The second chapter deals with damping and how it is obtained in meters. Permanent magnet instruments, iron core instruments, electrostatic meters, hot-wire meters and dynamometer meters each receive one chapter.

Watt-hour meters are discussed at some length; the errors in reading due to friction, short circuits, etc., are illustrated by experimental results. Magnetic testing apparatus is described and typical results given. The last chapter deals with the Wheatstone bridge, the Kelvin double bridge, and the potentiometer, for both continuous and alternating current circuits.

The writer knows of no book on electrical measuring instruments which is its equal in value to the advanced engineering student. A companion volume dealing with oscillographs, ondographs and other special devices is promised by the authors for the near future; it should receive a hearty welcome.

J. H. M.

SPECIAL ARTICLES

A NEW METHOD FOR THE GRAPHICAL SOLUTION OF ALGEBRAIC EQUATIONS

THE writer recently devised a graphical method for the solution of algebraic equations that seems to be of such general interest and importance as to be worthy of publication in this journal.

Let us consider first an equation of the type

$$f(u) \cdot f(x) + f(v) \cdot F(x) + f(y) = 0,$$

where $f(u)$, $f(v)$, $f(x)$ and $f(y)$ are the same or different functions of u, v, x and y. Construct a chart (shown in outline in Fig. 1) consisting of three vertical axes, P, Q and R, any convenient distance apart, intersected by a horizontal axis H. Along the right side of the axis P plot the calculated values of $f(x)$, positive values being laid off *upward* and negative values *downward* from the horizontal axis at a rate of A units per centimeter, A being so taken that the values of $f(x)$ likely to be met will fall within the limits of chart.

In a similar way lay off values of $F(x)$ along the left side of the axis R, positive values being measured off *upward* and negative values *downward* from the horizontal axis, at the rate of B units per centimeter, B being so taken that the values of $F(x)$ likely to be met will fall within the limits of the chart.

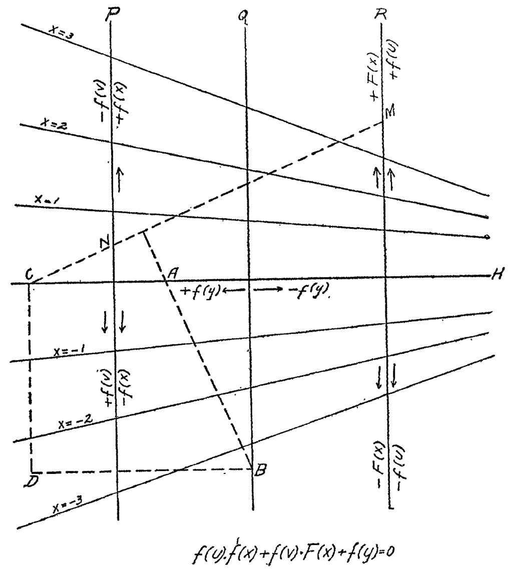

FIG. 1.

Along the horizontal axis lay off values of $f(y)$ at the rate of C units per centimeter, positive values of $f(y)$ being laid off to the *left* of the middle axis Q, and negative values of $f(y)$ to the *right* of that axis; C, being so taken that the values of $f(y)$ likely to be met will not lie too far to the right or to the left of the middle axis Q. Label the points thus located with the values of y used in calculating those of $f(y)$.

Values of $f(v)$ are to be laid off along the axis P in a way similar to that employed in laying off values of $f(x)$, positive values being measured *downward* and negative values *upward* from the horizontal axis H, at the rate of C/mB units per centimeter, where m is the perpendicular distance in centimeters between the outside axes P and R. Label the points thus located with the values of v to which they correspond.

In the same way, calculated values of $f(u)$ are to be laid off along the axis R, positive values being measured *upward*, and negative values *downward*, at the rate of C/mA units per centimeter. Label the points thus located with corresponding values of u. To finish the construction of the chart, connect each value of $f(x)$ on the axis P with the corresponding value of $F(x)$ on the axis R by means of a straight line of indefinite length, which is labelled with the value of x to which it corresponds. In Fig. 1, several of such lines have been drawn and marked with the values $x=1$, $x=2$, etc.

Let it be supposed that the values of u, v and y in any particular example are known, and that the value of x is to be calculated. Locate the point M on the scale of $f(u)$ (axis R), marked with the given value of u. Locate the point N on the scale of $f(v)$ (axis P) marked with the given value of v. Connect M and N, and note the point of intersection, C, of this line or its prolongation with the horizontal axis H. Locate a point A on the scale of $f(y)$ corresponding to the given value of y. From A, draw a line perpendicular to the line MN, and note where its prolongation intercepts the middle axis at B. From B, draw a horizontal line, and from C a vertical line, intersecting at D. It will now be noted that D lies on a certain straight line, which is labelled with the value of x required; or it lies between two such lines, and the required value of x may be read by interpolation.

In practical work the chart shown in outline in Fig. 1 would be constructed on cross-section paper. We should need, in addition, a sheet of transparent paper or tracing cloth, having two perpendicular lines ruled on its surface. To solve an equation of the form we have been considering, simply move the transparent sheet back and forth over the chart until one of the two perpendicular lines appears to pass through the given value of u at M, and the given value of v at N, at the same time that the other perpendicular appears to pass through the point A corresponding to the given value of y. It will now be easy to note the apparent points of intersection of the perpendiculars with the vertical and horizontal axes at B and C respectively; and by following along the vertical and horizontal cross-sectioning from B and C we may locate the point D, and thus determine the required value of x, without actually drawing any construction lines on the chart itself.

As a special example of the use of such a chart, let us consider the calculation of the maximum temperature obtainable with natural gas burned without excess of air. The equation will be of the form $ax+bx^2=c$, where the value of the coefficients a, b and c depend on the composition of the gas, and the specific heat at various temperatures of the products of combustion. In a particular case, let the equation be

$$3.2044\,t + 0.00074057\,t^2 = 8{,}203 \text{ calories.}^1$$

The construction of the chart is simplified by the fact that the coefficients of t and t^2 to

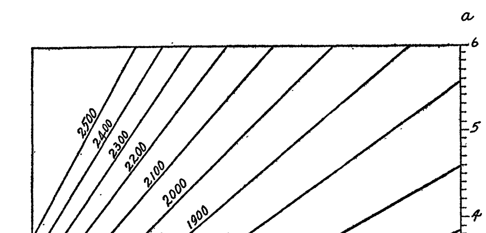

Fig. 2.

be considered will always be positive (Fig. 2). In this chart, the middle axis Q is moved to the left until it coincides with the left-hand

[1] Richard's "Metallurgical Calculations," Vol. I., p. 41.

axis, for the reason that the values of y (calories) to be considered will always be negative when transferred to the left-hand side of the equation, and will therefore lie to the right of Q. Since negative temperatures are not to be considered, the horizontal axis may be placed at the bottom of the chart.

In the construction of the chart let the vertical axes be taken 20 centimeters apart. Along the left-hand axis lay off values of t in an upward direction, at the rate of 100 units per centimeter ($A = 100$); in this way, the chart can be used for temperatures up to about 2500° C., if it be 30 centimeters high.

In a similar way, calculate the values of x^2 and lay them off in an upward direction along the right-hand axis, at the rate of 100,000 units per centimeter ($B = 100{,}000$). Since the maximum value of x to be considered is about 2,000, the graduations along the right-hand axis will extend about $2{,}000^2/100{,}000 = 40$ centimeters above the horizontal axis.

From left to right along the horizontal axis, lay down a scale for the various values of y, at the rate of 500 units per centimeter ($C = 500$). In this way, the maximum value of y (about 10,000) will lie about 20 centimeters from the left-hand end of the horizontal scale.

Along the left-hand axis, in a *downward* direction from a second horizontal axis (located at any convenient distance above the bottom of the chart), lay down a scale of coefficients of t^2, at the rate of

$$\frac{C}{mB} = \frac{500}{20 \times 100{,}000} = 0.00025 \text{ units per centimeter.}$$

Along the right-hand axis, lay off in an upward direction a scale of coefficients of t, at the rate of

$$\frac{C}{mA} = \frac{500}{20 \times 100} = 0.25 \text{ units per centimeter.}$$

To solve the particular quadratic equation given above, lay a transparent sheet bearing two perpendicular lines over the chart, so that the value of a (3.20), at M, the value of b (0.00074), at N, and the value of c (8203), at A, are crossed by the perpendicular lines of the transparent sheet. Note the point of intersection with the left-hand vertical axis at B, and with the auxiliary horizontal axis at C. From B and C follow along the horizontal and vertical cross-sectioning (not shown in Fig. 2) to locate the point D, where the required value of x (1805°) is read directly from the chart.

In Fig. 3 we have a further illustration of the use of such a chart in the solution of the equation

$$a \log x + b\sqrt{x} = c.$$

There are two values of $\sqrt{x}$, a positive and a negative one, for each value of x or $\log x$. There are accordingly two lines to be drawn from each value of $\log x$ on the left axis to connect with the corresponding values of $\sqrt{x}$ on the right axis. One of the two sets of lines thus formed has been shown in the figure by dashes.

Solution of particular equation

$$72.5 \log x + 6.25\sqrt{x} = 54$$

is shown in the chart, the point D representing the value of x required. It will be noticed that in this case there are three different sets of lines that cross the region in which D happens to fall. There are accordingly three real roots to the given equation, the values read from the chart being 3.8, 10.5 and 530.

It is apparent that the number of real roots to any equation of the general form $a \log x + b\sqrt{x} = c$ will depend upon the values of the coefficients a, b and c. In the region in which the point D is shown, there are always three real roots, one of these satisfying the equation if a positive value of $\sqrt{x}$ be taken; the other two if a negative value of $\sqrt{x}$ be taken. In the region of the chart in which the point B falls, there is but one real root of the equation. If negative values of the coefficient b are considered the chart may need to be extended to the left of the left-hand axis; there will be two real roots in this region. If negative values of the coefficient a are considered the chart may need to be extended to the right of the right-hand axis. The trend of the lines in the diagram indicates that in this region there will in general be but one real root of the equation when a is negative; but in certain special instances, as for example, when a is negative and c is very large, we may have two real roots; and there are other portions of the field where three real roots occur.

A use of the diagram, apart from the solving of equations, is thus to indicate the number of real roots that exist in the case of particular values of a, b and c. It is apparent that the chart will also indicate the effect of changes in the magnitude of the coefficients a, b and c on the absolute value or sign of x; and the reader will perceive that transcendental equations beyond the range of ordinary algebraic methods can be solved by this means.

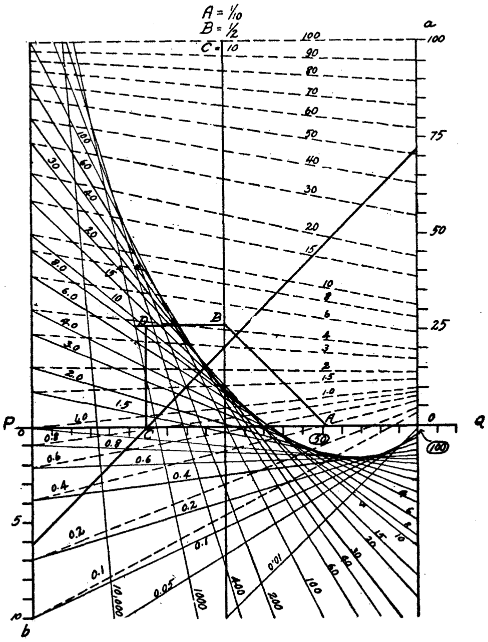

FIG. 3.

A further use for a chart of this kind is to suggest a proper empirical equation for the representation of experimental results. Thus, if the data collected in a series of experiments are believed to be expressible by an equation of the form $y = ax + bx^2$, the chart given in Fig. 2 may be used to determine the proper value of the coefficients a and b. The details of this procedure hardly require explanation; and other diagrams have already been published that constitute a graphical substitute for the method of least squares.

Returning now to the general case, it is evident that if $F(x) = 0$, we have

$$f(u) \cdot f(x) = f(y);$$

in this case the scale for $F(x)$ shrinks to a point at zero, through which all the lines representing different values of x must pass.

If $f(x)$ and $F(x)$ are constants the general equation takes the form

$$af(u) + bf(v) = f(y).$$

This may be charted as the so-called "alinement chart," well known to students of graphical mathematics.[2]

If $F(x)$ is replaced by $f(z)$ in the general equation we have five variables to consider. In this case $f(z)$ is plotted along the right-hand axis, the series of lines marked with the different values of x being omitted from the chart when the latter is first constructed. To use such a modified chart locate the point D in the usual way, then pass a straight line through D and that point on the right axis marked with the given value of z. The point of intersection of this line or its prolongation with the left axis gives the required value of x.

If two equations be given in which the values of x and z are to be determined, we locate two points D and D' in the usual way from the given values of u, v and y. Draw a straight line passing through D and D'. Its intersections with the left and right axes will give the values of x and z which simultaneously satisfy the equations. A set of three simultaneous equations of the general form

$$f(u) \cdot f(x) + f(v) \cdot f(z) + f(y) = 0$$

may be solved by an extension of this method.

It will be noticed that in the case last considered we are treating five variables, instead of the three that are included in the ordinary alinement chart. It was, indeed, by an extension of the principles of the alinement chart that the method presented in this paper was devised.

Exponential equations of all sorts may be handled by this method. Thus

[2] See, for example, Peddle, "The Construction of Graphical Charts," New York, McGraw-Hill Book Co.

$$x^u \cdot y^v = z$$

can be put in the logarithmic form

$$u \log x + v \log y = \log z,$$

and charted immediately.

It is even possible to combine two or more charts of the general type we have considered, enabling us to solve equations containing four or more terms. The method is thus almost one capable of handling algebraic equations in general; but further development of the subject would be out of place here.

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THE COORDINATION OF CHROMATOPHORES BY HORMONES[1]

The melanophores of the horned toad, *Phrynosoma cornutum* Harlan, become contracted during states of nervous excitement. All attempts to prevent this reaction locally by cutting various nerves have failed. It is thus suggested that the melanophores may be coordinated, in part, by a hormone produced during nervous excitation and carried to all parts of the skin by the circulation.

The skin of one leg may be isolated from the general circulation, without blocking its nerve supply, by tying a ligature snugly about the leg. When this is done the melanophores of the isolated leg remain expanded after the animal is thrown into a state of nervous excitement. The leg appears much darker than its mate. Upon removing the ligature the melanophores contract and the leg becomes pale. The effect is not due to a shortage of oxygen or the accumulation of metabolic products in the leg, for such effects do not influence the melanophores of a ligatured leg until much later and then they produce a *contraction* of the pigment cells. If blood drawn from a horned toad which is in a state of nervous excitement is injected into one of the subcutaneous lymph-spaces of a second animal, the skin above the lymph-space will become very much paler than that of the rest of the body. The injection of blood from a horned toad which has not been thrown into a state of nervous excitement does not have this effect. During states of nervous excitement the blood contains a substance which causes the pigment cells to contract.

What is this substance and where is it produced? The conception of a hormone coordinating melanophore activity is not altogether novel, for Fuchs (1914, pp. 1546–1547, 1651–1652) has attempted to explain the behavior of pigment cells in amphibian larvæ and reptiles by assuming that substances, perhaps internal secretions, which contract the melanophores, are produced in the body under the regulation of the pineal organ. Laurens (1916) has recently shown this hypothesis to be inapplicable to the phenomena observed by him in *Amblystoma punctatum*. That the pineal organ is not concerned, primarily at least, in the reaction in the horned toad is proved by the fact that removal of the entire brain anterior to the cerebellum does not prevent the melanophores from contracting during states of nervous excitation.

The studies of Cannon and his collaborators upon the physiology of the major emotions present a more promising clue to the nature of this hormone. Cannon and de la Paz (1911) have shown that during states of emotional excitement the adrenal glands are activated to such an extent that an increase in the adrenin content of the blood from the adrenal vein may be detected. Spaeth (1916) has amassed a formidable array of facts to prove that the melanophore is "a disguised type of smooth muscle cell." If Spaeth's contention be accepted, it would appear most probable that the melanophores should be controlled by adrenin, which occupies a particularly significant position in the physiology of smooth muscle (compare Elliott, 1905).

Adrenin has been shown to produce a contraction of the melanophores of the frog (Lieben, 1906) and of *Fundulus* (Spaeth, 1916). Very minute quantities have this effect upon the melanophores of the horned toad. Removal of the adrenal glands does not prevent

[1] Contributions from the Zoological Laboratory of the Museum of Comparative Zoology at Harvard College, No. 273.