Loria; Review of Pierpont's Functions of a Complex Variable, by H. P. Manning; "Shorter Notices": Snyder and Sisam's Analytic Geometry of Space, by R. M. Winger; Slichter's Elementary Mathematical Analysis, by L. C. Karpinski; Ford's Automorphic Functions, by A. Emch; Gibb's Interpolation and Numerical Integration and Carse and Shearer's Fourier's Analysis and Periodogram Analysis, by M. Bôcher; Herglotz's Analytische Fortsetzung des Potentials ins Innere der anziehenden Massen, by W. R. Longley; Lange's Das Schachspiel, by L. C. Karpinski; Ince's Descriptive Geometry and Photogrammetry, by V. Snyder; "Notes"; and "New Publications."

SPECIAL ARTICLES

THE PRESSURE OF SOUND WAVES

In his "Wärmestrahlung"[1] Planck, after proving from electromagnetic theory that the pressure of radiation equals the volume density of radiant energy, shows that the corpuscular theory of light would give a pressure twice as great. From this he infers that the Maxwell radiation pressure can not be deduced from energy considerations, but is peculiar to the electromagnetic theory and is a confirmation of that theory. The implied conclusion is that mechanical waves would not exert a pressure of this magnitude. It may be well to recall, therefore, that Lord Rayleigh has shown, from energy consideration,[2] that transverse waves in a cord exert a pressure equal to the linear energy density, and that sound waves in air must cause a pressure equal to the volume density of energy in the vibrating medium. Altberg[3] has made the conclusions of Rayleigh the basis of a method of determining the intensity of sounds.

As the pressure due to sound waves in a gas must be ultimately the result of molecular impacts, it would seem probable that the magnitude of this pressure may be determined from the elementary kinetic theory, and this proves to be the case. Consider an extended wave incident normally on a unit surface. According to the kinetic theory, the molecules which strike this surface are reflected with the same velocity that they had just before impact. As the surface is small in comparison with the extent of the wave front, we need not follow the history of these reflected molecules, which will immediately become dispersed in the passing wave in all directions. In other words, under these conditions no stationary waves will be formed by reflection, and we may confine our attention to the effect of the incident wave. Of course there will also be increased pressure on the rear surface due to the diffracted waves, but this will not affect the pressure on the front surface. At the instant that the wave front strikes the surface imagine the whole wave length divided into thin strips parallel to the surface, s in number and each of thickness x, so that sx is equal to one wavelength. The velocities of displacement due to the wave are mass effects, but it seems proper to add them to the different individual velocities of the gas molecules which move *en masse*. Let the velocities of wave displacement in the successive strips be $u_1, u_2, \ldots u_s$. The component velocities of translation of the gas molecules normal to the surface are U_1, $U_2, \ldots U_n$. The two other components contribute nothing to the pressure on the surface. The resultant velocity of the molecules having a velocity of translation U_1 in the first strip will be $U_1 + u_1$. As they are reflected with the same velocity, the change of momentum of each molecule is

$$2m(U_1 + u_1) = f.dt,$$

where m is the mass of each molecule and $f.dt$ the impulse of the force during collision. If N_1 is the number of molecules per unit volume having the velocity U_1, the number in the strip of thickness x is N_1x and if t_1 is the time required for the strip to move a distance x,

$$N_1x = N_1(U_1 + u_1)t_1.$$

Taking account of the fact that half the molecules of this class will be moving away from the surface, the total change of momen-

[1] "Wärmestrahlung," 2d ed., p. 58.

[2] *Phil. Mag.*, 3, 338, 1902.

[3] *Ann. der Phys.*, 11, 405, 1903.

tum of all the molecules of this class during the time t_1 is

$$N_1 m(U_1 + u_1)^2 t_1 = \Sigma f.dt.$$

The average pressure during the interval t_1 is $\Sigma f.dt/t_1$, therefore,

$$N_1 m(U_1 + u_1)^2 = p_1.$$

Similarly for all the strips as they successively strike the surface up to the last, where

$$N_1 m(U_1 + u_s)^2 = p_s.$$

Squaring and adding for all values of u from u_1 to u_s,

$$N_1 m(sU_1^2 + 2U_1 s\Sigma u_s + \Sigma u_s^2) = \Sigma p_s.$$

But Σu_s throughout the wave is zero, $\Sigma p/s$ is the average pressure during the impact of the whole wave, and $\Sigma u^2/s$ is u^2, the mean square velocity due to vibration, hence after dividing by s,

$$N_1 m(U_1^2 + u^2) = P_1$$

and the same is true of all the other classes of molecules with velocities from U_1 to U_n. If the total number of molecules of all classes is $N_1 = N_1 + N_2 + N_3$, etc., the final resultant effect after adding all the expressions for P_n will be

$$Nm(U^2 + u^2) = \Sigma P_n = P,$$

where U^2 is the mean square translational velocity $= \Sigma N_n U_n^2/N$. The kinetic theory shows that the pressure is NmU^2 when no sound waves are passing. Hence the increased pressure due to the waves is

$$Nmu^2 = \rho u^2,$$

where ρ is the density of the gas.

If the equation of the wave motion is

$$y = a \cos (2\pi/\lambda)(x - Vt),$$

$$u = dy/dt = a(2\pi/\lambda) V \sin (2\pi/\lambda)(x - Vt),$$

and, since the mean value of $\sin^2$ is 1/2,

$$u = \tfrac{1}{2}a^2(2\pi/\tau)^2 = \tfrac{1}{2}a^2\omega^2,$$

and the pressure due to the waves is $\rho u^2 = \frac{1}{2}\rho a^2\omega^2$, which also represents the maximum kinetic energy or mean total energy of the waves per unit volume, in agreement with Rayleigh's conclusion.

The same result might have been reached directly by assuming that the pressure of a gas is proportional to the mean square velocity of the molecules, however that velocity may be produced. The symmetrical positive and negative values of u would cause the products $U_n u_s$ to vanish in forming the squares of the resultant velocities, so that u^2 would be the increase in the mean square velocity, leading to the same result as that given above.

When we consider the propagation of sound waves in air in molecular rather than in mass terms the expression potential energy loses its meaning. The entire energy of the waves may be expressed in terms of molecular kinetic energy. The conclusion that $p = \rho u^2$ is equivalent to saying that the pressure due to sound waves is equal to twice the mean density of kinetic energy in the medium. When stated in this form, the results agree with those obtained by Planck for the corpuscular theory. The mean kinetic energy is twice as great in one case as in the other.

In the case of stationary waves, the energy density is evidently twice as great as in the incident waves alone; and the mean square velocity from node to node deduced from the mathematical expression for the wave disturbance, and hence the pressure, is likewise twice as great.

The absolute temperature of a gas is proportional to the mean square velocity of the molecules. Ordinarily we should limit this relation to the case where the motion is entirely chaotic, not *en masse*. In either progressive or stationary waves there is an increased mean square velocity in the direction of propagation which would record itself as an increase of temperature on any measuring instrument. In particular, at the loops of stationary waves where there are no density changes no lateral change of pressure would occur, while in the direction in which the waves travel there would be an increase of mean square velocity. In a sense there would be a state of polarized temperature. A thin bolometer strip would undoubtedly indicate a higher temperature when the waves are incident on its flat side than when they are incident on its edge. The maximum sound-wave pressure found by Altberg, for very intense stationary waves, was about .26 dyne. Since the pressure of a gas is proportional to the absolute temperature, $dT/T = dP/P$. From this it may be calculated that the increase of temperature indicated by a thin bolometer strip on which the waves exert a pressure of .26 dyne would be about .000075° at atmospheric pressure and a temperature of 17° C. or 290° absolute.

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RUDIMENTARY MAMMÆ IN SWINE A SEX-LIMITED CHARACTER[1]

The inheritance of the rudimentary mammæ found on the lower part of the scrotum of the boar and on the inside of the thighs to the rear of the inguinal pair in the sow, was reported as typically sex-limited by the writer in 1912 and 1913. Later, in 1914, due to the failure to discover a boar homozygous for the character, an attempt was made to classify the inheritance as sex-linked in nature. Certain more recent discoveries, due largely to a few selected matings, have cleared up the difficulties which in 1914 were believed to exist, and make the earlier interpretation more probable.

The case in point is as follows: A Duroc Jersey boar possessing the rudimentaries was mated to a grade black sow lacking them. A litter of nine pigs was farrowed, four of the boars having rudimentaries, and one lacking them, while three of the sows lacked rudimentaries and the fourth possessed them. Coupled with the evidence on the inheritance of this character published previously, this breeding performance indicates that both the Duroc Jersey boar and the grade black sow were heterozygous for this character.

One of the boars possessing rudimentaries from this litter was mated to the four sows of the litter with the following results:

[1] Paper No. 2 from the Laboratory of Animal Technology, Kansas Agricultural Experiment Station.

Record Number	Apparent Hereditary Constitution	Males		Females	
		With Rudimentaries	Without Rudimentaries	With Rudimentaries	Without Rudimentaries
Sow 26......	RR	4	0	3	0
Sow 27......	Rr	4	0	3	2
Sow 28......	rr	3	0	0	2
Sow 29	rr	4	0	0	4

This breeding performance very definitely indicates that the boar was homozygous for the rudimentary mammæ. All of the boar pigs that he sired possessed the character, even though two of the sows were of a type not to transmit it at all. If he were heterozygous for the character, then at least part of the seven male pigs from sows 28 and 29 should have lacked the rudimentaries; the chances of their all having them being one out of 128. The discovery of a boar homozygous for the rudimentaries removes the principal stumbling block to the simple sex-limited theory.

Davenport and Arkell have developed a scheme which bridges the discrepancies between sex-limited and sex-linked inheritance, even when apparently homozygous animals exist. Since, however, the sex-limited explanation advanced by Wood seems to cover all the facts that are involved in this case, and since it is much simpler, the writer prefers thus to interpret these results.

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THE NATIONAL ACADEMY OF SCIENCES

The sessions of the annual meeting of the National Academy of Sciences were held in the United States National Museum, Washington,