betterment is emphasized as well as its direct value in the advancement of knowledge, and strong claims are made for its support.

It certainly is the case that animal psychology in this country has, in the past decade, done very solid and instructive work with very little financial support from universities or research funds. The experiments here reported represent a gift of the time of one man of science and a gift of material resources from another. They are typical of the scientific devotion and self help which the public can profitably reward by any means in its power, and which any individual honors himself by supporting. EDWARD L. THORNDIKE

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RETROGRESSION IN AMERICAN LONGEVITY AT ADVANCED AGES

It is generally suspected among a limited group of scientific men that although we seem to be improving in matters of health we are doing so in spite of adverse conditions at the more advanced ages.

We have certainly improved on the whole, for the area in the United States from which acceptable records in mortality statistics are received annually (the registration area) has doubled in the number of states included, within the past decade (1900-1910), although it is still no more than half of the total number of the states of the Union, to the shame of such great states as Illinois, Iowa, Kansas, Nebraska, etc. That mortality conditions have improved in the neighborhood of the age of birth and in fact, at all the earlier ages, is so well established that it needs no comment. Also, the general death rate in this country has decreased more in the past decade (2.6) than in the previous two decades taken together (2.2).

But all this improvement is too deceiving; it covers up the fact that in some respects we are worse off now than we were twenty years or more ago. Stated concretely we expect to show in this paper that individuals between the ages of about 50 and 75 do not, on the average, live as long now as they did twenty years ago; and the extent of this retrogression is increasing. We shall refer to this period or interval of ages as the Period of Retrogression.

We hope to point out also slight indications of tendencies to "come back" at the still more advanced ages, say from 75 on. That the individuals at these extreme ages are "coming back" seems pretty firmly indicated by the results of this investigation, but not only is the "come back" small but it is also manifested at ages where statistical data are faulty; hence, we recommend that these indications be held in abeyance until they are more clearly verified by other investigations of similar nature.

The English statistician and actuary, Mr. George King, has explained a short method of constructing abridged mortality tables wherein only representative portions of the tables and the corresponding death rates and expectations of life are given. We have utilized this method to construct six abridged mortality tables based upon the mortality statistics of each of the sexes, and for the three single years 1890, 1900, and 1910. The year 1880 was not included because the population data and mortality statistics for that year which were reliable do not cover exactly the same area. The mortality statistics of all years previous to 1880 are worthless for our purpose. The essential purpose of this paper is to compare and discuss the results obtained through the construction of these mortality tables.

The fact that each mortality table is constructed from data covering but a single year absolutely prohibits the use of such tables except to point out general conclusions such as are indicated in this paper. Our attitude in this matter should not be forgotten.

As the explanation of the method of construction of the tables is technical and has no special bearing upon the interpretation of the final results, we shall merely refer the reader who desires further information to Mr. King's explanation—which, however, will bear much simplification—in the Registrar General's report for 1914.

The statistical data for the year 1890 com-

prise the population and deaths of the nine registration states of that year: Connecticut, Delaware, Massachusetts, New Hampshire, New Jersey, New York, Rhode Island, Vermont, and the District of Columbia. The statistics of the years 1900 and 1910 which were used in this investigation comprise those of the same states enumerated above, except Delaware, and of the states Indiana, Maine and Michigan.

The mortality tables were completed at the extreme and relatively unimportant ages not covered by reliable data, in the rather arbitrary manner discussed by Mr. King.

The abridged mortality tables are given here for visual comparison, but our discussion will be directed solely to the death rates and expectations of life given later.

1890			1900		1910	
Ages	Males	Females	Males	Females	Males	Females
12	100,000	100,000	100,000	100,000	100,000	100,000
17	98,026	97,715	98,390	98,182	98,537	98,678
22	94,558	94,613	95,596	95,466	96,281	96,665
27	90,219	90,793	92,115	92,066	93,487	94,089
32	85,655	86,559	88,414	88,379	90,344	91,220
37	80,876	82,122	84,450	84,594	86,618	88,036
42	75,777	77,484	80,108	80,625	82,281	84,485
47	70,309	72,670	75,247	76,289	77,246	80,445
52	64,259	67,318	69,998	71,206	71,394	75,532
57	57,290	61,084	63,556	64,932	64,260	69,204
62	49,359	52,181	55,178	57,292	55,255	61,098
67	40,571	43,990	45,167	48,036	44,707	50,974
72	31,000	34,582	33,670	36,844	32,651	38,674
77	20,826	24,192	21,806	24,707	21,358	25,362
82	11,308	14,022	11,151	13,372	11,643	13,677
87	4,579	6,244	4,013	5,240	4,212	5,678
92	1,388	1,976	898	1,347	1,068	1,671
97	339	464	108	198	227	316
102	68	84	6	14	40	34
107	11	12	0	0	6	2
_	0	1			0	. 0

MORTALITY TABLES

It is to be noticed that the ages in the neighborhood of the age of birth are ignored. This is practically necessary in the use of such short methods, considering the great variations in death rates at the ages of this period. However, examination of various mortality tables constructed upon mortality conditions in the United States will reveal little difference between the expectation of life at age twelve and that at the age of birth. Thus, the expectation of life at age twelve is a fair estimate of the average length of the whole of American life, especially when used for purposes of comparison of two or more sets of mortality conditions.

The abridged list of death rates and corresponding differences are as follows:

DEATH R	ATES	PER	100	,000
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Males

Ages	1890	Diff.	1900	Diff.	1910		
12	331	- 55	276	- 39	237		
17	546	- 64	482	- 89	393		
22	864	- 165	699	- 141	558		
27	1,015	- 230	785	- 156	629		
32	1,065	- 196	869	- 98	771		
37	1,260	- 273	987	- 40	947		
42	1,358	- 204	1,154	- 7	1.147		
47	1,696	- 313	1,383	+ 50	1.433		
52	1,961	- 306	1,655	+ 152	1,807		
57	2,757	- 283	2,474	+ 99	2,573		
62	3,277	+ 53	3,330	+ 311	3.641		
67	4,781	+ 159	4,940	+ 120	5,060		
72	6,172	+ 874	7,046	+ 597	7,643		
77	10,075	+ 475	10,550	+738	11,288		
82	13,893	+1,967	15,860	- 628	15,232		
87	20,324	+2,344	22,668	+ 25	22,693		
92	23,384	+7,475	30,859	-5,332	25,527		
Females							
		Fer	nales				
	382	Fen	nales 292	- 69	223		
12 17	382 574	Fen	nales 292 482	- 69 - 135	223 347		
12 17 22	382 574 745	Fen - 70 - 92 - 77	nales 292 482 668	- 69 - 135 - 165	223 347 503		
12 17 22 27	382 574 745 924	Fen - 70 - 92 - 77 133	nales 292 482 668 791	- 69 - 135 - 165 - 206	223 347 503 585		
12 17 22 27 32	382 574 745 924 987	$ \begin{array}{rrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrr$	nales 292 482 668 791 844	$ \begin{array}{rrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrr$	223 347 503 585 668		
12 17 22 27 32 37	382 574 745 924 987 1,136	Fer = -70 = -92 = -77 = -133 = -143 = -217	nales 292 482 668 791 844 919	$\begin{array}{rrrr} - & 69 \\ - & 135 \\ - & 165 \\ - & 206 \\ - & 176 \\ - & 146 \end{array}$	223 347 503 585 668 773		
12 17 22 27 32 37 42	382 574 745 924 987 1,136 1,195	Fer = -70 = -92 = -77 = -133 = -143 = -217 = -169	nales 292 482 668 791 844 919 1,026	$\begin{array}{rrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrr$	223 347 503 585 668 773 901		
12 17 22 27 32 37 42 47	$\begin{array}{c c} 382 \\ 574 \\ 745 \\ 924 \\ 987 \\ 1,136 \\ 1,195 \\ 1,417 \end{array}$	Fen - 70 - 92 - 77 - 133 - 143 - 217 - 169 - 181	nales 292 482 668 791 844 919 1,026 1,236	$\begin{array}{rrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrr$	223 347 503 585 668 773 901 1,113		
12 17 22 27 32 37 42 47 52	$\begin{array}{c c} 382\\ 574\\ 745\\ 924\\ 987\\ 1,136\\ 1,195\\ 1,417\\ 1,705\\ \end{array}$	Fen - 70 - 92 - 77 - 133 - 143 - 217 - 169 - 181 - 97	nales 292 482 668 791 844 919 1,026 1,236 1,608	$\begin{array}{rrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrr$	223 347 503 585 668 773 901 1,113 1,504		
12 17 22 27 32 37 42 47 52 57	$\begin{array}{c c} 382\\ 574\\ 745\\ 924\\ 987\\ 1,136\\ 1,195\\ 1,417\\ 1,705\\ 2,281\\ \end{array}$	Fen - 70 - 92 - 77 - 133 - 143 - 217 - 169 - 181 - 97 - 84	nales 292 482 668 791 844 919 1,026 1,236 1,608 2,197	$\begin{array}{rrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrr$	223 347 503 585 668 773 901 1,113 1,504 2,132		
12 17 22 27 32 37 42 47 52 57 62	$\begin{array}{c c} 382\\ 574\\ 745\\ 924\\ 987\\ 1,136\\ 1,195\\ 1,417\\ 1,705\\ 2,281\\ 2,846\\ \end{array}$	Fer = -70 = -92 = -77 = -133 = -143 = -217 = -169 = -181 = -97 = -84 = +118	nales 292 482 668 791 844 919 1,026 1,608 2,197 2,964	$\begin{array}{c cccc} - & 69 \\ - & 135 \\ - & 206 \\ - & 176 \\ - & 146 \\ - & 125 \\ - & 123 \\ - & 104 \\ - & 65 \\ + & 72 \end{array}$	223 347 503 585 668 773 901 1,113 1,504 2,132 3,036		
12 17 22 27 32 37 42 47 52 57 62 67	$\begin{array}{c} 382\\ 574\\ 745\\ 924\\ 987\\ 1,136\\ 1,195\\ 1,417\\ 1,705\\ 2,281\\ 2,846\\ 4,207\\ \end{array}$	Fer = -70 = -92 = -77 = -133 = -143 = -217 = -169 = -97 = -97 = -97 = -94 = +118 = +138	nales 292 482 668 791 844 919 1,026 1,236 1,608 2,197 2,964 4,345	$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	223 347 503 585 668 773 901 1,113 1,504 2,132 3,036 4,490		
12 17 22 27 32 37 42 47 52 57 62 67 72	$\begin{array}{c} 382\\ 574\\ 745\\ 924\\ 987\\ 1,136\\ 1,195\\ 1,417\\ 1,705\\ 2,281\\ 2,846\\ 4,207\\ 5,637\\ \end{array}$	Fen = 100 + 100	nales 292 482 668 791 844 919 1,026 1,236 1,608 2,197 2,964 4,345 6,565	$\begin{array}{rrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrr$	223 347 503 585 668 773 901 1,113 1,504 2,132 3,036 4,490 6,870		
12 17 22 27 32 37 42 47 52 57 62 67 72 77	$\begin{array}{c c} 382\\ 574\\ 745\\ 924\\ 987\\ 1,136\\ 1,195\\ 1,417\\ 1,705\\ 2,281\\ 2,846\\ 4,207\\ 5,637\\ 8,988\end{array}$	$\begin{array}{c c} Fer \\ \hline & - & 70 \\ - & 92 \\ - & 77 \\ - & 133 \\ - & 143 \\ - & 217 \\ - & 169 \\ - & 181 \\ - & 97 \\ - & 84 \\ + & 118 \\ + & 138 \\ + & 138 \\ + & 928 \\ + & 663 \end{array}$	292 482 668 791 844 919 1,026 1,236 1,608 2,197 2,964 4,345 6,565 9,651	$\begin{array}{cccc} - & 69 \\ - & 135 \\ - & 206 \\ - & 176 \\ - & 146 \\ - & 125 \\ - & 123 \\ - & 104 \\ - & 65 \\ + & 72 \\ + & 145 \\ + & 305 \\ + & 437 \end{array}$	223 347 503 585 668 773 901 1,113 1,504 2,132 3,036 4,490 6,870 10,088		
12 17 22 27 32 37 42 47 52 57 62 67 72 77 82	$\begin{array}{c c} 382\\ 574\\ 745\\ 924\\ 987\\ 1,136\\ 1,195\\ 1,417\\ 1,705\\ 2,281\\ 2,846\\ 4,207\\ 5,637\\ 8,988\\ 12,610\\ \end{array}$	$\begin{array}{c c} Fer \\ \hline & - & 70 \\ - & 92 \\ - & 77 \\ - & 133 \\ - & 217 \\ - & 169 \\ - & 181 \\ - & 97 \\ - & 84 \\ + & 118 \\ + & 138 \\ + & 928 \\ + & 663 \\ + & 1,106 \\ \end{array}$	nales 292 482 668 791 844 919 1,026 1,608 2,197 2,964 4,345 6,565 9,651 13,716	$\begin{array}{cccc} - & 69 \\ - & 135 \\ - & 206 \\ - & 176 \\ - & 146 \\ - & 125 \\ - & 123 \\ - & 104 \\ - & 65 \\ + & 72 \\ + & 145 \\ + & 305 \\ + & 437 \\ + & 464 \end{array}$	$\begin{array}{c} 223\\ 347\\ 503\\ 585\\ 668\\ 773\\ 901\\ 1,118\\ 1,504\\ 2,132\\ 3,036\\ 4,490\\ 6,870\\ 10,088\\ 14,180\end{array}$		
12 17 22 37 32 37 42 47 52 57 62 67 72 77 82 87	$\begin{array}{c c} 382\\ 574\\ 745\\ 924\\ 987\\ 1,136\\ 1,195\\ 1,417\\ 1,705\\ 2,281\\ 2,846\\ 4,207\\ 5,637\\ 8,988\\ 12,610\\ 18,496 \end{array}$	$\begin{array}{c c} Fer \\ \hline & - & 70 \\ - & 92 \\ - & 77 \\ - & 133 \\ - & 217 \\ - & 169 \\ - & 181 \\ - & 97 \\ - & 84 \\ + & 118 \\ + & 138 \\ + & 928 \\ + & 663 \\ + & 1,106 \\ + & 2,367 \end{array}$	nales 292 482 668 791 844 919 1,026 1,608 2,197 2,964 4,345 6,565 9,651 13,716 20,863	$ \begin{array}{c cccc} - & 69 \\ - & 135 \\ - & 206 \\ - & 176 \\ - & 146 \\ - & 125 \\ - & 123 \\ - & 104 \\ - & 65 \\ + & 72 \\ + & 145 \\ + & 305 \\ + & 437 \\ + & 464 \\ - & 867 \end{array} $	223 347 503 585 668 773 901 1,113 1,504 2,132 3,036 4,490 6,870 10,088 14,180 19,996		
12 17 22 27 32 37 42 47 52 57 62 67 72 77 82 87 92	$\begin{array}{c c} 382\\ 574\\ 745\\ 924\\ 987\\ 1,136\\ 1,195\\ 1,417\\ 1,705\\ 2,281\\ 2,846\\ 4,207\\ 5,637\\ 8,988\\ 12,610\\ 18,496\\ 23,437\\ \end{array}$	Fer = 100 + 1000 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100	nales 292 482 668 791 844 919 1,026 1,608 2,197 2,964 4,345 6,565 9,651 13,716 20,863 28,386	$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	223 347 503 585 668 773 901 1,113 1,504 2,132 3,036 4,490 6,870 10,088 14,180 19,996 32,868		

In the table of death rates given above, attention is called not so much to the absolute values and their differences—for the results lack graduation—but rather to the trend of mortality conditions as indicated by them. This trend among both the males and females is unquestionably forward at all ages below 60, except in the decade 1900–1910, in which the advance among the males is terminated atabout age 45. We might go farther and say that for this same decade the males did not advance as much at the ages at which they did advance as they did in the previous decade. On the other hand, the females maintained or even excelled their record of 1890–1900 in the decade 1900–1910.

The most important feature of the table of death rates is the group or period of ages at which both males and females have retrogressed. This retrogression is significant in value wherever indicated, and that it is not due to faulty statistics or errors is clearly shown by the fact that it appears *in both decades in both sexes*. Further, the retrogression is not spasmodic, but continues firmly from about age 60 on to the end of the table.

> EXPECTATIONS OF LIFE Males

Ages	1890	Diff.	1900	Diff.	1910		
12	46.30	+2.19	48.49	+.59	49.08		
17	42.18	+2.06	44.24	+.53	44.77		
22	38.63	+1.82	40.45	+.30	40.75		
27	35.37	+1.52	36.89	0	36.89		
32	32.12	+1.21	33.33	24	33.09		
37	28.86	+.91	29.77	37	29.40		
42	25.64	+.60	26.24	42	25.82		
47	22.43	+.35	22.78	45	22.33		
52	19.30	01	19.29	34	18.95		
57	16.34	35	15.99	29	15.70		
62	13.56	54	13.02	10	12.92		
67	10.94	60	10.34	+.02	10.36		
72	8.54	54	8.00	+.26	8.26		
77	6.49	49	6.00	+.33	6.33		
82	4.51	07	4.44	+.15	4.59		
87	3.73	48	3.25	+.34	3.59		
92	2.79	51	2.28	+.76	3.04		
Females							
12	47.90	+1.56	49.46	+1.83	51.29		
17	43.96	+1.36	45.32	+1.62	46.94		
22	40.31	+1.23	41.54	+1.32	42.86		
27	36.90	+1.08	37.98	+ .98	38.96		
32	33.58	+.88	34.46	+.65	35.11		
37	30.26	+.63	30.89	+.40	31.29		
42	26.92	+.37	27.29	+.21	27.50		
47	23.54	+.15	23.69	+.06	23.75		
52	20.21	01	20.20	07	20.13		
57	17.00	10	16.90	17	16.73		
62	14.47	66	13.81	21	13.60		
67	11.69	71	10.98	19	10.79		
72	9.18	72	8.46	05	8.41		
77	7.04	65	6.39	+.13	6.52		
82	5.37	62	4.75	+ 28	5.03		
07	4.14	61	3.53	+.33	3.86		
81							

There are a few widely varying values at the terminal ages, but, as mentioned above, the statistics at these ages are so faulty that little or no interpretation is possible.

Summarizing the results indicated by the table of death rates, mortality conditions seem to have been improved at ages below sixty during the two decades 1890–1900 and 1900– 1910 among both the males and the females, steadily so among the females but not so much so among the males. At ages sixty and above, both males and females seem to have retrogressed, particularly the males whose period of retrogression during the decade 1900–1910 began as far back as age 45. This period of retrogression among death rates for both sexes continues steadily toward the last ages of human life.

As indicated in the table of expectations of life given above, the average future life time of males at age twelve seems to have lengthened 2.19 years in the decade 1890-1900 and only .59 of a year in the decade 1900-1910, or 2.78 years in both decades. The gain of only .59 is rather difficult to explain, for even the general death rate suffered a relapse in 1910, and no one seems to know exactly why. It is possible that the fact that the period of retrogression encroached upon the earlier ages might offer at least a partial explanation. The period of retrogression among the expectations of life of the males is seen to begin about age fifty in the decade 1890-1900 and about age thirty in the decade 1900-1910.

That the initial ages of the period of retrogression in both decades precede the corresponding ages in the table of death rates from 10 to 15 years is what might be expected and is really quite important in that it emphasizes the fact that a retrogression in death rates at any period of ages will affect the expectation of life of *all* those living at any earlier ages. The two initial ages, fifty and thirty, mentioned above, differ from earlier ages only in the fact that these are the first ages at which the effect of retrogression at the advanced ages outweighs the effect of improvement at the earlier ages. The amount of retrogression in the expectation of life of the males rarely exceeds half of a year, but the mere fact that individuals at ages above fifty do not live as long now as they did several decades ago is of tremendous significance. If this period or retrogression could be made to vanish, so much more would the expectation of life at the earlier ages be increased. It would be serious enough if no advance were registered in this period; an actual retrogression amounts to a calamity.

It is very remarkable that the period of retrogression of the males in the decade 1900-1910 ends about age sixty-five and from that age on we notice a tendency to "come back," a tendency not found in the decade 1890-1900. The value of this "come back" is small, it is true, but the values give no indications of uncertainty by interposing occasionally a negative value (a retrogression). Whether this period of advance at the most extreme ages actually exists or not, we shall not presume to say, but the above figures are highly suggestive.

The period of retrogression among the expectations of life of the females also begins about age fifty, but there is quite a difference between the two decades considered. In the decade 1900-1910 the females seem to have overcome to a great extent the retrogression registered in the decade 1890-1900; this fact is not true of the males. Moreover, this period is now restricted to only about twenty years, whereas before it seemed to extend firmly to the end of the table.

Here again, we see evidences of an effort to "come back" appearing at the extreme ages. The fact that this period of "come back" appears among the expectations of life of the females in the same decade (1900–1910) as it does among the males adds strength to the probability of its actual existence.

The casual reader may have wondered how the period of retrogression among the death rates could extend to the end of the table while that of the corresponding expectations of life could end at some age such as 75. This is perfectly possible, for in obtaining the expectation of life at any age we divide the total number of years lived, by the population at that age, and this total number of years may be lessened without decreasing the expectation of life if the population at the given age is also lessened in the proper proportion.

In this paper we have pointed out a great field for work; we have pointed out the exact location of a serious problem. It still remains for others to diagnose the trouble, and that task might well be left to those familiar with the diseases operative at the ages covered by this period of retrogression. However, we dare suggest that far the greater part of the trouble is due to a peculiar state of indifference and ignorance in regard to the ordinary laws of nature, and therefore can be overcome best by a systematic plan of education along lines of elementary hygiene.

Every one knows that few individuals between the ages of thirty and sixty take any constructive forethought for their physical welfare; few carry out any definite plans for regular daily exercise or proper breathing of fresh air. Fewer still have even a fair conception of their own physical make-up or their condition at any particular time; this fact is due likely both to lack of time and to reluctance to face the truth.

One of the best ways to arouse interest in practical hygiene would be through the organization of a National Health League which would hope ultimately to have a representative organization in every large community. It should be the duty of such a body to encourage right living among its members and all individuals associated with them. This work should be supplemented by a systematic and regular program of study and discussion. For local organizations made up of individuals who insist they are too busy to make a personal study of the subject, practical lectures could be arranged at regular intervals, calculated to keep interest aroused. The lecturers could be obtained among broadminded and altruistic physicians or the faculty of the state university. The central organization, whether state or national could employ a part of its time and energy in no better way than in providing a complete corps of efficient lecturers who could answer the call to some local organization.

There are many individuals who are looking for a field in which they can utilize their executive powers in a worthy way. There are many wealthy people who are ready and even anxious to donate funds to a worthy cause. We believe a no more worthy cause exists than the one just suggested.

Much work has already been accomplished by organization such as the Y. M. C. A. to encourage right living among young men, but little of it touches the group of busy individuals who are the victims as well as the causes of the Period of Retrogression.

ANN ARBOR, MICH.

C. H. FORSYTH

SPECIAL ARTICLES

A METHOD OF PLOTTING THE INFLECTIONS OF THE VOICE

Some time ago, while the writer was engaged in the study of the "tones" of certain oriental languages, it became desirable to represent visually the tonal movements or figures executed by the voice in actual speech. Records of native speech were taken by the Rousselot apparatus, and the wave-lengths in each tracing were measured throughout, resulting in a series of numbers for each utterance—which series we may for the moment suppose as included within the compass of two octaves, from 10 to 40 of our scheme.

In default of any record of previous attempts of this kind, the following scheme was first tried as the most obvious and simple. Beginning at the top, the unit lines of the coordinate paper were numbered in succession downward from 10 to 40. Then beginning at the left-hand margin the measured numbers from the record were plotted in order, each upon its numbered line, but each advanced beyond its predecessor by a constant interval chosen after experiment as best suited to bring out the features of the voice-inflection. A continuous line drawn through the series of plotted points would then represent the movement of voice as regards pitch. Finally the whole was brought into relation with concert pitch by measuring the wave-length of the record of a C-fork and marking its place among the numbered lines, and computing the positions of the other notes of the scale according to the well-known ratios of the diatonic scale.

The results seemed convincing; but a study of them revealed a certain distortion of vertical values similar in kind to the horizontal distortion of Mercator's maps. This was due to the fact that the number-intervals were equally spaced, whereas to our thought and visual imagination the semitone intervals are equal. The first step toward remedying the difficulty was obvious and easy. The letters of the twelve semitones took the places of the integers on the unit-lines of the chart. The next step-to find the new places of these integers-was not so easy. After some fumbling and groping the following points became clear.

1. Each semitone of the series brings with it to its new place the same numerical value which it had in its former position as a definite term of a geometrical progression of twenty-four terms between 10 and 40, with $\sqrt[2]{2}$ for the common ratio. In Table I. below are given these values for the upper octave. Those for the lower octave are simply twice these. These numbers were entered on the chart against their respective semitones.

2. The integral numbers must next be assigned to their proper stations within this decimal series. Indeed 10, 20 and 40 already appear in that series, and so are assigned to position; while 15 and 30 are so close to semitone positions as to be practically coincident with them. A rough determination of the other positions might be made by the method of proportional parts, but the only real determination is by solving the equation of the geometrical progression just described. That equation is $y = a^x$, in which y and x are variables, and a is constant, namely the common ratio ${}^{12}\sqrt{2}$. The values of y are the integral numbers from 10 to 40. By applying these values in succession to the equation, the corresponding values of x are obtained, that is the