Interior, has planned an investigation in which L. M. Prindle and J. M. Hill, geologist, will visit places in this country where commercial deposits of these metals may be found.

Native platinum, the metal and sperrylite (platinum arsenide) have been found in basic