At the last meeting of the corporation of the Massachusettts Institute of Technology promotions and appointments were made to the instructing staff as follows: From assistant to associate professor Daniel F. Comstock (theoretical physics), George L. Homer (topographical surveying), C. L. E. Moore (mathematics), Ellwood B. Spear (inorganic chemistry), William E. Wickenden (electrical engineering). Instructors were promoted to assistant professorships as follows: James M. Barker (structural engineering), Ralph G. Hudson and Waldo V. Lyon (electrical engineering), Earl B. Millard (theoretical chemistry). Dr. Frederick G. Keyes was appointed associate professor of physico-chemical research.

DISCUSSION AND CORRESPONDENCE

SOME FUNDAMENTAL DIFFICULTIES OF MECHANICS

A long and interesting exchange of views on the fundamental equation of mechanics, which has taken place in the columns of Science, has led me to review some old notes in that connection. It has seemed to me that the question may be viewed from two different points, that of the systematizer and that of the teacher. The former desires an equation, fundamental in that from it he can develop the science most easily. The latter must consider as the fundamental principles those which appeal most directly and forcibly to the student, which enable the student to progress most easily, with rapidity and security. By *the student* I mean the average student, who has much experience of a mechanical nature, but is unaccustomed to logic and cares little about unity.

To the teacher of mechanics students in masses, that is, to nearly every mechanics or physics teacher, even in college and technical school, the first-named viewpoint is unimportant as compared with the second. His business is to diagnose the student's difficulties, and then to obviate or remove them. Some of these difficulties are inherent in the laws of mind and matter.

Any teacher will admit that to the average student the descriptive, phenomenological, attitude toward mechanics is quite too rarefied, too impersonal. Professor C. R. Mann has well said:

> To a beginner pushes and pulls are the real forces.

The beginner can imagine himself pushing or pulling, exerting an effort and taking an interest. Descriptively, it has been questioned whether the concept of force is of much value in mechanics; but the sense and memory of effort give the student his starting point, and the teacher must begin kinetics with force as well as with acceleration and mass.

When we exert effort we observe we either change the motion of bodies, or change the relative positions of bodies or of their parts, hence the forms of bodies. During such changes of position or form, more or less temporary changes of motion occur.

Hence we all quite unnecessarily infer that when the motions of bodies are changed, or their relative positions, or their forms, there must be something going on analogous to an effort; this we call force, and we say that the above effects of effort are the effects of force.

Moreover, we observe that while the changes of relative position or form of bodies due to our effort may persist after we have ceased to exert effort, on the contrary the motion which has been produced by an effort does not continue, it always diminishes and finally ceases. We note that the effort needed for the production or increase of motion depends on the contact of the body acted on with other things, as soil, pavement, ice; water, if floating; oil, if lubricated; air, if swinging suspended; and also on the form of the body, flat or jagged or round. In some cases the production of motion is harder, in others easier, the duration of the motions is shorter or longer, but sooner or later the motions end in rest. If we want a thing to keep going we have to keep pushing or pulling; and this without exception in all our bodily experience.

Hence we hastily but naturally conclude that rest is the natural state of all bodies, and that for the maintenance of even constant motion continuous effort, or force, is necessary.

It has been pointed out that the scholastic dictum about the necessity of force for the maintenance of motion is thus a consequence of common experience, a deduction of "common sense," which is the result of common experience. And while the common experience of boys and young men is changeable from age to age and different from one culture level to another, while men in the age of stone clubs or in the days of the stage coach had a range of common experience vastly different from what they have in an era of electricity and gasolene, nevertheless this element of terrestrial experience persists in them all—to maintain motion force must be continuously exerted; force lacking, rest supervenes.

Galileo's principle of inertia, then, Newton's first law of motion, is not a deduction of common sense, because it contradicts common experience. Only uncommon experience, interpreted by an uncommon mind, could arrive at it; and it is a fact that the world waited many ages for a genius to arise, fly in the face of common terrestrial experience, announce that the immediate consequence of force is acceleration, and interpret the inevitable extinction of unsupported terrestrial motions by the hypothesis of a force of friction, always opposing the existing motion and producing a negative acceleration. And the clear grasp of the inertia principle could only follow the study of a frictionless system.

Here we have the first difficulty of kinetics; its first law contradicts the student's common sense and all his ingrained mechanical experience. I doubt that many students, seeing the experiment for coefficient of friction, with horizontal slab, pulley and cord, actually interpret the slow uniform motion of the block in terms of two equal and opposed horizontal forces, producing each its own acceleration. It seems too far fetched; rather say, if you stop pulling the slab stops—and have done with it. And so with all the movements of wind and water; they go on because somehow they are driven. And so also Kepler interpreted the motion of the planet Mars in its orbit as due to a forward tangential force arising no doubt in the sun; and the schoolmen said that bodies fall with speeds proportional to their weights—which is roughly true for snowflakes and raindrops.

Change of motion, quantitatively called acceleration, is an idea rather remote from common experience. Every player of games is familiar with it in a crude way, but that it is a measurable quantity, or worth measuring, never entered any head before Galileo's. This is not at all remarkable, when we consider that speed is not given us by direct measurement, but only by simultaneous direct measurements of distance and time; much less are we given the rate of change of speed. The beginner has no real experience with acceleration as a measurable quantity; it is the rate of change of a rate of change, and too abstract for most people. It does have a connection with effort; to throw a ball fast is harder than to throw it slow; but I doubt if the average beginner ever has gone beyond that—and certainly many a student of calculus never connects this rough experience with d^2x/dt^2. In fact, we can not get differential expressions by measurement; Kepler's planetary laws and Galileo's laws of falling bodies are either integral expressions representing their tables of length[1] and time measurements, or are deduced from these integral expressions. Beginners do not of their own accord take the trouble to construct such tabulations or to differentiate twice the resulting integral expressions; in fact, few can do this, or at first realize what it all means when they are made to do it.

Our most continuous effort is to keep ourselves or other objects off the ground; the next most familiar, to set objects in motion upward, a motion which, unless some obstacle prevents, is sooner or later reversed into a motion downward. We say, as if an antagonistic effort were opposing ours, that the earth exerts a downward force upon us and all things near it; it is able to change their forms or to set them in motion downward.

While our sensations of effort are only qualitative, telling us of more and less, but not of how much, we assign measure to this earth effort, or force, or weight, by saying that its

[1] Angles are measured by arcs of graduated circles.

size is twice as great when it pulls on two exactly like objects together as it is when it pulls on only one of them; and conversely we use this pull to measure the elastic force of a spring, the relative magnitudes of different bodies, etc. This notion, that the magnitude of earth pull is proportional to the number of otherwise equal things on which it acts, is fundamental, and so familiar as to seem axiomatic; it is instinctive, as E. Mach would say.

The study of the downward motion of bodies affected by their own weight and only slightly by friction was a lifelong interest of Galileo. Directly or indirectly he showed two things; that they fall equal distances in equal times, and that unequal distances of fall are proportional to the squares of the times of fall. Differentiation of the latter showed that the gravitational acceleration is constant during the time of fall; the former showed it to be the same for all things, independently of their weight or material.

The last conclusion leads to an appreciation of another difficulty in the study of mechanics, if we take into account a law of psychology, well stated in the following quotation from William James:

. . . any number of impressions, from any number of sensory sources, falling simultaneously on a mind which has not yet experienced them separately, will yield a single undivided object to that mind. The law is that all things fuse that can fuse, and that nothing separates except what must.

The singling out of elements in a compound. It is safe to lay down as a fundamental principle that any total impression made on the mind must be unanalyzable so long as its elements have never been experienced apart or in other combinations elsewhere. The components of an absolutely changeless group of not-elsewhere-occurring attributes could never be discriminated. If all cold things were wet, and all wet things cold, if all hard things pricked our skin, and no other things did so: is it likely that we should discriminate between coldness and wetness, and hardness and pungency, respectively? If all liquids were transparent and no non-liquid were transparent, it would be long before we had separate names for liquidity and transparency. If heat were a function of position above the earth's surface, so that the higher a thing was the hotter it became, one word would serve for hot and high. We have, in fact, a number of sensations whose concomitants are invariably the same, and we find it accordingly impossible to analyze them out of the totals in which they are found.

Now to lift a stone vertically we have to exert an effort, neutralizing the earth's pull upon it, its weight. To throw the same stone horizontally, to accelerate it, we have also to exert effort; and the harder the stone is to lift, the harder it is to throw. (If we refine this crude observation by experiment, we find an exact proportionality between the weights of objects and the efforts or forces required to accelerate them equally.) Hastily generalizing, but most naturally, we say that stones are hard to throw, gates hard to swing, not in proportion as, but *because* they are heavy. To ordinary observation the accelerating and the gravitational efforts always increase and decrease exactly together; they do not tend to become discriminated, we do not abstract them separately.

To exact observation, however, a difference does show itself. The same stone weighed in a spring balance would elongate the spring less in low latitudes than in high (we tell our classes this; did any one ever try it?). The same pendulum vibrates more slowly in low latitudes than in high, as Richer found in 1672–3. We can imagine a man lifting and throwing a ball at the bottom and again at the top of a tower four thousand miles high, observing a notable change in the weight of the ball and yet none at all in the difficulty of throwing it. But such observations under terrestrial conditions have to be accurate to less than $\frac{1}{2}$ per cent., far more accurate than the unaided sense memory can be. To the average man a heavy thing is also hard to throw, because it is heavy; a fact which stands as a formidable obstacle to a clear grasp on the idea of mass; to most students mass and weight are forever identical, except that the book says to divide weight by g to get mass.

In an old copy of Wells' "Natural Philosophy" I find the following problem and answer, which may serve as an illustration:

Why will a large ship, moving toward a wharf with a motion hardly perceptible, crush with great force a boat intervening?

Because the great mass and weight of the vessel compensates for its want of velocity.

Which shows that the author of this famous book did not discriminate between mass and weight in a case where weight as force does not enter.

This confusion of mass and weight can not be helped by pseudo-definitions which attempt to evade the essentially kinetic nature of the mass concept. As is well known, Newton, in the "Principia," defined mass as the product of density and volume, and equivalent to quantity of matter. Neither of these statements has any value, as neither brings out the essential fact that a body subject to acceleration displays a constant characteristic property, which is the core of Newton's own treatment of the problem of accelerated motion. Another more recent definition states that mass is the result obtained by weighing with a balance scale. This can not help a student very much. The balance scale was known for centuries before Newton, and had mass been so easily defined it would hardly have been left for him to discover the fact of its existence and importance. The fact is, that mass is a concept of kinetics, not to be reached at all by static experiments, and not to be clearly discovered by kinetic experiments affected by friction. It came into science by way of Mars and the moon, and was then read into terrestrial experience. The "balance scale" gives us mass not directly, but by interpretation, even as does the Jolly balance. It is not always true that "in physics sensible people define things the way they do them."

Students in general seem to have no serious difficulty with the equality of push and counter-push, of friction and counter-friction, of action and reaction. Trouble does come up in the identification of actions and reactions, and in the realization that these always act upon different things, in opposite directions in the same straight line.

As illustrations, take two quotations, the first from Wells's "Natural Philosophy," of the sixties, the other from a recent book:

The centrifugal force is that force which impels a body moving in a curve to move outward or fly off from a center. The centripetal force is that force which draws a body moving in a curve toward the center, and compels it to move in a bent, or curvilinear course. In circular motion the centrifugal and centripetal forces are equal, and constantly balance each other. If the centrifugal force of a body revolving in a circular path be destroyed, the body will immediately approach the center; but if the centripetal force be destroyed, the body will fly off in a straight line, called a tangent.

Suppose the horse drawing a sled increases his speed. Two reactions now oppose the pull applied to the sled. One, friction, opposes the slipping of the sled over the ground; the other, due to inertia, opposes increase of speed. These two together are equal and opposite to the pull exerted on the sled.

These are only cases of confusion such as come up in every physics or mechanics classroom; centrifugal and centripetal forces balancing each other in circular motion, both acting on the same thing; friction and the "vis inertiæ" are the reactions to the pull exerted by a horse.

When one has endeavored to point out the nature of a difficulty, it is natural to ask him for the remedy. I am not pretending that I have found remedies for the difficulties mentioned above, some of which seem to be imposed upon us by the constitution of our minds and the environment in which the race has grown up. The only thing to do is to make every endeavor to break up the satisfaction of the student with the concepts which he has unconsciously formed, to try to contrive striking experiments which shall, for example, make plain that something more than the notion of weight is needed for their explanation, and, especially, to familiarize him with the concept acceleration and the various ways of arriving at its value, theoretically and practically. The teacher has almost to strive against instinct in the treatment of the laws of motion, and some people can never be expected to grasp them.

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