rived; also by aid of (4) and Newton's third law, that "action and reaction are always equal and contrary" the problem of impact of two particles can be solved.

8. By pursuing the course outlined above, the student has to learn and thoroughly understand, only two simple formulas, $M = W/g$, $F = Ma$.

Wm. Cain

Chapel Hill, N. C.

## GRAVITATION AND ELECTRICAL ACTION

In a paper to be published by the Academy of Science of St. Louis, evidence will be presented which appears to show conclusively, that gravitational attraction is diminished by electrical charges on the acting masses. The suspended masses of the Cavendish experiment are wholly enclosed in a shield of sheet metal. The small observation window is covered with wire gauze. When a knob terminal connected with the influence machine is moved towards or away from a knob terminal connected with the large attracting masses, the suspended masses slowly move to and fro around the vertical line of suspension. No disruptive discharges occur. It is found that gravitational attraction is decreased by either positive or negative electrification. By the to-and-fro movement of the knob terminal, the amplitude of vibration can be gradually increased from 2.5 minutes of arc to 50 minutes. It has been established by experimental methods that these results are not due to heat effects.

Francis E. Nipher

## THE PRODUCTION OF RADIUM

To the Editor of Science: On page 799 of the June 2, 1916, issue of Science a statement is made in regard to the production of radium by the Standard Chemical Co. in the year 1915, which is not in accord with facts, and I wish to make this correction. The actual amount of radium produced by the Standard Chemical Co. during 1915 was slightly more than 3 grams of radium element and of this the larger proportion was produced in the first three months of the year from radium which was in process of treatment during the latter part of 1914.

In this same article the production of radium at a cost of $37,599 per gram by the National Radium Institute Inc. working in cooperation and under the supervision of the Bureau of Mines, is compared with the market price of radium of $120,000 a gram. The radium produced by the National Radium Institute was obtained from high-grade carnotite ore treated without concentration, and the cost of production under these conditions is not properly comparable to the cost of production or the selling price of radium from lower grade ore or concentrates.

Applying the Bureau of Mines process to unconcentrated ore containing about 1.5 per cent. of uranium oxide (which is higher than the average carnotite ore) makes the cost of production nearer $70,000 than $40,000 per gram. Since this is practically the condition under which commercial producers of radium must operate, it would be fairer to compare cost of production by the Bureau of Mines process on this basis, rather than on the basis of the uncommercial and somewhat artificial conditions, connected with the treatment of the 1,000 tons of high-grade ore. Concentration of the low-grade ore, if practised, naturally reduces the efficiency of extraction, and in this way would raise the cost of production.

While it is true that the war cut off practically the entire European market to radium producers, it must be added that the growing American market for radium has been very adversely influenced by the widespread publishing of statements, from the United States Bureau of Mines, similar to the statement in Science which we are criticizing. The general effect of these statements has been to lead prospective purchasers of radium to believe that radium would soon be available at enormously reduced prices. Emphasis being laid by the Bureau of Mines on the exceptionally low cost of production, and in general no mention being made of the fact that this low cost of production was in a large measure due to the abnormal and uncommercial conditions under which the Bureau operated.

As regards ore concentration it is also interesting to note that the method used by the Bureau of Mines is one which has been used by the Standard Chemical Company for the past four and a half years, and on the basis of figures published by Dr. Charles L. Parsons in the May number of the *Journal of Industrial and Engineering Chemistry*, it is not evident that the method is satisfactorily efficient, when applied to the treatment of low-grade carnotite ore.

CHARLES H. VIOL

PITTSBURGH, PA.,
June 3, 1916

## SCIENTIFIC BOOKS

### RECENT BOOKS IN MATHEMATICS

*Algebraic Invariants.* By LEONARD EUGENE DICKSON, Professor of Mathematics, University of Chicago. New York, John Wiley and Sons, 1914. Pp. 100. $1.25.

*A Treatise on the Theory of Invariants.* By OLIVER E. GLENN, Ph.D., Professor of Mathematics in the University of Pennsylvania. Boston, Ginn and Company, 1915. Pp. 245.

*Contributions to the Founding of the Theory of Transfinite Numbers.* By GEORG CANTOR. Translated and Provided with an Introduction and Notes by PHILIP E. B. JOURDAIN. Chicago and London, The Open Court Publishing Company, 1915. Pp. 211. $1.25.

*Problems in the Calculus. With Formulas and Suggestions.* By DAVID D. LEIB, Ph.D., Instructor in Mathematics in the Sheffield Scientific School of Yale University. Boston and New York, Ginn and Company, 1915. Pp. 224.

*Diophantine Analysis.* By ROBERT D. CARMICHAEL, Assistant Professor of Mathematics in the University of Illinois. New York, John Wiley and Sons, 1915. Pp. 118.

*Historical Introduction to Mathematical Literature.* By G. A. MILLER, Professor of Mathematics in the University of Illinois. New York, The Macmillan Company, 1916. Pp. 295.

An invariant is any thing—a property or a relation or an expression or a configuration—that remains unaltered when other things connected with it suffer change. In this very comprehensive but essential meaning of the term, the notion is probably as ancient as the human intellect. Certainly in historic time the appeal of the idea has been universal. It has been said that science may be defined as the quest of invariance. Doubtless that quest is an essential mark of science but it is not peculiar to science. For the problem of invariance, the problem of finding permanence in the midst of change, arises out of the flux of things to confront man in all departments of life. And so it is that the search for what abides is not confined to science but is and always has been the chief enterprise of philosophy and theology and art and jurisprudence. It is, however, in mathematics that the notion of invariance has come to the clearest recognition of its character and significance. In this respect the notion in question has had a history like that of all other great ideas that have slowly and at length become available for the processes of logic.

The oldest and now most elaborate portion of the mathematical doctrine of invariance is about as old as American independence. Though now an imposing theory, its beginning was like a mustard seed. It began, not in ratiocination, but in an observation—mathematics indeed depends even more upon observation than upon formal reasoning. It began in what was in itself a very small observation, an observation (1773) by Lagrange that the discriminant of the quadratic form $ax^2 + 2bxy + cy^2$ remains unaltered on replacing $x$ by $x + \lambda y$. The next important step was taken by Gauss in 1801 and the next by Boole in 1841. Incited by Boole's beautiful results, the English mathematicians, Cayley and Sylvester, entered the field, the former producing in rapid succession his great memoirs on Quantics and the latter his brilliant investigations in what he conceived more poetically as the Theory of Forms. The interest so aroused quickly passed to the continent engaging the great abilities of such mathematicians as Aronhold, Hermite, Clebsch, Gordan and others. The result is the colossal doctrine variously styled the algebra of quantics, the theory of algebraic invariants and covariants, and the theory of forms.

It is to this doctrine that Professor Dick-