Astronomy" has called attention to the *unexplained fact* that the *full moon* tends to disperse the clouds under it. This follows as a necessary consequence of gravitation; but it is not restricted to the *full moon*, but is in active operation at all times by both sun and moon. The fact is however most easily observed at the time when the sun is absent.

Incidentally we may mention that were the moon's orbit in the plane of the ecliptic, the eclipse conditions of the tenth of August would be mostly repeated at each new moon; but the tidal phenomena would be fundamentally different. In the supposed case the crossing of the two tidal waves would be constantly at the pole of the ecliptic during the whole lunation, and the high tides would be confined to the latitudes of the arctic and antarctic circles. If, at the same time, the earth's equator were shifted into the ecliptic, there would be a *constant elevation* of water at both poles of the earth, while all other places on the surface of the earth would have four simple tidal waves each day. The general problem of the height of the tidal wave at any time and place on the earth's surface can not be considered here, but tables for that purpose have already been computed, though still unpublished.

We see from this exposition of the subject that all the infinite variety of tidal phenomena are fully explained by the operation of the forces of gravitation as developed under existing conditions in the solar system. The eclipse of August 10 represents a case in which the forces of the sun and moon act in perfect harmony during a few minutes of time; but it recurs at such infrequent and uncertain intervals that nothing useful can be learned from a single performance unless there be some known theoretical connection with preceding and subsequent events. The problem of the tides, which has been aptly called the "*Riddle of the Ages*," and designated in despair by an ancient philosopher as "*the tomb of human curiosity*," may therefore now be considered as completely solved.

JOHN N. STOCKWELL

CLEVELAND,
November 4, 1915

ON THE DEGREE OF EXACTNESS OF THE GAMMA FUNCTION NECESSARY IN CURVE FITTING[1]

THE note by Mr. P. F. Everitt in a recent number of this journal[2] discussing an earlier note by the present writer[3] seems so likely to obscure the essential point and purpose for which the earlier note was written that it appears desirable to advert to the subject once more.

In practical biometric work the gamma function is *chiefly* (though of course not entirely) used in connection with the fitting of Pearson's skew frequency curves, where such function appears in the expression for y_0. In other words, the exactness of approximation to the gamma function in these cases can affect nothing but the calculation of the ordinates and areas of the fitted curve. The writer finds it difficult to conceive of such circumstances in the ordinary prosecution of practical statistical researches as would necessitate or warrant the calculation of the ordinates or areas of a frequency curve to more than two places of decimals. This being the case, it seemed desirable, in the earlier paper, to call attention to the fact that a quite sufficiently "exact" approximation to the values of the gamma functions could be made by simple interpolation in a table of log $\lfloor n$.

In order that the statistical worker may form his own judgment as to what degree of exactness in approximating the gamma function is necessary in calculating y_0, Table I. is presented. This table shows, for four different skew frequency curves, the change produced in y_0 by altering[2] the logarithm of the term involving gamma functions by the following amounts: .0000001, .000001, .00001, .0001 and .001. The curves used for illustration are taken from Pearson's memoir "On the Mathematical Theory of Errors of Judgment, with Special Reference to the Personal Equation."[4]

The curve marked I. in the table is Pear-

[1] Papers from the Biological Laboratory of the Maine Agricultural Experiment Station, No. 90.

[2] SCIENCE, N. S., Vol. XLII., pp. 453–455, 1915.

[3] SCIENCE, N. S., Vol. XLI., pp. 506–507, 1915.

[4] *Phil. Trans.*, Vol. 198*A*, pp. 235–299, 1902.

son's "Bright-line" (3) curve[5] and has the equation

$$y = 53.359 \left(1 + \frac{x}{11.0856}\right)^{3.96821} \left(1 - \frac{x}{14.4504}\right)^{5.17262}.$$

Curve II. is Pearson's "Bisection" (3) curve[6] and has the equation

$$y = 71.56246 \left(1 + \frac{x}{11.349400}\right)^{5.41665} \left(1 - \frac{x}{6.998136}\right)^{3.33995}.$$

Curve III. is Pearson's "Bisection" (2–3) curve[7] and has the equation

$$y = 59.43126 \left(1 + \frac{x}{16.16672}\right)^{9.763885} \left(1 - \frac{x}{13.55284}\right)^{8.185180}.$$

Curve IV. is Pearson's "Bisection" (1–2) curve[8] and has the equation

$$y = 56.2136 \left(1 + \frac{x}{28.15012}\right)^{35.390655} \left(1 - \frac{x}{37.69023}\right)^{47.384605}.$$

It will be noted that these are all Type I curves and represent a rather wide range of values of the a's and m's. The expression for y_0 in a Type I curve is

$$y_0 = \frac{N}{b} \cdot \frac{m_1^{m_1} m_2^{m_2}}{(m_1 + m_2)^{m_1 + m_2}} \cdot K,$$

where

$$K = \frac{\Gamma(m_1 + m_2 + 2)}{\Gamma(m_1 + 1)\,\Gamma(m_2 + 1)}.$$

The table shows the change in the maximum ordinate, y_0, produced by altering $\log K$ to the amount indicated.

TABLE I

Showing the Maximum Effect on an Ordinate of the Curve Produced by a Change in the Value of the Log Gamma Term of the Indicated Amount

Curve	Amount of Change in Log K				
	.0000001	.000001	.00001	.0001	.001
I......	.00002	.00013	.00123	.01229	:12303
II.....	.00002	.00016	.00164	.01647	.16496
III...	.00001	.00014	.00137	.01369	.13700
IV....	.00001	.00013	.00130	.01295	.12958

From this table it is evident that:

1. An alteration of as much as one in the third decimal place in $\log K$ makes a change in the maximum ordinate of between 1 and 2 in the first decimal place, an amount practically negligible in many curve-fitting studies.

2. A degree of approximation to $\log \Gamma(n)$ such as is obtained by interpolation from a table of $\log \lfloor n$, when *only second differences* are used in the interpolation,[9] involves errors in the fourth decimal place in $\log \Gamma(n)$, or the fifth for values of $n > 25$ *circa*. These mean errors of the order of .02 *ca.* in the maximum ordinate (and, of course, smaller absolute errors in all other ordinates).

3. Interpolation from a table of $\log \lfloor n$ using second differences is, as we concluded in the earlier paper, quite sufficiently exact for all practical curve-fitting purposes. If any one desires to use ten-place logarithms or some other method, and make all his computations precisely exact to seven (or for the matter of that to 15, 20 or 50) places of figures he may, of course, do so. It is reasonably open to question, however, whether the *additional* contributions to knowledge which may fairly be expected to accrue from such procedure are likely to be of such magnitude or originality as to justify the labor.

RAYMOND PEARL

THE ORIGIN OF LOST RIVER AND ITS GIANT POTHOLES

IN a short article in SCIENCE in 1913,[1] Mr. Robert W. Sayles, of Harvard University, described and sought to explain the block-filled gorge and giant potholes of Lost River, in the Kinsman Notch, New Hampshire. During a first visit to the place, last summer, I saw certain features which seem worthy of attention, in formulating any working hypothesis of the origin of the phenomena.

As Mr. Sayles stated, Lost River is a small stream which flows eastward from the notch between Mt. Moosilauke and Mt. Kinsman, eddying and cascading beneath a deep pile of huge angular blocks and rifted ledges for a

[5] *Loc. cit.*, p. 287.

[6] *Loc. cit.*, p. 288.

[7] *Loc. cit.*, p. 288.

[8] *Loc. cit.*, p. 289.

[9] *Cf.* Table I. of the writer's earlier paper.

[1] Vol. XXXVII., pp. 611–613.