SPECIAL ARTICLES

A NEW FUNDAMENTAL EQUATION IN OPTICS

All the elementary text-books in physics are still using an equation for the conjugate foci of spherical lenses which is too inaccurate for the calculation of microscope objectives, applying with approximate accuracy only to very thin lenses. The same equation is all that is given in the more advanced treatises except those securing a closer approximation by the methods of higher mathematics.

It is possible, however, to develop a simple and rigorously accurate equation by geometry applicable to lenses of any thickness by the simple expedient of measuring the focal distances from the center of curvature of the lens instead of measuring it from the surface as has hitherto been the practise.

The equation is

$$\frac{n}{f}+\frac{n'}{f'}=\frac{n-n'}{r}\cdot\frac{\cos a}{\cos b},$$

in which n and n' are the indices of refraction, f and f' the focal distances of conjugate foci, r the radius of the lens, a the focal angle and b the radial angle. The meaning of these terms will be further explained below.

In the figure EQC' represents the paths of a ray of light refracted at the point Q on the surface of a lens whose radius is OQ.

OF and OF' are drawn parallel with QC' and OE respectively and in the triangle OQF

$$\frac{OF}{QF}=\frac{n}{n'}$$

since they are the sides opposite the angles OQF ($=180°$—angle of incidence) and QOF ($=OQF'$ the angle of refraction).

DD' is drawn through O, making $QD=QD'$, and the angle QOD is the one designated above as the radial angle. QP is perpendicular to DD' and

$$\cos b=\frac{OP}{r}.$$

Since OFD is similar to $OF'D'$ and $QF=OF'$,

$$\frac{OD}{OD'}=\frac{n}{n'},\quad OP=\frac{OD-OD'}{2}=\frac{OD}{2}\left(1-\frac{n'}{n}\right)$$

and

$$\cos b=\frac{OD}{2}\,\frac{n-n'}{rn}$$

or

$$OD=2\cos b\,\frac{rn}{n-n'}.$$

EE' and CC' are drawn through O, making equal angles with DD'. EOD is the angle designated above as the focal angle for the focus E. GH is drawn perpendicular to DD' and lines are drawn from G and H parallel to EQ. Since these three parallel lines are equidistant along GH, $GC=CJ$. The triangles OJH and OCE are similar and

$$\frac{OC}{OE}=\frac{OC-CG}{OC+CG}=\frac{2OC}{OC+CG}-1.$$

Dividing by OC and substituting OG for $OC+CG$ gives

$$\frac{1}{OE}=\frac{2}{OG}-\frac{1}{OC}.$$

The

$$\cos a=\frac{OD}{OG(=OH)}$$

and

$$\frac{1}{OE}+\frac{1}{OC}=2\frac{\cos a}{OD}.$$

Substituting the value of OD found in the last paragraph gives

$$\frac{1}{OE}+\frac{1}{OC}=\frac{n-n'}{rn}\cdot\frac{\cos a}{\cos b}.$$

From similar triangles OCD and $OE'D'$ we have

$$\frac{OC}{OE'}=\frac{n}{n'}$$

or

$$\frac{1}{OC}=\frac{n'}{nOE'}$$

which substituted in the above equation and multiplying by n gives the form of equation desired,

$$\frac{n}{OE}+\frac{n'}{OE'}=\frac{n-n'}{r}\cdot\frac{\cos a}{\cos b},$$

and it only remains to be shown that E and E' are conjugate foci. If the triangle EQE' is rotated upon EE' as an axis, at all points on the circle described by the point Q on the surface of the lens the light radiating from one

focus will be refracted towards the other and the two points E and E' are therefore conjugate foci, and f and f' may be substituted for OE and OE'.

A very similar solution which need not be given here can be obtained for the cases where one focus lies between Q and F or F' and the other on QE or QE' produced and which result in virtual instead of real images.

This equation applies to the refraction at one lens surface. For simple lenses or for lens systems two or more equations, according to the number of refractions, must be combined.

When the cone of light is narrow and does not diverge far from the optical axis the last factor $\cos a/\cos b$ becomes practically 1. This produces the simplest form of the equation. It can be used in calculating the foci of thick lenses in case the aberrations are neglected.

For the study of aberration the angles a and b can be calculated by solving the two triangles EDO and QDO in which EO and QO remain constant and the other sides vary according to the refractive index of the color of the ray of light investigated in the study of the chromatic aberration, or according to the position of Q when studying spherical aberration.

The usual equation found in the books can not be employed for either of the foregoing calculations when more than approximate results are required.

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ANTHROPOLOGY AT THE WASHINGTON MEETING

II

A New Type of Ruin Recently Excavated in the Mesa Verde National Park, Colorado: J. WALTER FEWKES.

An account of the excavation and repair of a new type of ruin on the point of a mesa opposite Cliff Palace, conducted under the auspices of the Interior Department and the Bureau of American Ethnology. Before the work was begun, the existence of a large building was indicated by a large mound, the surface of which was strewn with artificially fashioned stones, partly covered with soil, with a few feet of wall showing at one point. On top of the mound, at a place found later to indicate the highest wall, grew a large cedar tree, a cross-section of which revealed 360 annual rings. The building excavated is D-shaped, measuring 122 feet on the straight side and 64 feet broad. The standing walls now contain 120,000 cubic feet. The facing of the walls is artificially pecked with stone implements, and in many instances rubbed smooth. Many stones set in the walls or found in the débris bear incised ornamentation, the beginning of mural embellishment. The masonry is not only among the best in any prehistoric building north of Mexico, but the building itself is the most mysterious yet brought to light in our southwest.

There are evidences that it was neither completed nor inhabited, and evidently it was not intended for habitation. Its ground-plan exhibits a unity in design and a strict adherence to that plan throughout the construction of the building. It is believed to have been constructed by the neighboring cliff-dwellers; it is prehistoric and regarded as more modern than Cliff Palace. A fossil leaf of a palm in relief on the upper surface of the cornerstone at the western end of the building is believed to be a sun symbol, and the walls about it a solar shrine. The building is regarded as a sun temple of the neighboring cliff-dwellers, and is the first of its type yet excavated in the Mesa Verde National Park.

The Passing of the Indian: JAMES MOONEY.

The subject of the aboriginal population of America, and more particularly of the United States, at the first coming of the white man, has been a matter of much speculation, but of very little detailed investigation. There has been about as much error and loose statement on one side as on the other, some theorists claiming for the pre-Columbian period a dense population for which there is no evidence in fact; while others, largely those interested in various civilizing schemes, maintain that the Indian has held his own or is even actually increasing. The claim for a dense earlier population is based chiefly on ignorance of Indian living habit and the error of assuming as contemporaneous in occupancy settlement remains belonging to widely separated periods. The argument for stability or increase of the Indian population rests in part on the error of beginning the calculation with the beginning of federal relations with the tribes, ignoring the centuries of colonization and disturbance which preceded that period, and is also colored to some extent by a desire to draw good results from philanthropic and civilizing efforts.