

thors seem to have a mania for scattering literary fragments and may cultivate foreign journals merely for the sake of personal advertising.

Entomological News, 28: 141, after mentioning four journals which lasted an average of five years each, says:

In general it seems that the number of specialists in any one or two orders of insects is not sufficient to support a special journal, and we know of none such provided with an endowment fund guaranteeing its permanency. In this matter we must still be entomologists, apparently, and yet the record of general entomological journals contains many a short-lived periodical.

The significant point here is that, while we are specialists as regards the literature we desire, we are general entomologists as regards the literature we have to pay for.

As educational institutions the university is local, while the printing press is cosmopolitan, the only cosmopolitan university. The publication of scientific literature should not be supported by requiring specialists to pay for literature they do not need, any more than the university should depend for its entire support upon the tuition of its students.

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FUNDAMENTAL CONCEPTIONS OF MODERN MATHEMATICS

TO THE EDITOR OF SCIENCE: In your issue of August 4, there appeared a review of the first part of our "Fundamental Conceptions of Modern Mathematics," from the pen of Professor G. A. Miller. Against a hostile criticism, giving a portrayal of at least some of the main theses of our book and attempting to controvert them, we would have no inclination to protest. But all the important issues raised by our treatise are ignored by Professor Miller, who dwells upon features having no bearing upon any of the arguments of our work, or upon any of the doctrines which it is the purpose of the arguments to uphold.

Surely a reviewer can be justly expected to take up at least one or two of the principal doctrines of a treatise of which he disapproves,

and show that these doctrines are erroneous. Our book contains an account of quantities and their classification; an investigation into what the symbols used by mathematicians really stand for. We set forth the classification of quantities into what we call sorts, kinds and varieties, and show the importance of this classification in the subdivision (originally conceived by De Morgan) of algebraic science into single algebra, double algebra, etc. A precise statement is given of what we apprehend to be the nature of the quantities dealt with in quaternions and other systems of vector analysis, and of their relation to the quantities of ordinary algebra. We attempt to show that any really scientific treatment of ordinary imaginary quantities must be based on vector analysis, all imaginary and complex abstract quantities (save those of zero value) being, in fact, relations between vectors. This is, we hold, the only way to ascend, from a blind use of imaginary and complex expressions without any clear apprehension of what they denote, to a rational comprehension of the matter; in other words, from mere computation, and manipulation of symbols, to true science. We show further that the mathematicians who look upon a variable as a quantity and those who regard it as a symbol are equally in the wrong; a variable being represented by a symbol and being composed of quantities. We consider the arrangement of the quantities of a variable, and show the importance of this commonly neglected attribute. We discuss the peculiar arrangements which must be at hand to justify the application of the theory of monogenic functions, and show the relation of these multiplex arrangements (as we call them) to the arrangements of the elements of the aggregates designated by Cantor as *mehrfach geordnet*. As the simplest of variables we put forward the ordinary progressions of elementary mathematics which are not usually recognized as variables at all. We attempt to show clearly just what distinctions should be drawn between a progression and a series; and, including all progressions and all series under the head of sequences, lay down the conditions under which a variable is to be

classified as a sequence. Passing to the question of functional relation we take up independent and dependent variables, and show that these names correspond to at least three separate distinctions, a fact not ordinarily recognized by mathematicians. We give a somewhat elaborate discussion of functional relations, showing that what is ordinarily put forward as the Dirichlet definition of function does not adequately characterize a functional relation, and moreover is not really the definition given by Dirichlet. We lay down what we deem to be the conditions under which two or more variables may be said to be in functional relation with each other; and show that previous authors, in their treatment of functions, have not attained to a clear and precise view of the essential characteristics of a functional relation.

To these salient features of our work Professor Miller gives no heed whatsoever, though assuredly they comprise topics of fundamental importance in mathematics. He is content to dismiss our inquiries by stigmatizing them as relating chiefly "to definitions and the choice of words." We plead that our work is concerned chiefly with the unfolding of the conceptions which words should awaken in our minds, and not with the words themselves. To purely verbal questions we give scant attention. In our endeavors to attain to distinct and exact conceptions of what is fundamental to the inquiries of mathematics, we have found that the portrayal of these conceptions, as set forth by mathematicians of the highest eminence, are not free from great imperfections. We have spared no labor in obtaining and in stating in full "the definitions given by those who have made important advances in the fields" into which we go; and when unable to assent to these definitions, we have carefully set down our reasons for holding that they do not truly depict the lineaments of the conceptions which they purport to unfold. And Professor Miller, though manifesting his disapproval of our criticisms, makes not the slightest attempt to show that our charges of error are baseless, and that Baire, Pringsheim, Riemann, Russell, Weber, and the other

authorities whom we controvert are not guilty of the errors we ascribe to them.

Three passages of our work are specifically condemned by Professor Miller. All of these are trivial and could be removed from the work without affecting any of its doctrines or any major or minor argument put forward in defense of them. One fault that is imputed to us is that "on page 177 and elsewhere, the common erroneous assumption according to which the word function was used as synonymous with power is repeated." We merely say that "the word function *is said* to have been used by the older analysts as synonymous with power." We took care to insert the qualifying phrase *is said*, and so worded our remark is neither an assumption nor an error. And no reference to this usage occurs elsewhere. We are also rebuked for saying that "The only mathematician that we recall as making a specific distinction between quotient and ratio is Hamilton." We must acknowledge that such a distinction has been made by others, but we deny that the distinction we endeavor to enforce is as common as Professor Miller would have it appear. Finally we are chided for applying to imaginary and complex quantities the distinction between positives and negatives. Yet, if precedent is to be a guide, we can plead that both Gauss and Weierstrass used the two adjectives with respect to imaginary quantities.

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QUOTATIONS

SCIENTIFIC SNOBBERY

ONE reason for the neglect of science is that scientific men themselves frequently misrepresent the objects for which they work. For example, they often pretend that they perform their labors merely for their own amusement. We once heard it wittily said of such a man that he takes out his watch before dinner and exclaims, "Ha! I have half an hour before I must dress for dinner; I will just step over to my laboratory and make a discovery." But the public is not so easily deceived and therefore thinks in its dull way that the man of