SCIENCE

FRIDAY, FEBRUARY 4, 1916

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PONCELET POLYGONS[1]

THERE is nothing which can not be known. Such at least is the postulate of science. Wide as is the universe of matter, numberless as are the objects and the events in the world of either dead matter or living organisms, yet the scientist must have faith that all can be observed, classified, named; that a finite number of terms and a finite system of laws will suffice ultimately for the summing up of what we call the external universe. A dream, if one regards it as a positive expectation! Yet how far it has gone in the direction of realization in certain obvious horizons! In our solar system it is not frequently that a major planet is discovered. In the chemist's domain, does any one concede that the unknown elements are more in number than the known? Does any physicist really expect to come upon a new kind of activity at all comparable in importance with the Röntgen rays? Though the ideal of complete knowledge and perfect explanation may be destined never to be reached, yet how prone are we to imagine that it must be not far away!

In a certain contrast to the material world stands the world of intellect and reason, a contrast partly at least fictitious, but also in part intrinsic. It is in this world that geometry exists. Whatever else be true about geometry, it is plain from experience and from history that its objects are ideas or notions; that they are comple-

[1] Address of the retiring vice-president and chairman of Section A of the American Association for the Advancement of Science, Columbus, December 30, 1915.

mentary to, not extracted from, the material world. Knowable they are, therefore, by their very constitution. But who can ever conceive of them as limited in number? Who can imagine that ever in the future it could come to pass that there should be no more geometric concepts to be investigated; that a point might be attained where the mind of the mathematician should rest satisfied, all its curiosity appeased?

Connected with this contrast in the source of its objects is the slowness with which new objects in geometry emerge and diffuse into general knowledge. Called into being by shifting stimuli, multitudes of new systems of relations are invented and named and investigated; but most of them are speedily forgotten (or perhaps only dimly apprehended even by the discoverer), and very few in a century are those which survive to become the valued heritage of later generations.

There are many occasions when we meet to discuss only what is new. The present, however, is a fitting occasion for reviewing together some of the treasures handed on to us by geometricians of the past, and for stimulating our own ardor by the rehearsal of the fortunes and successes of earlier workers in our part of the field of science. The polygons of Poncelet were new a hundred years ago, and are not yet forgotten, but seem rather to attract increasingly the interest and attention of geometricians. I invite you to enjoy with me, since though not unknown they are not yet in the class of familiar objects, a rapid survey of their character and development.

For many centuries before Euler students of geometry had found interest in circles inscribed in a triangle and circumscribed to it. Usually their centers do not coincide. One circle may be kept stationary, while the triangle varies, and with it vary also the center of the other circle and its radius. Euler may have been the first to write out the relation that connects these three quantities, the two radii and the distance of the centers: $R^2 = 2Rr + d^2$, or it may have been discussed a hundred times before. Publication of this relation led to the study of analogous relations for polygons of more sides, Fuss in St. Petersburg, and some years later Steiner in Berlin, carrying the problem farthest, finding results for polygons of 4, 5, 6, 7 and of 8 sides. The case of regular polygons, for which the inscribed and circumscribed circles are concentric ($d = 0$) is elementary, and will always stimulate interest in the more general problem.

While attention was directed to finding an algebraic relation corresponding to a given geometric diagram, for a long time no one seems to have inquired whether this relation was merely a necessary condition, or whether it might also be a sufficient condition for the construction of the diagram. If two circles are drawn, satisfying the condition for a triangle: $R^2 = 2Rr + d^2$, can one always determine the triangle inscribed in the circle radius R and having its sides all tangent to the circle of radius r? And is there only one such triangle in each case, or some finite number greater than one? What of the case where the triangle (or polygon of 4, 5 or more sides) is regular—is it exceptional that for that case there are an infinite number of polygons which satisfy the requirements, provided there is one such?

It is not easy to apprehend the state of geometric knowledge in 1796, when Fuss wrote on this subject. He certainly supposed that a triangle could occur singly, and was unaware that others can always be inscribed and circumscribed to the same pair of circles. It would seem as though the roughest kind of experimentation would have shown the truth, or at least would have given grounds for a hypothesis. But Fuss limited his investigation, so Jacobi states, to the case where the polygon is symmetrical with respect to the common diameter of the two circles. Symmetrical-irregular polygons, he calls them; and this Fuss supposed to be essentially a restriction upon the generality of the problem, and hence he believed that he had solved only under limitations the problem proposed. This misapprehension apparently persisted for 26 years, until the appearance in 1822 of Poncelet's memorable work: "Traité des Propriétés Projectives des Figures." Indeed there is indirect evidence to this effect in an essay by Poncelet himself, of the date 1817, in which he challenges his correspondent to solve the problem of inscribing in a given conic a polygon of n sides, the sides to be tangent to a second given conic. This problem as stated is, as we now know, misleading, implying that there is a solution, and that the number of solutions is finite. Poncelet would hardly have ventured to publish such a problem had he not been sure that the mathematical public of that day would accept it in good faith.

It would be quite certain also, even if we had no direct knowledge of the fact otherwise, that the relations of collinearity and correlativity or reciprocity with respect to a conic were not at all commonly understood prior to 1822. The employment of transformations to derive one solution of a problem from another was not yet a recognized preliminary to all discussion. The student of conics to-day will reflect at once that two conics not specialized in situation have one self-polar triangle in common, and are transformed into themselves by three collineations or projectivities besides the identity, and are transformed simultaneously into each other by four reciprocities or polarities with respect to a third fixed conic. Thus to-day we should see in advance that any one triangle, or one pentagon, inscribed in one conic and circumscribed to another, implies seven others of the same sort. Solutions of Poncelet's problem must occur at least in sets of eight; but this fact, apparent from Poncelet's own discoveries, appears to have escaped his attention, and still less was it present to the minds of his contemporaries.

Knowledge of the investigations of Fuss and of Euler would have been almost useless to Poncelet. For the far superior generality of his problem, that of two conics in place of two circles, his method of projection is responsible. This allowed him to use metric properties of circles and draw conclusions concerning any two curves of the second order. But the discovery of his famous theorem on polygons was nothing less than a stroke of genius. Many have been quoted as authors of the saying that invention or discovery is the principal thing in geometry, while the proof is a relatively easy matter. In this case, however, the proof also is ingenious, carried on by the exclusively synthetic method. But the perception of the theorem, preceding its proof, escapes explanation from anything that had gone before. Were that his only contribution to our knowledge of geometry, it would ensure him grateful recognition from later students—as the compeer of Apollonius who gave us the foci of a conic, Desargues who first perceived poles and polars, Newton who described the organic construction of conics, and the immortal Pascal with his hexagon. Let us rehearse the theorem which gives a generic name to Poncelet polygons.

Of two given conics, call one the first and consider its points; call the other the second and consider its tangents. Form a broken line by taking a point of the first curve, a line of the second that passes through it, then another point of the first on this line, and so forth. It may be that this process will close, the last line passing through the first point. If it does close, forming a polygon of n sides with vertices on the first conic and sides tangent to the second, then every point of the first is a vertex of one such closed polygon, and every tangent of the second is a side of one such polygon of n sides.

That is the first part of the theorem. The second is this. Diagonals of all these closed polygons, which omit the same number of consecutive vertices of the polygon, are tangent to a fixed third conic; and the dual statement is true concerning points of intersection of non-consecutive sides. *This latter part of the theorem is true even if the polygon is not closed.* From some points of view this scholium exceeds in importance the principal theorem.

These statements give us a specific attitude toward the conics. We look upon the first as a groove prepared to guide a set of sliding points, and the second as a directrix for lines joining the points. If the lines are indefinitely extended, there will be outlying systems of crossings; a first extra set whose motion will describe a first extra conic; a second extra set with its conic locus, and so forth. The case where the polygon is closed is that in which one of these extra loci coincides with the first conic.

We may digress to notice a curious fact. The sides of an inscribed hexagon meet in 15 points, namely, six on the conic, three on the Pascal line and six which we may term for the moment extra points. These six extra points are vertices of a hexagon *circumscribed* to a second conic. If now the first hexagon, already inscribed to one conic, becomes circumscriptible, then the hexagon of the extra points, already circumscribed to a conic, becomes inscriptible to another. This separation of two properties which occur together in all polygons of the Poncelet type is a situation deserving further attention.

To return to Poncelet: His discovery of the mobility, or the infinite multitude, of these polygons upon two fixed conics, published in 1822, must have seemed to mathematicians of that day as startling as the announcement of a new genus of vertebrates by a traveler returning from distant lands. Its exact character had to be ascertained and settled. The possibilities of variation must be examined; as, for example, whether all the sides of the polygon need be required to touch the same conic. Here it was seen by Poncelet himself that if all conics concerned pass through the same four basis points, then it is sufficient for the purpose if each side in its order touches its own assigned conic—all the vertices will still be movable on their common track. After this, it seems like a new proposition to assert that the order in which the fixed conics are touched by successive sides may be varied, and still the polygon will close in the same number of sides. And it is a new proposition, as announced within the last few years by Rohn, provided not merely their order, but also their *cyclic* order, is altered. Whether in this generalized figure the extra points still describe loci of the same family, that I do not remember seeing demonstrated.

The fertile mind of Jacobi seized the germ idea of periodicity in this closed figure, so closely resembling sets of arguments of the elliptic functions differing by aliquot parts of a period. This suggestion was the more natural because of the geometrical diagrams current in the definition of elliptic arguments. Only six years after the date of Poncelet's book, we find (1828) in Crelle's *Journal*, Vol. III., Jacobi's brief and elegant essay on these polygons for the case of two circles. Steiner had but recently written on the same topic, apparently unaware that it had been approached before. Jacobi was able now in the light of Poncelet's theorem to vindicate the claims of Fuss to priority, since his *irregular-symmetric* polygons were particular cases in every infinite set of Poncelet polygons on the same pair of circles. Jacobi further applies the recursion formulæ arising in the iterated addition of a constant to the elliptic integral of the first kind. Note his compact and expressive formulæ. If the radii are R and r, the distance of their centers a, and the n-gon encircles the centers i times before closing, all this is duly contained in the three formulæ

$$\int_0^{\alpha} \frac{d\varphi}{\sqrt{(1 - kk\sin^2\varphi)}} = \frac{i}{n}\int_0^{\pi} \frac{d\varphi}{\sqrt{(1 - kk\sin^2\varphi)}};$$

$$\cos\alpha = \frac{r}{R + a};$$

$$kk = \frac{4aR}{(R + a)^2 - rr}.$$

By this apparatus he verified the conditions already calculated for the closure in 3, 4, 5, 6, 7 sides, and confirmed for 8 sides the result of Fuss in opposition to Steiner's formula.

Certainly there is something satisfactory in seeing similar steps in geometric construction replaced by successive additions of one fixed quantity to an elliptic argument. But the problem was originally one of algebraic geometry, in so far as the conic represents a quadric form and the conditions of incidence and contact are algebraic; hence it was to be expected that there would be investigators who would not be satisfied with this transcendental elucidation of Jacobi, but would insist upon algebraic treatment throughout. Moreover, when once the projective treatment of figures had acquired prestige in pure geometry, it made inroads rapidly in the analytic territory. It was then desirable to solve the problem in its generality, for two conics whose equations are given arbitrarily, not restricting them to be circles; and to use processes and nomenclature that would not be affected by linear substitutions upon the coordinates or collineation. These last two desiderata appealed to Cayley not long after 1850, and from time to time he worked out parts of the problem: to express in invariants of two quadrics the condition that a broken line inscribed in the one and circumscribed to the other shall close in n sides. The results are not stated in terms of *rational* invariants, but they have the very great merit of being quickly and easily perceived, and of requiring only invariant terminology. The discriminant of a quadric is perhaps the best known of all invariants. For a quadric with one linear parameter he requires the discriminant to be calculated—namely for $F + K\phi$, where $F = 0$ and $\Phi = 0$ are the equations of the two conics, respectively. This is of degree 3 in the parameter.

$$D = A + bK + cK^2 + dK^3.$$

Next, the square root of this discriminant is developed formally in ascending powers of K;

$$\sqrt{D} = \sqrt{A} + B_1K + B_2K^2 + C_1K^3$$
$$C_2K^4 + D_1K^5 + D_2K^6 + \text{etc.}$$

The conditions of closure are now, in form at least, simplicity itself, namely, the vanishing of a determinant whose constituents are coefficients in this development. For an odd number of sides in the polygon, the leading constituent is C_1; for an even number, C_2, thus:

For 3 sides,

$$C_1 = 0,$$

For 5 sides,

$$\begin{vmatrix} C_1 & C_2 \\ C_2 & D_1 \end{vmatrix} = 0,$$

For 7 sides,

$$\begin{vmatrix} C_1 & C_2 & D_1 \\ C_2 & D_1 & D_2 \\ D_1 & D_2 & E_1 \end{vmatrix} = 0;$$

For 4 sides,

$$C_2 = 0,$$

For 6 sides,

$$\begin{vmatrix} C_2 & D_1 \\ D_1 & D_2 \end{vmatrix} = 0,$$

For 8 sides,

$$\begin{vmatrix} C_2 & D_1 & D_2 \\ D_1 & D_2 & E_1 \\ D_2 & E_1 & E_2 \end{vmatrix} = 0, \text{ etc.}$$

When one of these conditions is satisfied, the corresponding polygons are inscribed in the conic $\Phi = 0$, and circumscribed to the other. To test two given conics by this method would evidently involve considerable labor, but it would have the merit of being straightforward work, all of one kind—the calculation of determinants. Only one such would enter, the square root of the discriminant of the conic that carries the tangents, hence rationalization would be easy. It is hardly likely that results more elegant will be reached by any method; yet there are later researches, that I have not yet been able to examine, highly praised by reviewers. It does not appear that Cayley has given any account of the modifications necessary in these conditions when the sides touch different curves of the pencil.

Two other questions, however, were started by Cayley. The first is that of the relations in terms of the two invariant cross-ratios of the two conics—those belonging to the four common points or the four common tangents in the one conic and in the other. Conditions that exhibit a recursive law of formation in one domain of rationality are quite certain to do the same in a different domain, and Halphen has carried out the solution of this problem to completion (if that is a possibility) in his Elliptic Functions, Part 2. His interest in the geometry of the figure led him to propose the question, How many conics in a *linear system* can serve as loci for the vertices of a polygon of m sides, the sides to be tangent to a fixed conic? The answer is, for a polygon of 3 sides, 2 conics; for 5 sides, 6 conics; for 6 sides, 6 conics; in general

$$\frac{m^2}{4}\left(1-\frac{1}{p^2}\right)\left(1-\frac{1}{q^2}\right)\left(1-\frac{1}{r^2}\right)\cdots,$$

where all the prime factors of m are p, q, r, etc.

Cayley's second new problem in this connection was one concerning curves other than conics. If m_i denotes the order; μ_i the class of any curve, and it is required to describe a closed polygon beginning from a vertex A upon a curve of order m_1, drawing a side that shall touch a curve of class μ_1 and meet in a second vertex a curve of order m_2, and so on, then the number of solutions is twice the continued product of the m's and the μ's. This implies that the curves are all different, and calls for modification when coincidences are required.

Cayley initiated, but Hurwitz carried to completion, an algebraic explanation of the mobility of the Poncelet polygon whenever it actually exists. This, which is much the simplest method of attack, is by means of a correspondence upon a rational curve or line. The conic is a rational curve, and its points or its tangents can be given by quadric functions of a single parameter. In the presence of a second conic to carry the tangents, any point of the first corresponds primarily to two others, namely those two points in which the first conic is cut again by tangents to the second conic drawn from the first point. Such a correspondence is symmetrical two-to-two or (2, 2). Points further removed from any given first point are related to it secondarily, or more remotely, by a derivative

(2, 2) correspondence between the parameters. Hence there should be $2+2$ closures, whatever the degree of remoteness demanded between first and last points. But exactly four, improper indeed, are supplied by the participation of the four common points or the four common tangents. The relevant algebraical equation for the parameters will always have four roots relating to improper or degenerate polygons. If it has any more than these four, it admits all parameter-values as roots. Hence one actual *proper* polygon of m sides proves the existence of countless others. This brief but conclusive reasoning gives the problem its true setting in advance, but leaves for other methods the question of the existence of the all-important *first proper* polygon.

Gino Loria, in his memorable work, *Il passato id il presente delle principali teorie geometriche,* makes mention of these papers of Hurwitz at the climax of his paragraphs on theorems of closure; and says of the earlier essay, that in it "we do not know whether to wonder more at the immensity of the view, or at the perfection of its beauty; and so with this we bring to an end this digression, for which we should seek in vain a close more worthy." I have preferred however to summarize it earlier, in order to make clear with the greater brevity certain other applications that depend upon the same principle.

It is hardly needful to remind you that the (2, 2) correspondence leads inevitably to elliptic functions, as Euler long ago pointed out. If we picture the situation by means of a Riemann surface, it must have two leaves and four branch-points; and is therefore of deficiency one, whence all functions belonging to the surface are doubly periodic. Of course in the foregoing survey we have been thinking mainly of real points and lines and loci, and so have neglected the second period—the first being real. The use of elliptic functions enables us to understand the situation involving imaginary arguments, as when the point locus is completely enclosed by the line-locus, so that a *real* polygon is obviously impossible, and yet the invariant conditions may be satisfied. The one essential premise is in every case, that the things under consideration are algebraically connected, two values of either to every one value of the other.

First let me recall the chain of circles devised by Steiner, most recently so interestingly treated by Professor Emch by the aid of his mechanical linkages. Let two circles enclose a ring-shaped area in the plane, and draw any one circle in that ring tangent to the first two. Let a second be drawn touching both the directors and the last mentioned circle, then a third touching in the same way the second, and so on. If a last circle ever appears in the series, say the nth in order, touching the first one, call the chain closed. This chain is now like the Poncelet polygon in the essential feature, in that every member (circle) is preceded by one and followed by one definite member of the series: the correspondence is certainly algebraic and (2, 2). Therefore the chain will close with n circles, no matter what one be selected for the first. Both Hurwitz and Emch have stated weaker conditions that lead to the same conclusion; but it would seem, if the analogy of the polygon porism is valid, that many other variations of conditions ought yet to be attempted.

There are Steiner's polygons on a plane cubic, with alternate sides passing through one of two selected fixed points on the curve. This curve, with points represented in elliptic functions of a parameter, might seem out of place among conics and other rational curves, but the next example will remind us of the natural connection. Let the base points be A and B. Choose at random a point 1 on the cubic curve, and draw in their order the lines $A12$, $B23$, $A34$, $B45$, etc., all the numbered points lying on the curve. If this series never closes, the same would be true if point 1 were chosen elsewhere; or if it does close after $2n$ sides, the same will be true for every position of point 1. Here of course the relation of the base points is decisive, and the fact that elliptic arguments of three points in a line sum up congruent to zero makes the proper choice of the point B a mere matter of arithmetic, *i. e.*, division of a period of the functions for that cubic curve.

Projection from any point of a twisted quartic curve gives in the plane a cubic curve. But also from one of four special points, the quartic projects into a conic double. At the same time the generators of any quadric surface containing the quartic curve are projected into tangents of a second conic. Any Poncelet polygon of $2n$ sides on those two conics is then the projection, if we please, of a system of generators from the two families on the quadric, alternating, n from each family. On the plane cubic the same set of lines would be projected from a point P as the alternating sides of a Steiner polygon, where points A and B are projected by the two generators through the point P. As all generators of one family meet every generator of the other family, this makes clear the intimate connection between Steiner's and Poncelet's polygons.

To vary the object, look at Hurwitz's plane quartic curve with two cusps and a node. It has two inflexions and a double tangent, and is therefore of class 4, dual to itself. On such a curve let a tangent be drawn, and through each intersection with the curve the second possible tangent from that point; we have clearly another (2, 2) correspondence, and are prepared for the discovery that closure in a finite number of sides, starting from any one tangent or vertex, implies closure in the same number of sides, whatever the point of beginning. In place of two conics we have here the one quartic, but the essential (2, 2) correspondence is in evidence, and the same mobility of figure results from it.

Not to be confounded with these examples is the particular plane quartic curve investigated by Lüroth, which admits the inscription of a complete pentagon. There is a resemblance, it is true, in the fact that it too is a problem of closure, and in the variability of the pentagon. For if the sides of one such pentagon are given by equations, $p=0$, $q=0$, etc., so that the quartic equation is

$$pqrs + qrst + rstp + stpq + tpqr = 0,$$

then these five sides are tangents of a unique conic, and every tangent to that conic is one of a set of five constituting an inscribed pentagon of the same quartic. But the correspondence is (4, 4), and the circumscribed locus is not a rational curve. It is, however, in one direct line of descent from Poncelet's triangles. Those triangles mark, on the conic-bearing tangents, sets of three points in involution; and any cubic involution of tangents has for locus of the vertices of its triangles a second conic. So when the number of tangents in each set is increased, we have the *involution-curves*. It is an involution of the fifth order which generates for its locus this quartic curve of Lüroth, each tangent intersecting the four in its own set. Such an involution is the equivalent of a (4, 4) correspondence, which might in special cases degenerate into two (2, 2) correspondences, and carry Lüroth's quartic curve with it into two distinct conics, each containing a system of inscribed Poncelet triangles.

A somewhat different kind of curve arising from a (2, 2) correspondence was that investigated some years ago by Holgate. Starting with a pencil of conics, normalized to a system of coaxial circles, he gave to every point in their plane an index, usually ∞, but for certain points finite. From the point any line is drawn. It touches two circles of the system. One of these has a second tangent through that same point, and that tangent touches a third circle, etc. If after n steps of this kind the first line is reached again, the index of that point is n. Holgate determined the locus of points whose index is 3 as a parabola; that for index 4 as a nodal quartic, and laid out the general method for higher indices. One should react from this experiment to something more like the original Poncelet object; to one fixed conic as support of its tangents, and a double infinity or net of conics. A simple infinity of conics in this net would contain Poncelet triangles with respect to the fixed conic: their index would be 3, and their envelope would take the place of Holgate's parabola. And for the dual problem, there is ready at hand the well-known system of confocal conics, in which the indices of all straight lines should be studied, and the envelopes of lines for each integral index.

The number of different treatments of this same problem increases, not rapidly, but steadily; its fascination is exerted upon the successive generations of mathematicians, and some of their works of art stand out from the mass, some for a little time, some longer. I shall pass over most of them, these images, in geometric shape, of the algebraic (2, 2) correspondence; and describe only one more related object, an image of a (3, 3) correspondence. Franz Meyer studied it and elaborated it in detail, years ago as a docent at Tübingen, in his book on *Apolarität*. Studying the quartic involution, he began with the (3, 3) correspondence among points upon a twisted cubic curve, the simplest rational curve in space of three dimensions. For comparison, remember the cubic involution on a conic in two-space. There we had this theorem on Poncelet triangles: If a conic be circumscribed to one triangle which is circumscribed about a fixed conic, then there are ∞^1 other triangles similarly related to the two conics. Meyer found the theorem, surprising by contrast: If a tetrahedron be formed of four planes which osculate one fixed cubic curve in three-space, and a second cubic curve be passed through its four vertices, then that pair of cubics may have, or *may not have*, a second tetrahedron similarly related to them. If, however, there is a second tetrahedron, then there is a simply infinite set of such. Many other remarkable facts in the geometry of twisted cubic curves he developed, most of which still wait for diffusion among the geometric public.

Such a discrepancy between conic and cubic does not exist in regard to periodic sets of lines and planes, respectively, of period seven. Whether it is found for periods five and six, no one has yet undertaken to determine. Yet a cleavage so marked, and so unexpected, is certainly a challenge to geometricians to explore further the so-called *norm curves* of hyperspace, and the involutions of point sets of low orders upon them.

Also the half-forgotten fact deserves recognition and exploitation, that all those Poncelet systems are associated with *linear involutions* upon rational curves. In that feature, possibly, lies even more promise of generalizations and discoveries than in Jacobi's brilliant and beautiful depiction by the aid of periodic functions.

Not every creation of the geometric mind finds an environment ready in which it can live and grow. Some remain, immortal but alone, like the ancient theorem of Pythagoras or perhaps in recent years Morley's Pentacle, that creation of tantalizing beauty and illusory simplicity. Most new ideas in geometry die early, or pass, by publication, into the condition of mummies or fossils; let our grateful recognition and praise follow then those fortunate worthies like Poncelet, whose genius has given us the fruitful ideas, problems and theories with a significance stretching far beyond their accidental first form, reappearing through the years in new embodiments, and so achieving a life if not perpetual, at least as long enduring as the present era of intellectual culture.

H. S. White

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THE CONTEST WITH PHYSICAL NATURE[1]

I fancy that if Christopher Columbus is able at this time to survey this world and see what is happening that he is well pleased at his venturesome voyage. While the nations of the world that he left have their knives at each other's throats the peoples of this new world have sent their most learned men, their philosophers, their scientists, inventors and engineers to talk with one another as to how this new land may become wiser, richer and be made more useful. This is surely a contrast. It is a condition for which my knowledge of history offers no parallel.

There are times I know when nations who believe in themselves must fight. But let us not delude ourselves with the notion that civilization is the product of arms. The only excuse for war is to secure peace, that men of thought, resourcefulness and skill may have opportunity to make themselves masters of the secrets of nature.

For the real battle of the centuries is not between men or between nations or between races. The one fight, the enduring contest, is between man and physical nature.

There is no denying the fact that we live in a world that is hostile and secretive. It is organized to destroy us if it can. Our enemies have cunning and ferocity. We have but to fold our arms and the beasts, the flies, the rats, the mosquitoes and the vermin would make us their easy prey. And if they could not win by force, they would bring death by starvation. This world was made for a fighting man and for none other. Softness is not to be our portion, because nature knows no holiday. So man must battle with nature that he may secure that physical peace necessary to give his spirit a chance to show itself in things of beauty and deeds of goodness.

And this is what we call civilization—this triumph over the down-pull of nature. We make her yield. We master her secrets. With wooden club and stone axe, with bow and arrow and with fire man mastered his wild enemies and then with seed and water man mastered the surface of the earth. The sea challenged him and he discovered the floating log, the paddle and then the sail, until he made himself master also of the surface of the sea. These things it took ages to do. Nature revealed nothing. Man had to observe and reflect that he might discover or invent. Was there ever such a discovery as that a planted seed would sprout and yield? Or that the wind would drive a hollowed log?

But these things happened long ago. And now we have made not only the surface of the land and sea our own, but their depths as well. The wind not only fills our sails, but we master the air itself. We make our own lightning and harness it to work for us, to push and to pull, to lift and to turn. We have found the great secret that nature can be made to fight nature. But we must fight with her for our weapons. They are not handed to us; they are hidden from us. If man is to have dominion over this earth, he is committed to an unending search. He must bore and burrow, dig and blast, crush and refine, distill and mix, burn and compress until he forces nature to yield her locked and buried treasures.

[1] Address before the Mining and Geological Section of the Pan-American Scientific Congress.