

you in your various fields of that which I have called the larger background of knowledge. It is only by this that we can see things in their true perspective. Our respective sciences and our special fields of research become of value only when their wider relations are apprehended. And may I without unduly magnifying mine office as a biologist call your attention to the fact that biology has contributed no unworthy share to the means of progress in the sister sciences. The contributions of biologists, especially the workers in physiological chemistry, to the general advance of chemical science does not require to be mentioned; nor do I need to refer to the usefulness to physical chemistry of the fertile ideas of Pfeffer and De Vries in the explanation of osmotic pressure. The physicists do not need to be told that by far the most sensitive galvanometer for the measurement of minute currents of short duration is the device of the physiologist Einthoven, designed primarily for use in the study of living organs. The engineers will recall that the method of recording progressive changes on a revolving drum is the application of the kymographion invented by Ludwig for the recording of blood pressures; but now employed in securing graphic records of a great variety of natural phenomena.

Or let me reverse the picture and remind you that the physiologist, the pathologist and the physician are laboring to apply the results of your researches in the explanation of the normal life processes, and to use them in the discovery of the causes of pain and suffering and disease, to the end that these causes may be overcome. Toward this result all lines of scientific effort are contributory.

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*THE ERRORS IN PRECISE LEVELING DUE
TO IRREGULAR ATMOSPHERIC
REFRACTIONS¹*

VERY accurate determinations of elevations above some datum have been made possible by the great improvements in the wye or spirit level which have taken place during the last half century. In 1867 the International Geodetic Association defined precise leveling as that which has a probable accidental error of not more than 3 mm. per kilometer. The leveling run to establish the controlling or fundamental elevations in the interior of the countries, during the decades which followed, showed these limits to be too liberal. In 1912 the International Geodetic Association adopted a resolution calling for a still higher grade of leveling called "leveling of high precision." This is defined as leveling in which every line, set of lines or net is run twice in opposite directions on different dates, as far as possible, and whose errors, computed by prescribed formulas, do not exceed ± 1 mm. per kilometer for the probable accidental error and ± 0.2 mm. per kilometer for the probable systematic error.

This class of leveling is easily secured with the modern instruments and methods. In fact the greater portion of the leveling done with the older instruments and methods in the United States by the Coast and Geodetic Survey and by other organizations came within these limits.

The datum or plane of reference which has been adopted in this and in other countries is mean sea level, that is the surface of the oceans with the water assumed to be at rest and affected only by gravity. This surface may be determined with relation to fixed points on land by long series of tidal observations. The mean surface varies in height from day to day, month to month, and even from year to year. Whether there are secular changes is not definitely known. The disturbing influences are the sun and moon, prevailing winds and varying atmospheric pressures. The configuration of the shore may have some

¹ Read before the Washington Philosophical Society, March 13, 1915.

effect, but this would probably be very slight at points on the open coast and would no doubt be constant and not show in the series of tidal results.

The following table gives the yearly averages for the tidal stations at the Presidio, San Francisco, just inside the Golden Gate.

Year	Height, Ft.	Year	Height, Ft.	Year	Height, Ft.	Year	Height, Ft.
1898	8.30	1902	8.57	1906	8.58	1910	8.42
1899	8.44	1903	8.53	1907	8.66	1911	8.61
1900	8.50	1904	8.63	1908	8.43	1912	8.49
1901	8.46	1905	8.65	1909	8.53	1913	8.51

The mean sea level for 16 years (1898 to 1913) equals 8.519 feet on the staff. The staff was frequently referred to a substantial bench mark near the tidal station and corrections were applied to take account of any variation in the elevation of the zero of the staff referred to this bench mark.

The total range during the 16 years was 0.36 foot. The greatest difference from the mean is 0.22 foot, while the average difference is 0.075 foot. It is seen that the mean value for any three consecutive years does not differ from the mean for the sixteen years by more than .110 foot. There are 14 3-year groups with an average difference from the mean of 0.04 foot, about 1.3 centimeters.

Ordinarily when a tidal station is established solely for determining mean sea level from which to extend a line of precise levels a series of tidal observations extending through only three years is made. Judging from the San Francisco records, we may therefore expect an uncertainty of the plane of 0.04 foot or 0.013 meter.

Whether or not the mean sea levels at different parts of the same ocean and of different oceans lie in the same equipotential surface is a question which has not been solved in the United States. It is true that the several transcontinental lines of levels indicate that the Pacific is higher than the Atlantic and Gulf, but this may be due to accumulated errors in the thousands of miles of levels involved. The results of careful leveling across the Isthmus of Panama show that the mean

sea levels of the Atlantic and Pacific are in the same equipotential surface within 17.8 centimeters. This difference may be largely due to the unavoidable errors in the leveling and the determination of the sea levels of the two oceans. The mean sea level on the Pacific was determined by observations through only one year and may be in error several tenths of a foot.

The leveling across Florida is not strong and its results are not conclusive. There are four lines between St. Augustine on the Atlantic and Cedar Keys on the Gulf, with a total range of 0.85 meter in the difference in elevation of the two places. At each of those points tidal observations are being made and within a year or two a new line of levels will be run between them. We hope to obtain then some definite data in regard to the relative elevations of the two bodies of water.

In the absence of conclusive information we have assumed in our level net adjustment that the surfaces of the three great bodies of water touching this country are in the same equipotential surface and the starting points of the various lines of levels are consequently given zero elevations.

There are two ways in which the errors of leveling show; one in the closing of circuits, and the other in the disagreement in the difference in elevation between each two consecutive bench marks as determined by the two runnings of the line between them. We shall not have time to consider at length the closing errors. The closing errors of the circuits of levels run entirely with the new instruments and methods² are seldom greater than 0.20 millimeter per kilometer. This is a clear indication that the accumulative errors in a long line are small.

The following table shows the principal facts in regard to the closing errors of the 84 circuits forming the net which was adjusted in 1912.

There are many sources of error in leveling, of a systematic nature which may be made to

² See the following C. and G. S. publications: Appendix No. 3, Report for 1903, and Special Publications Nos. 18 and 22.

Rate per Km. in Mm.	Rate per Mile in Thousandths of Foot	No. of Lines	No. Km.	No. Miles	Per Cent. of Whole
.00 to .10	0.00 to 0.52	56	17,251	10,719	40.1
.10 " .20	0.52 " 1.05	36	8,708	5,411	20.3
.20 " .30	1.05 " 1.57	30	9,732	6,047	22.6
.30 " .40	1.57 " 2.10	7	1,480	920	3.4
.40 " .50	2.10 " 2.62	14	1,488	925	3.5
.50 " .60	2.62 " 3.15	4	492	306	1.1
.60 " .70	3.15 " 3.67	5	595	370	1.4
.70 " .80	3.67 " 4.20	1	312	194	0.7
.80 " .90	4.20 " 4.72	1	150	93	0.3
.90 " 1.00	4.72 " 5.25	5	647	402	1.5
1.00 " 2.00	5.25 " 10.50	13	2,076	1,290	4.8
Over 2.00	Over 10.50	1	48	30	0.1
		173	42,979	26,707	99.8

act as accidental errors in a long line of levels. The methods followed in the Coast and Geodetic Survey have this in view. One of the most troublesome errors encountered in the past was due to the changes in the relation between the line of sight and the axis of the bubble caused by rapid and unequal temperature changes in the different parts of the instrument. The older instruments were made of metals having large coefficients of expansion, and the bubble was at a considerable distance from the center of the telescope tube. It was found that the error of a line was a function of its direction or azimuth. This error is probably eliminated in leveling run with the Coast and Geodetic Survey level which has been in use about fifteen years. It is made of nickel-iron with a coefficient of expansion of only .000004 and its bubble is set into the tube of the telescope near the line of sight.

We will not consider the sources of errors which are well known and which are largely eliminated by the methods employed, but will confine ourselves to some interesting errors apparently due to variations in the vertical atmospheric refraction on steep grades, and even these can only be touched upon. They are considered at some length in a recent publication of the Survey.³

All leveling by the Survey is run in both directions, forward and backward, the line is divided into sections approximately one kilometer in length, and the two runnings of a sec-

³ See Special Publication No. 22, of the C. and G. Survey.

tion are made in different days in order to vary the atmospheric conditions. Usually one running is made in the morning and the other in the afternoon. For a number of years the observers have kept records of the time of day of the runnings of the different sections and the weather conditions which obtained. Five lines of levels were selected for a study of the possible relation between the errors of leveling and the conditions of the weather, the time of the observations and the steepness of the grade. These lines are:

No.	Line	Distance, Kms.	Direction of Progress	Average Length of Section, Kms.
1	San Francisco, Cal. to Marmol, Nev.	497	Eastward	0.8
2	Beowawe to Marmol, Nev.	451	Westward	0.9
3	Brigham, Utah, to Beowawe, Nev.	486	do.	0.8
4	Butte to Devon, Mont.	461	Northward	0.8
5	Pocatello, Idaho, to Butte, Mont.	415	do.	1.1
	Total	2310		

Mean grade per section, steep slopes. 16.6 meters
Mean grade per section, low slopes... 3.4 meters
Mean grade per section, all sections... 6.4 meters

The total length of these lines is 2,310 kilometers. As it was impracticable to investigate the relations between the size and sign of the discrepancy between the results of the two runnings of the sections and the many different grades, the leveling was separated into only two classes: First, those having grades exceeding ten meters and, second, those with smaller grades. As the average length of a section is about one kilometer a ten-meter grade corresponds to a grade of approximately one per cent. The average grade for the first class is 16.6 meters, for the second 3.4 meters, and for all the sections 6.4 meters.

We shall first see whether there is any difference in the elevation between the two ends of a section by the two runnings where one running is in the morning and the other in the afternoon. The direction of the slope is not considered. In the following table A

stands for the morning and *P* for the afternoon running. The discrepancies are given in millimeters.

MORNING AND AFTERNOON RUNNINGS COMPARED,
WEATHER CONDITIONS IGNORED

	Steep Grade	Low Grade
Number of sections	188	761
<i>P-A</i> , total, positive	+ 496.4	+ 1,645.1
Mean discrepancy	+ 2.64	+ 2.16
Number of sections	144	629
<i>P-A</i> , total, negative	- 340.4	- 1,277.8
Mean discrepancy	- 2.36	- 2.03
Number of sections	332	1,390
Mean discrepancy	2.52	2.10
Accumulated discrepancy ..	+ 156.0	+ 367.3
Mean accumulation per section	+ 0.47	+ 0.26

For both classes the sections with positive values of *P-A* predominate while the mean accumulated discrepancy per section for the low grades is only 0.55 of that for the steep grades.

The next table includes the morning and afternoon runnings, which were made in sunshine.

MORNING AND AFTERNOON RUNNINGS, ALL IN SUNSHINE

	Steep Grade	Low Grade
Number of sections	131	529
<i>P-A</i> , total, positive	+ 358.6	+ 1,140.8
Mean discrepancy	+ 2.74	+ 2.15
Number of sections	87	456
<i>P-A</i> , total, negative	- 203.0	- 935.5
Mean discrepancy	- 2.33	- 2.05
Number of sections	218	985
Mean discrepancy	2.58	2.11
Accumulated discrepancy ..	+ 155.6	+ 205.3
Mean discrepancy per section	+ 0.71	+ 0.21

The evidence here is similar to that of the preceding table but the accumulated discrepancy for the steep grades is three and one half times as great as that for the lower grades.

The investigation does not indicate whether the morning or the afternoon running gives a value nearer the truth, but it is the speaker's opinion that the afternoon is freer from error than the morning running.

It is the speaker's opinion that the afternoon running gives on an average a difference which is closer to the truth than the morning running. In the afternoon the temperatures of the ground and of the air are more nearly the same and a layer of air of uniform density should be concentric or nearly so with the sea-level surface. If this is true the refraction on the front and back sights should be about the same. The leveling of the U. S. Coast and Geodetic Survey is seldom done after 5 o'clock in the afternoon. So the afternoon running is not materially affected by the abnormal refraction of the late afternoon when a line of sight on a grade would pass through layers of colder and denser air which would tend to be concentric with the surface of the ground. In the late afternoon the earth cools more rapidly than the air and the air near the earth's surface becomes colder than the air above and consequently denser than normal.

In the morning on a clear day the air is receiving radiated heat from the earth's surface. This decreases the density of the air close to the ground, and forms layers which tend to be concentric with the surface of the ground rather than with the sea level surface. (The air near the earth is of course not at rest but tends to rise, owing to the decreased density.) It may be assumed that the line of sight to the observer from the rod held down the grade is not affected abnormally while the sight to the rod held up the grade is usually close to the ground and must pass through the layers of lower density near the earth's surface. This sight would be less refracted than the one down the grade and may even be negatively refracted, therefore the morning running would give too small a difference between the zeroes of the rods sighted on from one station. It is the speaker's belief that, other things being equal, a line of levels run over steep grades in two directions in the afternoon, from noon to about one hour before sundown, will give results closer to the truth than levels with both runnings in the forenoon or with one leveling in the forenoon and the other in the afternoon. It is believed that this also applies to leveling over slopes of moderate grade.

The following table gives data for leveling done under different conditions of the sky. The letter *C* stands for cloudy and *S* for sunshine or clear.

RUNNINGS IN CLOUDY AND CLEAR WEATHER

	Steep Grades	Low Grades
Number of sections	56	217
<i>C-S</i> , total, positive	+ 159.2	+ 473.1
Mean discrepancy	+ 2.84	+ 2.18
Number of sections	45	228
<i>C-S</i> , total, negative	- 85.1	- 482.3
Mean discrepancy	- 1.89	- 2.12
Number of sections	101	445
Mean discrepancy	2.42	2.15
Accumulated discrepancy ..	+ 74.1	- 9.2
Mean accumulation per section	+ 0.73	- 0.02

The mean accumulated discrepancy here is +0.73 for the steep grades while for the low grades it is practically zero.

It is the general belief among geodesists that the leveling under a cloudy sky is practically free from systematic errors resulting from atmospheric conditions. Therefore it would appear that the leveling under a clear sky causes the observed differences in elevation on steep grades to be too small.

In the following table are given data for the steep sections which had one running in clear and the other in cloudy weather, but the data are arranged in two groups, one where the running in sunshine was made in the morning called (*SA*) while the other has the running in sunshine made in the afternoon (*SP*):

RUNNINGS WHEN CLOUDY AND ON CLEAR MORNINGS

	Steep Grades	Low Grades
Number of sections	56	240
<i>C-SA</i> , accumulation per section. +	0.24	+ 0.11

RUNNINGS WHEN CLOUDY AND ON CLEAR AFTERNOONS

	Steep Grades	Low Grades
Number of sections	45	215
<i>C-SP</i> , accumulation per section. +	1.34	- 0.21

The number of sections on steep grades is too small to warrant our drawing any definite conclusions from the data given. The indication from the steep section is that the afternoon running gives a value lower than the morning value.

The average accumulated values of *C-SA* and *C-SP* for the sections with low grade are small, +0.11 millimeter per section in the former and -0.21 millimeter per section in the latter. These sections are quite numerous as compared with the number of steep sections, and should no doubt be given some consideration before coming to a decision as to whether the morning or afternoon runnings in sunshine give the larger differences.

The data in the following table show some relations between the systematic errors of leveling and calm and windy weather. The letter *C* stands for calm and *W* for wind.

RUNNINGS IN CALM AND IN WIND

	Steep Grades	Low Grades
Number of sections	63	277
<i>C-W</i> , total, positive	+ 140.0	+ 544.3
Mean discrepancy	+ 2.22	+ 2.0
Number of sections	75	345
<i>C-W</i> , total, negative	- 199.8	- 757.4
Mean discrepancy	- 2.66	- 2.20
Number of sections	138	622
Mean discrepancy	2.46	2.11
Accumulated discrepancy ..	- 59.8	- 203.1
Mean accumulation per section	- 0.43	- 0.33

Both for the steep and low grades the runnings during wind give on an average greater differences in elevation between the ends of a section than the runnings during calm. The average is almost as large for the low grades as for the steep ones. This can not be considered to be a general rule for other factors may and probably do influence the results. All of the lines are in the western portion of the United States where it is usually more windy in the afternoon than in the morning. Calm is infrequent there in the afternoon. Therefore the value of *C-W* would be somewhat confused with the value of *P-A*.

If both runnings are made in the forenoon or both in the afternoon, then the values of *C-W* should be practically free from the effect of the time of the day. In the following table there are given the data for such sections, the amount of grade not being considered.

SECTIONS WITH BOTH RUNNINGS IN THE MORNING OR BOTH IN THE AFTERNOON

Number of sections	85
$C-W$, total, positive	+ 177.2
Mean discrepancy	+ 2.08
Number of sections	90
$C-W$, total, negative	— 222.3
Mean discrepancy	— 2.47
Number of sections	175
Accumulated discrepancy	— 45.1
Mean accumulation per section	— 0.26

The effect of morning and afternoon conditions being eliminated (but not the cloudy or clear sky) we have a result which shows a larger value on an average for the running in wind than the one in calm.

There are 495 sections, each of which had one running in the morning and one running in the afternoon with both runnings made during calm. These sections should have values for $P-A$ which are free from the effect of calm and wind. The data for these sections are shown below. The grade is not considered.

SECTIONS RUN IN BOTH DIRECTIONS DURING CALM

Number of sections	259
$P-A$, total, positive	+ 603.5
Mean discrepancy	+ 2.33
Number of sections	236
$P-A$, total, negative	— 497.5
Mean discrepancy	— 2.11
Number of sections	495
Accumulated discrepancy	+ 106.0
Mean accumulation per section	+ 0.21

The results in the above table are free from the effects of wind and calm, but may be and probably are somewhat affected by cloudy or clear sky. But the indication is that the afternoon running is greater than the forenoon, on an average.

If it is assumed that the running in wind is free from error, then the data for the sections shown below should give an indication as to whether an afternoon or forenoon running of a section will give the greater difference in elevation. The amount of grade is not considered.

The term $(C-W)A$ represents calm minus

wind, with the calm running in the forenoon, while $(C-W)P$ is the same, except that the calm running is in the afternoon.

Mm.

Number of sections 256, total	
positive value	$(C-W) A + 499.2$
Number of sections 330, total	
negative value	$(C-W) A - 759.0$
Number of sections 94, total	
positive value	$(C-W) P + 221.7$
Number of sections 87, total	
negative value	$(C-W) P - 182.3$
Mean accumulated discrepancy	
per section for	$(C-W) A - 0.44$
	$(C-W) P + 0.22$

The indications from these data are that the difference from the calm running in the forenoon is too small and from the calm running in the afternoon too great, upon the assumption that the running in wind is without error. This bears out the conclusion stated earlier in this paper that the afternoon running gives a greater difference in elevation than the morning running.

CONCLUSIONS

While the data used in the investigation into the sources of error in precise leveling are not sufficient to warrant any definite statements, yet they seem to justify the following conclusions as probable.

1. The average size of the discrepancy between the values of the difference in elevation determined twice under different conditions does not give a clear idea of the magnitude of the accidental errors which may be produced by certain conditions, as the custom is to make the length of sight as great as the conditions will permit. Therefore, the extra length of sight may offset otherwise favorable conditions and give a large difference between two runnings of a section.

2. For sections run twice under different conditions the average accumulated value of the discrepancy is greater for the sections with steep grades than with low grades; the direction of the running being ignored and only the actual difference in elevation between the ends of a section being considered.

3. On all grades, but more especially the

steep ones, the difference in elevation determined in the afternoon is on an average greater than that determined in the forenoon.

4. On an average, a running during wind gives a greater difference in elevation than one during calm. The amount of this difference is somewhat greater for the steep than for the low grades.

5. On an average a running when the sky is cloudy gives a larger difference in elevation between two points, on a steep grade, than a running while the sun is shining. For low grades there is practically no difference, on an average, between the runnings under the two conditions.

6. For steep grades (about 10 meters per kilometer) the probability is that the afternoon running gives, on an average, a result closer to the truth than the forenoon running. The afternoon running should be ended sometime before sundown. The running in wind probably gives results on an average closer to the truth than a running in calm.

7. While the data in the tables given above make these conclusions justifiable, yet, owing to the fact that there are so many conditions to be considered, it is impracticable to obtain at present any reliable numerical values for the effect of any given atmospheric condition or set of conditions.

8. It is believed that, other things being equal, the running in the afternoon (if not within about an hour of sunset) gives, on an average, more accurate results than the forenoon running; also that, other things being equal, a running in wind is more accurate, on an average, than one in calm; and, that other things being equal, a running with a cloudy sky will be more accurate, on an average, than one in sunshine. Hence, the ideal condition would be an afternoon with a moderate wind and a cloudy sky.

9. It is believed that the mere fact of running backward or forward has no real effect on the result of a running, as the value of $B-F$ may vary in sign for different lines and even for different parts of a single line.

WILLIAM BOWIE

U. S. COAST AND GEODETIC SURVEY

BESSEY HALL AT THE UNIVERSITY OF NEBRASKA

DR. BESSEY is gone, but he leaves with us an imperishable memory. He was the first professor in the natural science group to remain long with the University of Nebraska and to leave an indelible mark upon it. It is fitting that the permanent home of two fundamental natural sciences in the university should be named in his honor.

The writer believes that he first suggested naming such a building after Dr. Bessey when he penned for the approval page 21 of the biennial report of 1911-12. This report contains the sentence:

The inadequate and dangerous building known as Nebraska Hall should be removed and an adequate building called Bessey Hall in honor of Dr. Bessey erected to house the natural sciences.

Nevertheless when he wrote these words it was then as now the writer's opinion that in general no building built at public expense should be named after the living. Dr. Bessey was great enough so that this exception was planned, but his lamented death prevented the exception being made. Let us now render his memory a special honor by resolving that hereafter no building shall be named for any one until his life's work is complete. This is in harmony with the regents' act in deciding that hereafter the title of head dean shall not be awarded.

Some of the special friends of Dr. Bessey are disappointed that the building is not to be located on a more conspicuous site. To these I would say that Dr. Bessey insisted on only one thing—north light for the use of his microscopes. He was, however, pleased to have the building located away from the noise and dust of heavy traffic. The location as now determined met his critical approval. The building will have north windows along its main side and will be so located that no other university building can by any possibility obstruct the view.

The building will have three stories above ground. The basement, not to be used for class-room purposes, will be utilized for lockers, toilet rooms, store rooms, constant tem-