saved together with part of the apparatus. Chancellor Elliott has announced that a new chemistry building will be erected as soon as possible.

FINAL plans have been drawn for a head house for the school of applied science of the Carnegie Institute of Technology, which is to cost $300,000. A portion of the building will be four stories high and the remainder ten. Construction work will start as soon as steel deliveries can be made. The structure will house the executive offices and library of the engineering school, and the departments of modern languages, machine design and commercial engineering.

DR. H. E. EGGERS has been appointed professor of pathology and bacteriology, Dr. Amos W. Peters, assistant professor of biochemistry, and Dr. John T. Myers, instructor in bacteriology, in the college of medicine of the University of Nebraska, Omaha.

PROFESSOR J. VERSLUYS, who has held the chair of zoology and comparative anatomy at Giessen since 1907, has been appointed to the corresponding chair in the new Flemish University at Genth.

THE *Journal* of the American Medical Association indicates that negotiations are pending that may bring Professor R. Bárany, of Vienna, to the University of Stockholm as professor of otology and rhinolaryngology. He recently delivered at Stockholm the customary address describing his research when presented with the Nobel prize. It will be remembered that he was a war prisoner in Russia when notified that the prize in medicine had been conferred on him.

DISCUSSION AND CORRESPONDENCE

CAN A BODY EXERT A FORCE UPON ITSELF?

IN connection with our annual attempt to give our students a few clear ideas about elementary dynamics, the question of the meaning to be assigned to the word *force* perennially arises. May I call attention to a well-known phenemenon which seems well suited to serve as a shibboleth in distinguishing between clear and hazy conceptions of force?

Let a liquid be uniformly rotated in an open vessel. What are the forces acting on each surface particle? Why is the free surface parabolic?

In answering these questions one recent author finds it necessary unwittingly to deny all three of the laws of motion. He states that "When a liquid is at rest or in equilibrium *the resultant of all the forces acting on a particle in its free surface is perpendicular to the surface at that point*" [whereas according to the first law the resultant force must be zero]. In the case of a rotating liquid, we are told, "the resultant force acting on the surface particles is due not only to gravity, but to centrifugal force. . . . It will be noted that the resultant force [shown drawn perpendicular to the free surface] is greater at points higher up on the surface, so that a surface particle near the top presses against the surrounding liquid with far more force than it would if at the bottom of the curve." But according to the second law the resultant force must be in the direction of the resultant acceleration, which in this case is obviously centripetal; and according to the third law, if the particle presses against the surrounding liquid, the liquid must press back upon it with an equal and opposite force not mentioned by the author.

Such an explanation is evidently completely misleading. Yet another recent text-book does equal violence to the laws of motion in explaining the same phenomenon. "The resultant force," we are told, "is made up of two components; one of these is the weight of the particle, mg, the other is the reaction which the particle offers against acceleration toward the center by the centripetal force $mr\omega^2$."

Of course the trouble is that among mathematical physicists it has been customary to reduce such problems to purely statical ones by introducing centrifugal forces in accordance with D'Alembert's principle; but authors of elementary texts sometimes forget that the forces so introduced are purely imaginary.

Does not the third law mean this: A body A can not exert a force upon itself as a whole; any force acting on it must be due to, that is, associated with, the existence of, some other body or medium *B;* and that other body or medium *B* while exerting a force on *A,* is experiencing an equal and opposite force due to *A*; whenever the existence of a force on *A* is discovered we should immediately seek out the body or medium *B* which is the other party to the transaction; whenever a force is mentioned, the body or medium exerting the force should be clearly in mind.

Considered from this point of view, the answers to the above questions regarding a rotating liquid would run somewhat as follows: The forces acting on a water particle in the free surface are (1) its weight, *due to* the earth, (2) a force *due to* the liquid in contact with it, and (3) a force normal to the surface, *due to* the atmosphere. The resultant of these is a centripetal force since the acceleration is centripetal. If we can prove that the second force is normal to the free surface, then it follows immediately from the force triangle that the normal to the surface makes an angle with the axis of spin whose tangent is equal to the ratio of $r\omega^2$ to g, and that the section of the free surface is parabolic.

The proof we need is the following: Suppose a closed, cylindrical can, full of liquid and with its bottom horizontal, is uniformly rotating around the vertical axis of symmetry. On any co-axial cylindrical surface within the liquid with a radius r there is a pressure because of the rotation equal to $\frac{1}{2}\rho r^2\omega^2$ per cm.^{2}; at any height y above the bottom there is also a hydrostatic pressure due to gravity equal to $P - \rho gy$. The equation for a surface of constant pressure within the liquid is therefore

$$\tfrac{1}{2}\rho r^2\omega^2 + P - \rho gy = \text{constant},$$

$$r^2\omega^2 - 2gy = \text{constant}.$$

But the force on any particle due to the surrounding liquid is, of course, normal to the surface of constant pressure at that point. If we now suppose the can opened on top and all the liquid within a surface of constant pressure removed, the pressure formerly exerted by the removed liquid would be supplied by the atmosphere and the remaining liquid would continue to rotate exactly as before. Thus the free surface of our rotating liquid must coincide with a surface of constant pressure, and the force on a surface particle due to the liquid in contact with it (including surface tension), being normal to the surface of constant pressure, is normal to the free surface. In a similar manner the more general proposition may be proved that the free surface of any liquid whose particles remain at a constant distance from each other during any motion, is *normal to the force with which the liquid acts on the surface particles at each point,* and is not, as often stated, normal to the resultant force acting on them.

When a student finds in an elementary text the statement that "when a body *is* accelerated we may consider the force of reaction as one of the forces acting upon the body," and is told that one of the forces acting on one of the masses of an Atwood's machine, m_1, is "the reaction of the mass m_1 against its upward acceleration" [which is equivalent to the statement that a body when accelerated acts upon itself with a force *ma,* so that the resultant force is always zero]—when a student tries to reconcile such assertions with the laws of motion, is it surprising that he becomes confused and discouraged?

Why not use force only in the single definite sense implied in the laws of motion?

The fact that the two authors quoted are unusually experienced and successful teachers suggests that they are not the only ones who are making the path of freshmen unnecessarily difficult. I have taken the liberty of using them as "horrible examples" in this respect because their text-books are for the most part admirably clear, and because I know them to be men who are big enough not to resent well-meant criticism.

If there is any question as to the wisdom of the conclusion suggested above, let us thrash the matter out now. To avoid misunderstanding, let me add that in using the phrase "force due to—" for the sake of brevity, no relation of cause and effect is implied in any critical philosophical sense.

Gordon S. Fulcher
University of Wisconsin,
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