Once more, we must never lose sight of the fact that from the Early Aurignacian Period onwards a negroid element in the broadest sense of the word shared in this artistic culture as seen on both sides of the Pyrenees.

At least we now know that cave man did not suffer any sudden extinction, though on the European side, partly, perhaps, owing to the new climatic conditions, this culture underwent a marked degeneration. It may well be that, as the osteological evidence seems to imply, some outgrowth of the old Cro-Magnon type actually perpetuated itself in the Dordogne. We have certainly lengthened our knowledge of the Palæolithic. But in the present state of the evidence it seems better to subscribe to Cartailhac's view that its junction with the Neolithic has not yet been reached. There does not seem to be any real continuity between the culture revealed at Maglemose and that of the immediately superposed Early Neolithic stratum of the shell-mounds, which, moreover, as has been already said, evidence a change both in climatic and geological conditions, implying a considerable interval of time.

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(To be continued)

THE ORGANIZATION OF THOUGHT[1]

THE subject of this address is the organization of thought, a topic evidently capable of many diverse modes of treatment. I intend more particularly to give some account of that department of logical science with which some of my own studies have been connected. But I am anxious, if I can succeed in so doing, to handle this account so as to exhibit the relation with certain considerations which underlie general scientific activities.

It is no accident that an age of science has developed into an age of organization. Organized thought is the basis of organized action. Organization is the adjustment of diverse elements so that their mutual relations may exhibit some predetermined quality. An epic poem is a triumph of organization, that is to say, it is a triumph in the unlikely event of it being a good epic poem. It is the successful organization of multitudinous sounds of words, associations of words, pictorial memories of diverse events and feelings ordinarily occurring in life, combined with a special narrative of great events: the whole so disposed as to excite emotions which, as defined by Milton, are simple, sensuous and passionate. The number of successful epic poems is commensurate, or rather, is inversely commensurate with the obvious difficulty of the task of organization.

Science is the organization of thought. But the example of the epic poem warns us that science is not any organization of thought. It is an organization of a certain definite type which we will endeavor to determine.

Science is a river with two sources, the practical source and the theoretical source. The practical source is the desire to direct our actions to achieve predetermined ends. For example, the British nation, fighting for justice, turns to science, which teaches it the importance of compounds of nitrogen. The theoretical source is the desire to understand. Now I am going to emphasize the importance of theory in science. But to avoid misconception I most emphatically state that I do not consider one source as in any sense nobler than the other, or intrinsically more interesting. I can not see why it is nobler to strive to understand than to busy oneself with the right ordering of one's

[1] Address of the president of the Mathematical and Physical Science Section, British Association for the Advancement of Science, Newcastle-on-Tyne, 1916.

actions. Both have their bad sides; there are evil ends directing actions, and there are ignoble curiosities of the understanding.

The importance, even in practise, of the theoretical side of science arises from the fact that action must be immediate, and takes place under circumstances which are excessively complicated. If we wait for the necessities of action before we commence to arrange our ideas, in peace we shall have lost our trade, and in war we shall have lost the battle.

Success in practise depends on theorists who, led by other motives of exploration, have been there before, and by some good chance have hit upon the relevant ideas. By a theorist I do not mean a man who is up in the clouds, but a man whose motive for thought is the desire to formulate correctly the rules according to which events occur. A successful theorist should be excessively interested in immediate events, otherwise he is not at all likely to formulate correctly anything about them. Of course, both sources of science exist in all men.

Now, what is this thought organization which we call science? The first aspect of modern science which struck thoughtful observers was its inductive character. The nature of induction, its importance, and the rules of inductive logic have been considered by a long series of thinkers, especially English thinkers, Bacon, Herschel, J. S. Mill, Venn, Jevons and others. I am not going to plunge into an analysis of the process of induction. Induction is the machinery and not the product, and it is the product which I want to consider. When we understand the product we shall be in a stronger position to improve the machinery.

First, there is one point which it is necessary to emphasize. There is a tendency in analyzing scientific processes to assume a given assemblage of concepts applying to nature, and to imagine that the discovery of laws of nature consists in selecting by means of inductive logic some one out of a definite set of possible alternative relations which may hold between the things in nature answering to these obvious concepts. In a sense this assumption is fairly correct, especially in regard to the earlier stages of science. Mankind found itself in possession of certain concepts respecting nature—for example, the concept of fairly permanent material bodies—and proceeded to determine laws which related the corresponding percepts in nature. But the formulation of laws changed the concepts, sometimes gently by an added precision, sometimes violently. At first this process was not much noticed, or at least was felt to be a process curbed within narrow bounds, not touching fundamental ideas. At the stage where we now are, the formulation of the concepts can be seen to be as important as the formulation of the empirical laws connecting the events in the universe as thus conceived by us. For example, the concepts of life, of heredity, of a material body, of a molecule, of an atom, of an electron, of energy, of space, of time, of quantity, and of number. I am not dogmatizing about the best way of getting such ideas straight. Certainly it will only be done by those who have devoted themselves to a special study of the facts in question. Success is never absolute, and progress in the right direction is the result of a slow, gradual process of continual comparison of ideas with facts. The criterion of success is that we should be able to formulate empirical laws, that is, statements of relations, connecting the various parts of the universe as thus conceived, laws with the property that we can interpret the actual events of our lives as being our fragmentary knowledge of this conceived interrelated whole.

But, for the purposes of science, what is the actual world? Has science to wait for the termination of the metaphysical debate till it can determine its own subject-matter? I suggest that science has a much more homely starting-ground. Its task is the discovery of the relations which exist within that flux of perceptions, sensations and emotions which forms our experience of life. The panorama yielded by sight, sound, taste, smell, touch and by more inchoate sensible feelings, is the sole field of its activity. It is in this way that science is the thought organization of experience. The most obvious aspect of this field of actual experience is its disorderly character. It is for each person a *continuum*, fragmentary, and with elements not clearly differentiated. The comparison of the sensible experiences of diverse people brings its own difficulties. I insist on the radically untidy, ill-adjusted character of the fields of actual experience from which science starts. To grasp this fundamental truth is the first step in wisdom, when constructing a philosophy of science. This fact is concealed by the influence of language, moulded by science, which foists on us exact concepts as though they represented the immediate deliverances of experience. The result is that we imagine that we have immediate experience of a world of perfectly defined objects implicated in perfectly defined events which, as known to us by the direct deliverance of our senses, happen at exact instants of time, in a space formed by exact points, without parts and without magnitude: the neat, trim, tidy, exact world which is the goal of scientific thought.

My contention is that this world is a world of ideas, and that its internal relations are relations between abstract concepts, and that the elucidation of the precise connection between this world and the feelings of actual experience is the fundamental question of scientific philosophy. The question which I am inviting you to consider is this: How does exact thought apply to the fragmentary, vague *continua* of experience? I am not saying that it does not apply, quite the contrary. But I want to know how it applies. The solution I am asking for is not a phrase, however brilliant, but a solid branch of science, constructed with slow patience, showing in detail how the correspondence is effected.

The first great steps in the organization of thought were due exclusively to the practical source of scientific activity, without any admixture of theoretical impulse. Their slow accomplishment was the cause and also the effect of the gradual evolution of moderately rational beings. I mean the formation of the concepts of definite material objects, of the determinate lapse of time, of simultaneity, of recurrence, of definite relative position, and of analogous fundamental ideas, according to which the flux of our experiences is mentally arranged for handy reference: in fact, the whole apparatus of common-sense thought. Consider in your mind some definite chair. The concept of that chair is simply the concept of all the interrelated experiences connected with that chair—namely, of the experiences of the folk who made it, of the folk who sold it, of the folk who have seen it, or used it, of the man who is now experiencing a comfortable sense of support, combined with our expectations of an analogous future, terminated finally by a different set of experiences when the chair collapses and becomes fire-wood. The formation of that type of concept was a tremendous job, and zoologists and geologists tell us that it took many tens of millions of years. I can well believe it.

I now emphasize two points. In the first place, science is rooted in what I have just called the whole apparatus of common-sense thought. That is the *datum* from which it starts, and to which it must recur. We may speculate, if it amuses us, of other beings in other planets who have arranged analogous experiences according to an entirely different conceptual code—namely, who have directed their chief attention to different relations between their various experiences. But the task is too complex, too gigantic, to be revised in its main outlines. You may polish up common sense, you may contradict it in detail, you may surprise it. But ultimately your whole task is to satisfy it.

In the second place, neither common sense nor science can proceed with their task of thought organization without departing in some respect from the strict consideration of what is actual in experience. Think again of the chair. Among the experiences upon which its concept is based, I included our expectations of its future history. I should have gone further and included our imagination of all the possible experiences which in ordinary language we should call perceptions of the chair which might have occurred. This is a difficult question, and I do not see my way through it. But at present in the construction of a theory of space and of time, there seem insuperable difficulties if we refuse to admit ideal experiences.

This imaginative perception of experiences, which, if they occurred, would be coherent with our actual experiences, seems fundamental in our lives. It is neither wholly arbitrary, nor yet fully determined. It is a vague background which is only made in part definite by isolated activities of thought. Consider, for example, our thoughts of the unseen flora of Brazil.

Ideal experiences are closely connected with our imaginative reproduction of the actual experiences of other people, and also with our almost inevitable conception of ourselves as receiving our impressions from an external complex reality beyond ourselves. It may be that an adequate analysis of every source and every type of experience yields demonstrative proof of such a reality and of its nature. Indeed, it is hardly to be doubted that this is the case. The precise elucidation of this question is the problem of metaphysics. One of the points which I am urging in this address is that the basis of science does not depend on the assumption of any of the conclusions of metaphysics; but that both science and metaphysics start from the same given groundwork of immediate experience, and in the main proceed in opposite directions on their diverse tasks.

For example, metaphysics inquires how our perceptions of the chair relate us to some true reality. Science gathers up these perceptions into a determinate class, adds to them ideal perceptions of analogous sort, which under assignable circumstances would be obtained, and this single concept of that set of perceptions is all that science needs; unless indeed you prefer that thought find its origin in some legend of those great twin brethren, the cock and bull.

My immediate problem is to inquire into the nature of the texture of science. Science is essentially logical. The nexus between its concepts is a logical nexus, and the grounds for its detailed assertions are logical grounds. King James said, "No bishops, no king." With greater confidence we can say, "No logic, no science." The reason for the instinctive dislike which most men of science feel towards the recognition of this truth is, I think, the barren failure of logical theory during the past three or four centuries. We may trace this failure back to the worship of authority which in some respects increased in the learned world at the time of the Renaissance. Mankind then changed its authority, and this fact temporarily acted as an emancipation. But the main fact, and we can find complaints[2] of it at the very commencement of the modern movement, was the establishment of a reverential attitude towards any statement made by a classical author. Scholars became commentators on truths too fragile to bear translation. A science which hesitates to forget its founders is lost. To this hesitation I ascribe the barrenness of logic. Another reason for distrust of logical theory and of mathematics is the belief that deductive reasoning can give you nothing new. Your conclusions are contained in your premises, which by hypothesis are known to you.

In the first place this last condemnation of logic neglects the fragmentary, disconnected character of human knowledge. To know one premise on Monday, and another premise on Tuesday, is useless to you on Wednesday. Science is a permanent record of premises, deductions and conclusions, verified all along the line by its correspondence with facts. Secondly, it is untrue that when we know the premises we also know the conclusions. In arithmetic, for example, mankind are not calculating boys. Any theory which proves that they are conversant with the consequences of their assumptions must be wrong. We can imagine beings who possess such insight. But we are not such creatures. Both these answers are, I think, true and relevant. But they are not satisfactory. They are too much in the nature of bludgeons, too external. We want something more explanatory of the very real difficulty which the question suggests. In fact, the true answer is embedded in the discussion of our main problem of the relation of logic to natural science.

[2] *E. g.*, in 1551 by Italian schoolmen.

It will be necessary to sketch in broad outline some relevant features of modern logic. In doing so I shall try to avoid the profound general discussions and the minute technical classifications which occupy the main part of traditional logic. It is characteristic of a science in its earlier stages—and logic has become fossilized in such a stage—to be both ambitiously profound in its aims and trivial in its handling of details. We can discern four departments of logical theory. By an analogy which is not so very remote I will call these departments or sections the arithmetic section, the algebraic section, the section of general-function theory, the analytic section. I do not mean that arithmetic arises in the first section, algebra in the second section, and so on; but the names are suggestive of certain qualities of thought in each section which are reminiscent of analogous qualities in arithmetic, in algebra, in the general theory of a mathematical function, and in the analysis of the properties of particular functions.

The first section—namely, the arithmetic stage—deals with the relations of definite propositions to each other, just as arithmetic deals with definite numbers. Consider any definite proposition; call it "p." We conceive that there is always another proposition which is the direct contradictory to "p"; call it "not-p." When we have got two propositions, p and q, we can form derivative propositions from them, and from their contradictories. We can say, "At least one of p or q is true, and perhaps both." Let us call this proposition "p or q." I may mention as an aside that one of the greatest living philosophers has stated that this use of the word "or"—namely, "p or q" in the sense that either or both may be true—makes him despair of exact expression. We must brave his wrath, which is unintelligible to me.

We have thus got hold of four new propositions, namely, "p or q," and "not-p or q," and "p or not-q," and "not-p or not-q." Call these the set of disjunctive derivatives. There are, so far, in all eight propositions p, not-p, q, not-q, and the four disjunctive derivatives. Any pair of these eight propositions can be taken, and substituted for p and q in the foregoing treatment. Thus each pair yields eight propositions, some of which may have been obtained before. By proceeding in this way we arrive at an unending set of propositions of growing complexity, ultimately derived from the two original propositions p or q. Of course, only a few are important. Similarly we can start from three propositions, p, q, r, or from four propositions, p, q, r, s, and so on. Any one of the propositions of these aggregates may be true or false. It has no other alternative. Whichever it is, true or false, call it the "truth-value" of the proposition.

The first section of logical inquiry is to settle what we know of the truth-values of these propositions, when we know the truth-values of some of them. The inquiry, so far as it is worth while carrying it, is not very abstruse, and the best way of expressing its results is a detail which I will not now consider. This inquiry forms the arithmetic stage.

The next section of logic is the algebraic stage. Now, the difference between arithmetic and algebra is that in arithmetic definite numbers are considered, and in algebra symbols—namely, letters—are introduced which stand for any numbers. The idea of a number is also enlarged. These letters, standing for any numbers, are called sometimes variables and sometimes parameters. Their essential characteristic is that they are undetermined, unless, indeed, the algebraic conditions which they satisfy implicitly determine them. Then they are sometimes called unknowns. An algebraic formula with letters is a blank form. It becomes a determinate arithmetic statement when definite numbers are substituted for the letters. The importance of algebra is a tribute to the study of form. Consider now the following proposition,

The specific heat of mercury is 0.033.

This is a definite proposition which, with certain limitations, is true. But the truth-value of the proposition does not immediately concern us. Instead of mercury put a mere letter which is the name of some undetermined thing: we get

The specific heat of x is 0.033.

This is not a proposition; it has been called by Russell a propositional function. It is the logical analogy of an algebraic expression. Let us write $f(x)$ for any propositional function.

We could also generalize still further, and say

The specific heat of x is y.

We thus get another propositional function, $F(x, y)$ of two arguments x and y, and so on for any number of arguments.

Now, consider $f(x)$. There is the range of values of x, for which $f(x)$ is a proposition, true or false. For values of x outside this range, $f(x)$ is not a proposition at all, and is neither true nor false. It may have vague suggestions for us, but it has no unit meaning of definite assertion. For example,

The specific heat of water is 0.033

is a proposition which is false; and

The specific heat of virtue is 0.033

is, I should imagine, not a proposition at all; so that it is neither true nor false, though its component parts raise various associations in our minds. This range of values, for which $f(x)$ has sense, is called the "type" of the argument x.

But there is also a range of values of x for which $f(x)$ is a true proposition. This is the class of those values of the argument which *satisfy* $f(x)$. This class may have no members, or, in the other extreme, the class may be the whole type of the arguments.

We thus conceive two general propositions respecting the indefinite number of propositions which share in the same logical form, that is, which are values of the same propositional function. One of these propositions is

$f(x)$ yields a true proposition for each value of x of the proper type;

the other proposition is

There is a value of x for which $f(x)$ is true.

Given two, or more, propositional functions $f(x)$ and $\phi(x)$ with the same argument x, we form derivative propositional functions, namely,

$$f(x) \text{ or } \phi(x),\ f(x) \text{ or not-}\phi(x),$$

and so on with the contradictories, obtaining, as in the arithmetical stage, an unending aggregate of propositional functions. Also each propositional function yields two general propositions. The theory of the interconnection between the truth-values of the general propositions arising from any such aggregate of propositional functions forms a simple and elegant chapter of mathematical logic.

In this algebraic section of logic the theory of types crops up, as we have already noted. It can not be neglected without the introduction of error. Its theory has to be settled at least by some safe hypothesis, even if it does not go to the philosophic basis of the question. This part of the subject is obscure and difficult, and has not been finally elucidated, though Russell's brilliant work has opened out the subject.

The final impulse to modern logic comes from the independent discovery of the importance of the logical variable by Frege and Peano. Frege went further than Peano, but by an unfortunate symbolism rendered his work so obscure that no one fully recognized his meaning who had not found it out for himself. But the movement has a large history reaching back to Leibniz and even to Aristotle. Among English contributors are De Morgan, Boole and Sir Alfred Kempe; their work is of the first rank.

The third logical section is the stage of general-function theory. In logical language, we perform in this stage the transition from intension to extension, and investigate the theory of denotation. Take the propositional function $f(x)$. There is the class, or range of values for x, whose members satisfy $f(x)$. But the same range may be the class whose members satisfy another propositional function $\phi(x)$. It is necessary to investigate how to indicate the class by a way which is indifferent as between the various propositional functions which are satisfied by any member of it, and of it only. What has to be done is to analyze the nature of propositions about a class—namely, those propositions whose truth-values depend on the class itself and not on the particular meaning by which the class is indicated.

Furthermore, there are propositions about alleged individuals indicated by descriptive phrases: for example, propositions about "the present King of England," who does exist, and "the present Emperor of Brazil," who does not exist. More complicated, but analogous, questions involving propositional functions of two variables involve the notion of "correlation," just as functions of one argument involve classes. Similarly functions of three arguments yield three-cornered correlations, and so on. This logical section is one which Russell has made peculiarly his own by work which must always remain fundamental. I have called this the section of functional theory, because its ideas are essential to the construction of logical denoting functions which include as a special case ordinary mathematical functions such as sine, logarithm, etc. In each of these three stages it will be necessary gradually to introduce an appropriate symbolism, if we are to pass on to the fourth stage.

The fourth logical section, the analytic stage, is concerned with the investigation of the properties of special logical constructions, that is, of classes and correlations of special sorts. The whole of mathematics is included here. So the section is a large one. In fact, it is mathematics, neither more nor less. But it includes an analysis of mathematical ideas not hitherto included in the scope of that science, nor, indeed, contemplated at all. The essence of this stage is construction. It is by means of suitable constructions that the great framework of applied mathematics, comprising the theories of number, quantity, time and space, is elaborated.

It is impossible even in brief outline to explain how mathematics is developed from the concepts of class and correlation, including many-cornered correlations, which are established in the third section. I can only allude to the headings of the process which is fully developed in the work, "Mathematica Principia," by Mr. Russell and myself. There are in this process of development seven special sorts of correlations which are of peculiar interest. The first sort comprises one-to-many, many-to-one, and one-to-one correlations. The second sort comprises serial relations, that is, correlations by which the members of some field are arranged in a serial order, so that, in the sense defined by the relation, any member of the field is either before or after any other member. The third class comprises inductive relations, that is, correlations on which the theory of mathematical induction depends. The fourth class comprises selective relations, which are required for the general theory of arithmetic operations, and elsewhere. It is in connection with such relations that the famous multiplicative axiom arises for consideration. The fifth class comprises vector relations, from which the theory of quantity arises. The sixth class comprises ratio relations, which interconnect number and quantity. The seventh class comprises three-cornered and four-cornered relations which occur in geometry.

A bare enumeration of technical names, such as the above, is not very illuminating, though it may help to a comprehension of the demarcations of the subject. Please remember that the names are technical names, meant, no doubt, to be suggestive, but used in strictly defined senses. We have suffered much from critics who consider it sufficient to criticize our procedure on the slender basis of a knowledge of the dictionary meanings of such terms. For example, a one-to-one correlation depends on the notion of a class with only one member, and this notion is defined without appeal to the concept of the number one. The notion of diversity is all that is wanted. Thus the class α has only one member, if (1) the class of values of x which satisfies the propositional function,

x is not a member of α,

is not the whole type of relevant values of x, and (2) the propositional function,

x and y are members of α, and

x is diverse from y,

is false whatever be the values of x and y in the relevant type.

Analogous procedures are obviously possible for higher finite cardinal members. Thus, step by step, the whole cycle of current mathematical ideas is capable of logical definition. The process is detailed and laborious, and, like all science, knows nothing of a royal road of airy phrases. The essence of the process is, first to construct the notion in terms of the forms of propositions, that is, in terms of the relevant propositional functions, and secondly to prove the fundamental truths which hold about the notion by reference to the results obtained in the algebraic section of logic.

It will be seen that in this process the whole apparatus of special indefinable mathematical concepts, and special *a priori* mathematical premises, respecting number, quantity and space, has vanished. Mathematics is merely an apparatus for analyzing the deductions which can be drawn from any particular premises, supplied by common sense, or by more refined scientific observation, so far as these deductions depend on the forms of the propositions. Propositions of certain forms are continually occurring in thought. Our existing mathematics is the analysis of deductions, which concern those forms and in some way are important, either from practical utility or theoretical interest. Here I am speaking of the science as it in fact exists. A theoretical definition of mathematics must include in its scope any deductions depending on the mere forms of propositions. But, of course, no one would wish to develop that part of mathematics which in no sense is of importance.

This hasty summary of logical ideas suggests some reflections. The question arises, How many forms of propositions are there? The answer is: An unending number. The reason for the supposed sterility of logical science can thus be discerned. Aristotle founded the science by conceiving the idea of the form of a proposition, and by conceiving deduction as taking place in virtue of the forms. But he confined propositions to four forms, now named A, I, E, O. So long as logicians were obsessed by this unfortunate restriction, real progress was impossible. Again, in their theory of form, both Aristotle and subsequent logicians came very near to the theory of the logical variable. But to come very near to a true theory, and to grasp its precise application, are two very different things, as the history of science teaches us. Everything of importance has been said before by somebody who did not discover it.

Again, one reason why logical deductions are not obvious is that logical form is not a subject which ordinarily enters into thought. Common-sense deduction probably moves by blind instinct from concrete proposition to concrete proposition, guided by some habitual association of ideas. Thus common sense fails in the presence of a wealth of material.

A more important question is the relation of induction, based on observation, to deductive logic. There is a tradition of opposition between adherents of induction and of deduction. In my view, it would be just as sensible for the two ends of a worm to quarrel. Both observation and deduction are necessary for any knowledge worth having. We can not get an inductive law without having recourse to a propositional function. For example, take the statement of observed fact,

This body is mercury, and its specific heat is 0.033.

The propositional function is formed,

Either x is not mercury, or its specific heat is 0.033.

The inductive law is the assumption of the truth of the general proposition, that the above propositional function is true for every value of x in the relevant type.

But it is objected that this process and its consequences are so simple that an elaborate science is out of place. In the same way, a British sailor knows the salt sea when he sails over it. What, then, is the use of an elaborate chemical analysis of sea-water? There is the general answer, that you can not know too much of methods which you always employ; and there is the special answer, that logical forms and logical implications are not so very simple, and that the whole of mathematics is evidence to this effect.

One great use of the study of logical method is not in the region of elaborate deduction, but to guide us in the study of the formation of the main concepts of science. Consider geometry, for example. What are the points which compose space? Euclid tells us that they are without parts and without magnitude. But how is the notion of a point derived from the sense-perceptions from which science starts? Certainly points are not direct deliverances of the senses. Here and there we may see or unpleasantly feel something suggestive of a point. But this is a rare phenomenon, and certainly does not warrant the conception of space as composed of points. Our knowledge of space properties is not based on any observations of relations between points. It arises from experience of relations between bodies. Now a fundamental space relation between bodies is that one body may be part of another. We are tempted to define the "whole and part" relation by saying that the points occupied by the part are some of the points occupied by the whole. But "whole and part" being more fundamental than the notion of "point," this definition is really circular and vicious.

We accordingly ask whether any other definition of "spatial whole and part" can be given. I think that it can be done in this way, though, if I be mistaken, it is unessential to my general argument. We have come to the conclusion that an extended body is nothing else than the class of perceptions of it by all its percipients, actual or ideal. Of course, it is not any class of perceptions, but a certain definite sort of class which I have not defined here, except by the vicious method of saying that they are perceptions of a body. Now, the perceptions of a part of a body are among the perceptions which compose the whole body. Thus two bodies a and b are both classes of perceptions; and b is part of a when the class which is b is contained in the class which is a. It immediately follows from the logical form of this definition that if b is part of a, and c is part of b, then c is part of a. Thus the relation "whole to part" is transitive. Again, it will be convenient to allow that a body is part of itself. This is a mere question of how you draw the definition. With this understanding, the relation is reflexive. Finally, if a is part of b, and b is part of a, then a and b must be identical. These properties of "whole and part" are not fresh assumptions, they follow from the logical form of our definition.

One assumption has to be made if we assume the ideal infinite divisibility of space. Namely, we assume that every class of perceptions which is an extended body contains other classes of perceptions which are extended bodies diverse from itself. This assumption makes rather a large draft on the theory of ideal perceptions. Geometry vanishes unless in some form you make it. The assumption is not peculiar to my exposition.

It is then possible to define what we mean by a point. A point is the class of extended objects which, in ordinary language, contain that point. The definition, without presupposing the idea of a point, is rather elaborate, and I have not now time for its statement.

The advantage of introducing points into geometry is the simplicity of the logical expression of their mutual relations. For science, simplicity of definition is of slight importance, but simplicity of mutual relations is essential. Another example of this law is the way physicists and chemists have dissolved the simple idea of an extended body, say of a chair, which a child understands, into a bewildering notion of a complex dance of molecules and atoms and electrons and waves of light. They have thereby gained notions with simpler logical relations.

Space as thus conceived is the exact formulation of the properties of the apparent space of the common-sense world of experience. It is not necessarily the best mode of conceiving the space of the physicist. The one essential requisite is that the correspondence between the common-sense world in its space and the physicists' world in its space should be definite and reciprocal.

I will now break off the exposition of the function of logic in connection with the science of natural phenomena. I have endeavored to exhibit it as the organizing principle, analyzing the derivation of the concepts from the immediate phenomena, examining the structure of the general propositions which are the assumed laws of nature, establishing their relations to each other in respect to reciprocal implications, deducing the phenomena we may expect under given circumstances.

Logic, properly used, does not shackle thought. It gives freedom and, above all, boldness. Illogical thought hesitates to draw conclusions, because it never knows either what it means, or what it assumes, or how far it trusts its own assumptions, or what will be the effect of any modification of assumptions. Also the mind untrained in that part of constructive logic which is relevant to the subject in hand will be ignorant of the sort of conclusions which follow from various sorts of assumptions, and will be correspondingly dull in divining the inductive laws. The fundamental training in this relevant logic is, undoubtedly, to ponder with an active mind over the known facts of the case, directly observed. But where elaborate deductions are possible, this mental activity requires for its full exercise the direct study of the abstract logical relations. This is applied mathematics.

Neither logic without observation, nor observation without logic, can move one step in the formation of science. We may conceive humanity as engaged in an internecine conflict between youth and age. Youth is not defined by years, but by the creative impulse to make something. The aged are those who, before all things, desire not to make a mistake. Logic is the olive branch from the old to the young, the wand which in the hands of youth has the magic property of creating science.

A. N. WHITEHEAD

DR. HALDANE'S SILLIMAN LECTURES

DR. J. S. HALDANE, of the University of Oxford, gives the Silliman lectures at Yale University on October 9, 10, 12 and 13. The general subject of the lectures is: Organization and Environment as illustrated by the Physiology of Breathing. The topics of the separate lectures are:

Lecture I.—The problem presented by the coordinated maintenance of reactions between organism and environment—vitalistic and mechanistic attempts at explanation; The elementary facts relating to breathing; The respiratory center and the blood; Alveolar air and the exact regulation of its CO_2 percentage; Apnea and hyperpnea; Varying frequency of breathing; Physiological effects of varying pressures of gases; Effects of deprivation of CO_2; Effects of air of confined spaces and mines; Effects of compressed air in diving; Influence of the vagus nerves in breathing; Coordination of the responses to central and peripheral nervous stimuli, so that the respiratory apparatus acts as a whole.

Lecture II.—The gases of the blood; Oxyhemoglobin and the conditions of its dissociation; The combinations of CO_2 in the blood and their dissociation; Effects of oxygenation of hemoglobin on the dissociation of CO_2; Exact physiological regulation of the blood-gases; Evidence that CO_2 acts physiologically as an acid; Investigations of