

# SCIENCE

FRIDAY, NOVEMBER 12, 1915

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## THE HUMAN SIGNIFICANCE OF MATHEMATICS<sup>1</sup>

Homo sum; humani nil a me alienum puto.—*Terence.*

THE subject of this address is not of my choosing. It came to me by assignment. I may, therefore, be allowed to say that it is in my judgment ideally suited to the occasion. This meeting is held here upon this beautiful coast because of the presence of an international exposition, and we are thus invited to a befitting largeness and liberality of spirit. An international exposition properly may and necessarily will admit many things of a character too technical to be intelligible to any one but the expert and the specialist. Such things, however, are only incidental—contributory, indeed, yet incidental—to pursuit of the principal aim, which is, I believe, or ought to be, the representation of human things as human—an exhibition and interpretation of industries, institutions, sciences and arts, not primarily in their accidental or particular character as illustrating individuals or classes or specific localities or times, but primarily in their essential and universal character as representative of man. A world-exposition will, therefore, as far as practicable, avoid placing in the forefront matters so abstruse as to be fit for the contemplation and understanding of none but specialists; it will, as a whole, and in all its principal parts, address itself to the general intelligence; for it aims at being, for the multitudes of men and

<sup>1</sup> An address delivered August 3, 1915, Berkeley, Calif., at a joint meeting of the American Mathematical Society, the American Astronomical Society, and Section A of the American Association for the Advancement of Science.

women who avail themselves of its exhibitions and lessons, an exposition of humanity: an exposition, no doubt, of the activities and aspirations and prowess of individual men and women, but of men and women, not in their capacity as individuals, but as representatives of humankind. Individual achievements are not the object, they are the means, of the exposition. The object is humanity.

What is the human significance—what is the significance for humanity—of “the mother of the sciences”? And how may the matter be best set forth, not for the special advantage of professional mathematicians, for I shall take the liberty of having these but little in mind, but for the advantage and understanding of educated men and women in general? I am unable to imagine a more difficult undertaking, so technical, especially in its language, and so immense is the subject. It is clear that the task is far beyond the resources of an hour’s discourse, and so it is necessary to restrict and select. This being the case, what is it best to choose? The material is superabundant. What part of it or aspect of it is most available for the end in view? “In abundant matter to speak a little with elegance,” says Pindar, “is a thing for the wise to listen to.” It is not, however, a question of elegance. It is a question of emphasis, of clarity, of effectiveness. What shall be our major theme?

Shall it be the history of the subject? Shall it be the modern developments of mathematics, its present status and its future outlook? Shall it be the utilities of the science, its so-called applications, its service in practical affairs, in engineering and in what it is customary to call the sciences of nature? Shall it be the logical foundations of mathematics, its basic principles, its inner nature, its characteristic processes and structure, the differences and

similitudes that come to light in comparing it with other forms of scientific and philosophic activity? Shall it be the bearings of the science as distinguished from its applications—the bearings of it as a spiritual enterprise upon the higher concerns of man as man? It might be any one of these things. They are all of them great and inspiring themes.

It is easy to understand that a historian would choose the first. The history of mathematics is indeed impressive, but is it not too long and too technical? And is it not already accessible in a large published literature of its own? I grant, the historian would say, that its history is long, for in respect of antiquity mathematics is a rival of art, surpassing nearly all branches of science and by none of them surpassed. I grant that, for laymen, the history is technical, frightfully technical, requiring interpretation in the interest of general intelligence. I grant, too, that the history owns a large literature, but this, the historian would say, is not designed for the general reader, however intelligent, the numerous minor works, no less than the major ones, including that culminating monumental work of Moritz Cantor, being, all of them, addressed to specialists and intelligible to them alone. And yet it would be possible to tell in one hour, not indeed the history of mathematics, but a true story of it that would be intelligible to all and would show its human significance to be profound, manifold and even romantic. It would be possible to show historically that this science, which now carries its head so high in the tenuous atmosphere of pure abstractions, has always kept its feet upon the solid earth; it would be possible to show that it owns indeed a lowly origin, in the familiar needs of common life, in the homely necessities of counting herds and measuring lands; it would be possible to

show that, notwithstanding its birth in the concrete things of sense and raw reality, it yet so appealed to sheer intellect—and we must not forget that creative intellect is the human faculty *par excellence*—it so appealed to this distinctive and disinterested faculty of man that, long before the science rose to the level of a fine art in the great days of Euclid and Archimedes, Plato in the wisdom of his maturer years judged it essential to the education of freemen because, said he, there is in it a necessary something against which even God can not contend and without which neither gods nor demi-gods can wisely govern mankind; it would be possible, our historian could say, to show historically to educated laymen that, even prior to the inventions of analytical geometry and the infinitesimal calculus, mathematics had played an indispensable rôle in the “Two New Sciences” of physics and mechanics in which Galileo laid the foundations of our modern knowledge of nature; it would be possible to show not only that the analytical geometry of Descartes and Fermat and the calculus of Leibnitz and Newton have been and are essential to our still advancing conquest of the sea, but that it is owing to the power of these instruments that the genius of such as Newton, Laplace and Lagrange has been enabled to create for us a new earth and a new heavens compared with which the Mosaic cosmogony or the sublimest creation of the Greek imagination is but “as a cabinet of brilliants, or rather a little jewelled cup found in the ocean or the wilderness”; it would be possible to show historically that, just because the pursuit of mathematical truth has been for the most part disinterested—led, that is, by wonder, as Aristotle says, and sustained by the love of beauty with the joy of discovery—it would be possible to show that, just because of the disinterestedness of mathe-

matical research, this science has been so well prepared to meet everywhere and always, as they have arisen, the mathematical exigencies of natural science and engineering; above all, it would be feasible to show historically that to the same disinterestedness of motive operating through the centuries we owe the upbuilding of a body of pure doctrine so towering to-day and vast that no man, even though he have the “Andean intellect” of a Poincaré, can embrace it all. This much, I believe, and perhaps more, touching the human significance of mathematics, a historian of the science might reasonably hope to demonstrate in one hour.

More difficult, far more difficult, I think, would be the task of a pure mathematician who aimed at an equivalent result by expounding, or rather by delineating, for he could not in one hour so much as begin to expound, the modern developments of the subject. Could he contrive even to delineate them in a way to reveal their relation to what is essentially humane? Do but consider for a moment the nature of such an enterprise. Mathematics may be legitimately pursued for its own sake or for the sake of its applications or with a view to understanding its logical foundations and internal structure or in the interest of magnanimity or for the sake of its bearings upon the supreme concerns of man as man or from two or more of these motives combined. Our supposed delineator is actuated by the first of them: his interest in mathematics is an interest in mathematics for the sake of mathematics; for him the science is simply a large and growing body of logical consistencies or compatibilities; he derives his inspiration from the muse of intellectual harmony; he is a pure mathematician. He knows that pure mathematics is a house of many chambers; he knows that its foundations lie far beneath

the level of common thought; and that the superstructure, quickly transcending the power of imagination to follow it, ascends higher and higher, ever keeping open to the sky; he knows that the manifold chambers—each of them a mansion in itself—are all of them connected in wondrous ways, together constituting a fit laboratory and dwelling for the spirit of men of genius. He has assumed the task of presenting a vision of it that shall be worthy of a world-exposition. Can he keep the obligation? He wishes to show that the life and work of pure mathematicians are human life and work: he desires to show that these toilers and dwellers in the chambers of pure thought are representative men. He would exhibit the many-chambered house to the thronging multitudes of his fellow men and women; he would lead them into it; he would conduct them from chamber to chamber by the curiously winding corridors, passing now downward, now upward, by delicate passage-ways and subtle stairs; he would show them that the wondrous castle is not a dead or static affair like a structure of marble or steel, but a living architecture, a living mansion of life, human as their own; he would show them the mathetic spirit at work, how it is ever weaving, tirelessly weaving, fabrics of beauty, finer than gossamer yet stronger than cables of steel; he would show them how it is ever enlarging its habitation, deepening its foundations, expanding more and more and elevating the superstructure; and, what is even more amazing, how it perpetually performs the curious miracle of permanence combined with change, transforming, that is, the older portions of the edifice without destroying it, for the structure is eternal: in a word, he would show them a vision of the whole, and he would do it in a way to make them perceive and feel that, in thus

beholding there a partial and progressive attainment of the higher ideals of man, they were but gazing upon a partial and progressive realization of their own appetitions and dreams.

That is what he *would* do. But how? *Mengenlehre*, *Zahlenlehre*, algebras of many kinds, countless geometries of countless infinite spaces, function theories, transformations, invariants, groups and the rest—how can these with all their structural finesse, with their heights and depths and limitless ramifications, with their labyrinthine and interlocking modern developments—I will not say how can they be presented in the measure and scale of a great exposition—but how is it possible in one hour to give laymen even a glimpse of the endless array? Nothing could be more extravagant or more absurd than such an undertaking. Compared with it, the American traveler's hope of being able to see Rome in a single forenoon was a most reasonable expectation. But it is worth while trying to realize how stupendous the absurdity is.

It is evident that our would-be delineator must compromise. He can not expound, he can not exhibit, he can not even delineate the doctrines whose human worth he would thus disclose to his fellow men and women. The fault is neither his nor theirs. It must be imputed to the nature of things. But he need not, therefore, despair and he need not surrender. The method he has proposed—the method of exposition—that indeed he must abandon as hopeless, but not his aim. He is addressing men and women who are no doubt without his special knowledge and his special discipline, as he in his turn is without theirs, but who are yet essentially like himself. He would have them as fellows and comrades persuaded of the dignity of his *Fach*: he would have them feel that it is also theirs; he would

have them convinced that mathematics stands for an immense body of human achievements, for a diversified continent of pure doctrine, for a discovered world of intellectual harmonies. He can not *show* it to them as a painter displays a canvas or as an architect presents a cathedral. He can not give them an immediate vision of it, but he can give them intimations; by appealing to their *fantasie* and, through analogy with what they know, to their understanding, not only can he convince them that his world exists, but he can give them an intuitive apprehension of its living presence and its meaning for humankind. This is possible because, like him, they, too, are idealists, dreamers and poets—such essentially are all men and women. His auditors or his readers have all had *some* experience of ideas and of truth, they have all had inklings of more beyond, they have all been visited and quickened by a sense of the limitless possibilities of further knowledge in every direction, they have all dreamed of the perfect and have felt its lure. They are thus aware that the small implies the large; having seen hills, they can believe in mountains; they know that Euripides, Shakespeare, Dante, Goethe, are but fulfillments of prophecies heard in peasant tales and songs; they know that the symphonies of Beethoven or the dramas of Wagner are harbingered in the melodies and the sighs of those who garner grain and in their hearts respond to the music of the winds or the “solemn anthems of the sea”; they sense the secret by which the astronomy of Newton and Laplace is foretold in the shepherd’s watching of the stars; and knowing thus this plain spiritual law of progressiveness and implication, they are prepared to grasp the truth that modern mathematics, though they do not understand it, is, like the other great things, but a sublime fulfillment, the realization of

prophecies involved in what they themselves, in common with other educated folk, know of the rudiments of the science. Indeed, they would marvel if upon reflection it did not seem to be so. Our pure mathematician in speaking to his fellow men and women of his science will have no difficulty in persuading them that he is speaking of a subject immense and eternal. As born idealists they have intimations of their own—the evidence of intuition, if you please—or a kind of insight resembling that of the mystic—that in the world of mind there must be something deeper and higher, stabler and more significant, than the pitiful ideas in life’s routine and the familiar vocations of men. They are thus prepared to believe, before they are told, that behind the veil there exists a universe of exact thought, an everlasting cosmos of ordered ideas, a stable world of concatenated truth. In their study of the elements, in school or college, they may have caught a shimmer of it or, in rare moments of illumination, even a gleam. Of the existence, the reality, the actuality, of our pure mathematician’s world they will have no doubt, and they will have no doubt of its grandeur. They may even, in a vague way, magnify it overmuch, feeling that it is, in some wise, *more* than human, significant only for the rarely gifted spirit that dwells, like a star, apart. The pure mathematician’s difficulty lies in showing, in *his* way, that such is not the case. For he does not wish to adduce utilities and applications. He is well aware of these. He knows that if he “would tell them they are more in number than the sands.” Neither does he despise them as of little moment. On the contrary, he values them as precious. But he wishes to do his subject and his auditors the honor of speaking from a higher level: he desires to vindicate the worth of mathematics on the ground of its

sheer ideality, on the ground of its intellectual harmony, on the ground of its beauty, "free from the gorgeous trappings" of sense, pure, austere, supreme. To do this, which ought, it seems, to be easy, experience has shown to be exceedingly difficult. For the multitude of men and women, even the educated multitude, are wont to cry,

Such knowledge is too wonderful for me,  
It is too high, I can not attain unto it,

thus meaning to imply, What, then, or where is its human significance? Their voice is heard in the challenge once put to me by the brilliant author of "East London Visions." What, said he, can be the human significance of "this majestic intellectual cosmos of yours, towering up like a million-lustred iceberg into the arctic night," seeing that, among mankind, none is permitted to behold its more resplendent wonders save the mathematician alone? What response will our pure mathematician make to this challenge? Make, I mean, if he be not a wholly naïve devotee of his science and so have failed to reflect upon the deeper grounds of its justification. He may say, for one thing, what Professor Klein said on a similar occasion:

Apart from the fact that pure mathematics can not be supplanted by anything else as a means for developing the purely logical faculties of the mind, there must be considered here, as elsewhere, the necessity of the presence of a few individuals in each country developed in far higher degree than the rest, for the purpose of keeping up and gradually raising the *general* standard. Even a slight raising of the general level can be accomplished only when some few minds have progressed far ahead of the average.

That is doubtless a weighty consideration. But is it all or the best that may be said? It is just and important but it does not go far enough; it is not, I fear, very convincing; it is wanting in pungence and edge; it does not touch the central nerve of

the challenge. Our pure mathematician must rally his sceptics with sharper considerations. He may say to them: You challenge the human significance of the higher developments of pure mathematics because they are inaccessible to all but a few, because their charm is esoteric, because their deeper beauty is hid from nearly all mankind. Does that consideration justify your challenge? You are individuals, but you are also members of a race. Have you as individuals no human interest nor human pride in the highest achievements of your race? Is nothing human, is nothing humane, except mediocrity and the commonplace? Was Phidias or Michel Angelo less human than the carver and painter of a totem-pole? Was Euclid or Gauss or Poincaré less representative of man than the countless millions for whom mathematics has meant only the arithmetic of the market place or the rude geometry of the carpenter? Does the quality of humanity in human thoughts and deeds decrease as they ascend towards the peaks of achievement, and increase in proportion as they become vulgar, attaining an upper limit in the beasts? Do you not know that precisely the reverse is true? Do you not count aspiration humane? Do you not see that it is not the common things that every one may reach, but excellences high-dwelling among the rocks—do you not know that, in respect of human worth, these things, which but few can attain, are second only to the supreme ideals attainable by none?

How very different and how very much easier the task of one who sought to vindicate the human significance of mathematics on the ground of its applications! In respect of temperamental interest, of attitude and outlook, the difference between the pure and the applied mathematician is profound. It is—if we may liken spiritual

things to things of sense—much like the difference between one who greets a newborn day because of its glory and one who regards it as a time for doing chores and values its light only as showing the way. For the former, mathematics is justified by its supreme beauty; for the latter, by its manifold use. But are the two kinds of value essentially incompatible? They are certainly not. The difference is essentially a difference of authority—a difference, that is, of worth, of elevation, of excellence. The pure mathematician and the applied mathematician sometimes may, indeed they not infrequently do, dwell together harmoniously in a single personality. If our spokesman be such a one—and I will not suppose the shame of having the utilities of the science represented on such an occasion by one incapable of regarding it as anything but a tool, for that would be disgraceful—if, then, our spokesman be such a one as I have supposed, he might properly begin as follows: In speaking to you of the applications of mathematics I would not have you suppose, ladies and gentlemen, that I am thus presenting the *highest* claims of the science to your regard; for its highest justification is the charm of its immanent beauty; I do not mean, he will say, the beauty of appearances—the fleeting beauties of sense, though these, too, are precious—even the outer garment, the changeful robe, of reality is a lovely thing; I mean the eternal beauty of the world of pure thought; I mean intellectual beauty; in mathematics this nearly attains perfection; and “intellectual beauty is self-sufficing”; uses, on the other hand, are not; they wear an aspect of apology; uses resemble excuses, they savor a little of a plea in mitigation. Do you ask: Why, then, plead them? Because, he will say, many good people have a natural incapacity to ap-

preciate anything else; because, also, many of the applications, especially the higher ones, are themselves matters of exceeding beauty; and especially because I wish to show, not only that use and beauty are compatible forms of worth, but that the more mathematics has been cultivated for the sake of its inner charm, the fitter has it become for external service.

Having thus at the outset put himself in proper light and given his auditors a scholar's warning against what would else, he fears, foster a disproportionment of values, what will he go on to signalize among the utilities of a science whose primary allegiance to logical rectitude allies it to art, and which only incidentally and secondarily shapes itself to the ends of instrumental service? He knows that the applications of mathematics, if one will but trace them out in their multifarious ramifications, are as many-sided as the industries and as manifold as the sciences of men, penetrating everywhere throughout the full round of life. What will he select? He will not dwell long upon its homely uses in the rude computations and mensurations of counting-house and shop and factory and field, for this indispensable yet humble manner of world-wide and perpetual service is known of all men and women. He will quickly pass to higher considerations—to navigation, to the designing of ships, to the surveying of lands and seas, and the charting of the world, to the construction of reservoirs and aqueducts, canals, tunnels and railroads, to the modern miracles of the marine cable, the telegraph, the telephone, to the multiform achievements of every manner of modern engineering, civil, mechanical, mining, electrical, by which, through the advancing conquest of land and sea and air and space and time, the conveniences and the prowess of man have been multiplied a billionfold. It need not

be said that not all this has been done by mathematics alone. Far from it. It is, of course, the joint achievement of many sciences and arts, but—and just this is the point—the contributions of mathematics to the great work, direct and indirect, have been indispensable. And it will require no great skill in our speaker to show to his audience, if it have a little imagination, that, as I have said elsewhere, if all these mathematical contributions were by some strange spiritual cataclysm to be suddenly withdrawn, the life and body of industry and commerce would suddenly collapse as by a paralytic stroke, the now splendid outer tokens of material civilization would quickly perish, and the face of our planet would at once assume the aspect of a ruined and bankrupt world. For such is the amazing utility, such the wealth of by-products, if you please, that come from a science and art that owes its life, its continuity and its power to man's love of intellectual harmony and pleads its inner charm as its sole appropriate justification. Indeed it appears—contrary to popular belief—that in our world there is nothing else quite so practical as the inspiration of a muse.

But this is not all nor nearly all to which our applied mathematician will wish to invite attention. It is only the beginning of it. Even if he does not allude to the quiet service continuously and everywhere rendered by mathematics in its rôle as a norm or standard or ideal in every field of thought whether exact or inexact, he will yet desire to instance forms and modes of application compared with which those we have mentioned, splendid and impressive as they are, are meager and mean. For those we have mentioned are but the more obvious applications—those, namely, that continually announce themselves to our senses everywhere in the affairs, both great

and small, of the workaday world. But the really great applications of mathematics—those which, rightly understood, best of all demonstrate the human significance of the science—are not thus obvious; they do not, like the others, proclaim themselves in the form of visible facilities and visible expedients everywhere in the offices, the shops, and the highways of commerce and industry; they are, on the contrary, almost as abstract and esoteric as mathematics itself, for they are the uses and applications of this science in other sciences, especially in astronomy, in mechanics and in physics, but also and increasingly in the newer sciences of chemistry, geology, mineralogy, botany, zoology, economics, statistics and even psychology, not to mention the great science and art of architecture. In the matter of exhibiting the endless and intricate applications of mathematics to the natural sciences, applications ranging from the plainest facts of crystallography to the faint bearings of the kinetic theory of gases upon the constitution of the Milky Way, our speaker's task is quite as hopeless as we found the *pure* mathematician's to be; and he, too, will have to compromise; he will have to request his auditors to acquaint themselves at their leisure with the available literature of the subject and especially to read attentively the great work of John Theodore Merz dealing with the "History of European Thought in the Nineteenth Century," where they will find, in a form fit for the general reader, how central has been the rôle of mathematics in all the principal attempts of natural science to find a cosmos in the seeming chaos of the natural world. Another many-sided work that in this connection he may wish to commend as being in large part intelligible to men and women of general education and catholic mind is Enriques's "Problems of Science."

I turn now for a moment to the prospects of one who might choose to devote the hour to an exposition or an indication of modern developments in what it is customary to call the foundations of mathematics—to a characterization, that is, and estimate of that far-reaching and still advancing critical movement which has to do with the relations of the science, philosophically considered, to the sciences of logic and methodology. What can he say on this great theme that will be intelligible and edifying to the multitudes of men and women who, though mathematically inexperienced, yet have a genuine humane curiosity respecting even the profounder and subtler life and achievements of science? He can point out that mathematics, like all the other sciences, like the arts too, for that matter, and like philosophy, originates in the refining process of reflection upon the crude data of common sense; he can point out that this process has gradually yielded from out the raw material and still continues to yield more and more ideas of approximate perfection in the respects of precision and form; he can point out that such ideas, thus disentangled and trimmed of their native vagueness and indeterminateness, disclose their mutual relationships and so become amenable to the concatenative processes of logic; and he can point out that these polished ideas with their mutual relationships become the bases or the content of various branches of mathematics, which thus tower above common sense and appear to grow out of it and to stand upon it like trees or forests upon the earth. He will point out, however, that this appearance, like most other obvious appearances, is deceiving; he will, that is, point out that these upward-growing sciences or branches of science are found, in the light of further reflection, to be downward-growing as well, pushing their roots

deeper and deeper into a dark soil far beneath the ground of evident common sense; indeed, he will show that common sense is thus, in its relation to mathematics, but as a sense-litten mist enveloping only the mid-portion of the stately structure, which, like a towering mountain, at once ascends into the limpid ether far above the shining cloud and rests upon a base of subterranean rock far below; he will point out that, accordingly, mathematicians, in respect of temperamental interest, fall into two classes—the class of those who cultivate the upward-growing of the science, working thus in the upper regions of clearer light, and the class of those who devote themselves to exploring the deep-plunging roots of the science; and it is, he will say, to the critical activity of the latter class—the logicians and philosophers of mathematics—that we owe the discovery of what we are wont to call the foundations of mathematics—the great discovery, that is, of an immense mathematical *sub*-structure, which penetrates far beneath the stratum of common sense and of which many of even the greatest mathematicians of former times were not aware. But whilst such foundational research is in the main a modern phenomenon, it is by no means exclusively such; and to protect his auditors against a false perspective in this regard and the peril of an overweening pride in the achievements of their own time, our speaker may recommend to them the perusal of Thomas L. Heath's superb edition of Euclid's "Elements" where, especially in the first volume, they will be much edified to find, in the rich abundance of critical citation and commentary which the translator has there brought together, that the refined and elaborate logico-mathematical researches of our own time have been only a deepening and widening of the keen mathematical criticism of a few

centuries immediately preceding and following the great date of Euclid. Indeed but for that general declension of Greek spirit which Professor Gilbert Murray in his "Four Stages in Greek Religion" has happily characterized as "the failure of nerve," what we know as the modern critical movement in mathematics might well have come to its present culmination, so far at least as pure geometry is concerned, fifteen hundred or more years ago. It is a pity that the deeper and stabler things of science and the profounder spirit of man can not be here disclosed in a manner commensurate with the great exposition, surrounding us, of the manifold practical arts and industries of the world. It is a pity there is no means by which our speaker might, in a manner befitting the subject and the occasion, exhibit intelligibly to his fellow men and women the ways and results of the last hundred years of research into the groundwork of mathematical science and therewith the highly important modern developments in logic and the theory of knowledge. How astonished the beholders would be, how delighted, too, and proud to belong to a race capable of such patience and toil, of such disinterested devotion, of such intellectual finesse and depth of penetration. I can think of no other spectacle quite so impressive as the inner vision of all the manifold branches of rigorous thought seen to constitute one immense structure of autonomous doctrine reposing upon the spiritual basis of a few select ideas and, superior to the fading beauties of time and sense, shining there like a celestial city, in "the white radiance of eternity." That is the vision of mathematics that a student of its philosophy would, were it possible, present to his fellow men and women.

In view of the foregoing considerations it evidently is, I think, in the nature of the

case impossible to give an adequate sense of the human worth of mathematics if one choose to devote the hour to any one of the great aspects of it with which we have been thus far concerned. Neither the history of the subject nor its present estate nor its applications nor its logical foundations—no one of these themes lends itself well to the purpose of such an exposition, and still less do two or more of them combined. Even if such were not the case I should yet feel bound to pursue another course; for I have been long persuaded that, in respect of its human significance, mathematics invites to a point of view which, unless I am mistaken, has not been taken and held in former attempts at appreciation. I have already alluded to bearings of mathematics as distinguished from applications. It is with its bearings that I wish to deal. I mean its bearings upon the higher concerns of man as man—those interests, namely, which have impelled him to seek, over and above the needs of raiment and shelter and food, some inner adjustment of life to the poignant limitations of life in our world and which have thus drawn him to manifold forms of wisdom, not only to mathematics and natural sciences, but also to literature and philosophy, to religion and art, and theories of righteousness. What is the rôle of mathematics in this perpetual endeavor of the human spirit everywhere to win reconciliation of its dreams and aspirations with the baffling conditions and tragic facts of life and the world? What is its relation to the universal quest of man for some supreme and abiding good that shall assuage or annul the discords and tyrannies of time and limitation, withholding less and less, as time goes by, the freedom and the peace of an ideal harmony infinite and eternal?

In endeavoring to suggest, in the time remaining for this address, a partial answer

to that great question, in attempting, that is, to indicate the relations of mathematics to the supreme ideals of mankind, it will be necessary to seek a perspective point of view and to deal with large matters in a large way.

Of the countless variety of appetitions and aspirations that have given direction and aim to the energies of men and that, together with the constraining conditions of life in our world, have shaped the course and determined the issues of human history, it is doubtless not yet possible to attempt a confident and thoroughgoing classification according to the principle of relative dignity or that of relative strength. If, however, we ask whether, in the great throng of passional determinants of human thought and life, there is one supreme passion, one that in varying degrees of consciousness controls the rest, unifying the spiritual enterprises of our race in directing and converging them all upon a single sovereign aim, the answer, I believe, can not be doubtful: the activities and desires of mankind are indeed subject to such imperial direction and control. And if now we ask what the sovereign passion is, again the answer can hardly admit of question or doubt. In order to see even *a priori* what the answer must be, we have only to imagine a race of beings endowed with our human craving for stability, for freedom, and for perpetuity of life and its fleeting goods, we have only to fancy such a race flung, without equipment of knowledge or strength, into the depths of a treacherous universe of matter and force where they are tossed, buffeted and torn by the tumultuous onward-rushing flood of the cosmic stream, originating they know not whence and flowing they know not why nor whither, we have, I say, only to imagine *this*, sympathetically, which ought to be easy for us as men, and then to ask our-

selves what would *naturally* be the controlling passion and dominant enterprise of such a race—unless, indeed, we suppose it to become strangely enamored of distress or to be driven by despair to self-extinction. We humans require no Gotama nor Heraclitus to tell us that man's lot is cast in a world where naught abides. The universal impermanence of things, the inevitableness of decay, the mocking frustration of deepest yearnings and fondest dreams, all this has been keenly realized wherever men and women have had seeing eyes or been even a little touched with the malady of meditation, and everywhere in the literature of power is heard the cry of the mournful truth. "The life of man," said the Spirit of the Ocean, "passes by like a galloping horse, changing at every turn, at every hour."

"Great treasure halls hath Zeus in heaven,  
From whence to man strange dooms be given,  
Past hope or fear."

Such is the universal note. Whether we glance at the question in a measure *a priori*, as above, or look into the cravings of our own hearts, or survey the history of human emotion and thought, we shall find, I think, in each and all these ways, that human life owns the supremacy of one desire: it is the passion for emancipation, for release from life's limitations and the tyranny of change: it is our human passion for some ageless form of reality, some everlasting vantage-ground or rock to stand upon, some haven of refuge from the all-devouring transformations of the weltering sea. And so it is that our human aims, aspirations, and toils thus find their highest unity—their only intelligible unity—in the Spirit's quest of a stable world, in its endless search for some mode or form of reality that is at once infinite, changeless, eternal.

Does some one say: This may be granted,

but what is the point of it all? It is obviously true enough but what, pray, can be its bearing upon the matter in hand? What light does it throw upon the human significance of mathematics? The question is timely and just. The answer, which will grow in fullness and clarity as we proceed, may be at once begun.

How long our human ancestors, in remote ages, may have groped, as some of their descendants even now grope, among the things of *sense*, in the hope of finding *there* the desiderated good, we do not know—past time is long and the evolution of wisdom has been slow. We do know that, long before the beginnings of recorded history, superior men—advanced representatives of their kind—must have learned that the deliverance sought was not to be found among the objects of the *mobile* world, and so the spirit's quest passed from thence; passed from the realm of perception and sense to the realm of concept and reason: thought ceased, that is, to be merely the unconscious means of pursuit and became itself the quarry—mind had discovered mind; and there, in the realm of ideas, in the realm of spirit proper, in the world of reason or thought, the great search—far outrunning historic time—has been endlessly carried on, with varying fortunes, indeed, but without despair or breach of continuity, meanwhile multiplying its resources and assuming gradually, as the years and centuries have passed, the characters and forms of what we know today as philosophy and science and art. I have mentioned the passing of the quest from the realm of sense to the realm of conception: a most notable transition in the career of mind and especially significant for the view I am aiming to sketch. For thought, in thus becoming a conscious subject or object of thought, then began its destined course in reason: in ceasing to be

merely an unconscious means of pursuit and becoming itself the quarry, it definitely entered upon the arduous way that leads to the goal of rigor. And so it is evident that the way in question is not a private way; it does not belong exclusively to mathematics; it is public property; it is the highway of conceptual research. For it is a mistake to imagine that mathematics, in virtue of its reputed exactitude, is an insulated science, dwelling apart in isolation from other forms and modes of conceptual activity. It would be such, were its rigor absolute; for between a perfection and any approximation thereto, however close, there always remains an infinitude of steps. But the rigor of mathematics is not absolute—absolute rigor is an ideal, to be, like other ideals, aspired unto, forever approached, but never quite attained, for such attainment would mean that every possibility of error or indetermination, however slight, had been eliminated from idea, from symbol, and from argumentation. We know, however, that such elimination can never be complete, unless indeed the human mind shall one day lose its insatiable faculty for doubting. What, then, is the distinction of mathematics on the score of exactitude? Its distinction lies, not in the attainment of rigor absolute, but partly in its exceptional devotion thereto and especially in the advancement it has made along the endless path that leads towards that perfection. But, as I have already said, it must not be thought that mathematics is the sole traveler upon the way. It is important to see clearly that it is far from being thus a solitary enterprise. First, however, let us adjust our imagery to a better correspondence with the facts. I have spoken of *the* path. We know, however, that the paths are many, as many as the varieties of conceptual subject-matter, all of them converging towards the same high goal. We

see them originate here, there and yonder in the soil and haze of common thought; we see how indistinct they are at first—how ill-defined; we observe how they improve in that regard as the ideas involved grow clearer and clearer, more and more amenable to the use and governance of logic. At length, when thought, in its progress along any one of the many courses, has reached a high degree of refinement, precision and certitude, then and thereafter, but not before, we call it mathematical thought; it has undergone a long process of refining evolution and acquired at length the name of mathematics; it is not, however, the creature of its name; what is called mathematics has been long upon the way, owning at previous stages other designations—common sense, practical art perhaps, speculation, theology it may be, philosophy, natural science, or it may be for many a millenium no name at all. Is it, then, only a question of names? In a sense, yes: the ideal of thought is rigor; mathematics is the name that usage employs to designate, not attainment of the ideal, for it can not be attained, but its devoted pursuit and close approximation. But this is not the essence of the matter. The essence is that all thought, thought in all its stages, however rude, however refined, however named, owns the unity of being human: spiritual activities are one. Mathematics thus belongs to the great family of spiritual enterprises of man. These enterprises, all the members of the great family, however diverse in form, in modes of life, in methods of toil, in their progress along the way that leads towards logical rectitude, are alike children of one great passion. In genesis, in spirit and aspiration, in motive and aim, natural science, theology, philosophy, jurisprudence, religion and art are one with mathematics: they are all of them sprung from the hu-

man spirit's craving for invariant reality in a world of tragic change; they all of them aim at rescuing man from "the blind hurry of the universe from vanity to vanity": they seek cosmic stability—a world of abiding worth, where the broken promises of hope shall be healed and infinite aspiration shall cease to be mocked.

Such has been the universal and dominant aim and such are the cardinal forms that time has given its prosecution.

And now we must ask: What have been the fruits of the endless toil? What has the high emprise won? And what especially, have been the contributions of mathematics to the total gain? To recount the story of the spirit's quest for ageless forms of reality would be to tell afresh, from a new point of view, the history of human thought, so many and so diverse are the modes or aspects of being that men have found or fancied to be eternal. Edifying indeed would be the tale, but it is long, and the hour contracts. Even a meager delineation is hardly possible here. Yet we must not fail to glance at the endless array and to call, at least in part, the roll of major things. But where begin? Shall it be in theology? How memory responds to the magic word. "The past rises before us like a dream." As the long succession of the theological centuries passes by, what a marvelous pageant do they present of human ideals, contrivings and dreams, both rational and superrational. Alpha and Omega, the beginning and ending, which is, which was and which is to come; I Am That I Am; Father of lights with which is no variableness, neither shadow of turning; the bonitas, unitas, infinitas, immutabilitas of Deity; the undying principle of soul; the sublime hierarchy of immortal angels, terrific and precious, discoursed of by sages, commemorated by art, feared and loved by millions of men

and women and children: these things may suffice to remind us of the invariant forms of reality found or invented by theology in her age-long toil and passion to conquer the mutations of time by means of things eternal.

But theology's record is only an immense chapter of the vastly more inclusive annals of world-wide philosophic speculation running through the ages. If we turn to philosophy understood in the larger sense, if we ask what answers she has made in the long course of time to the question of what is eternal, so diverse and manifold are the voices heard across the centuries, from the East and from the West, that the combined response must needs seem to an unaccustomed ear like an infinite babel of tongues: the Confucian Way of Heaven; the mystic Tao, so much resembling fate, of Lao Tzu and Chuang Tzu; Buddhism's inexorable spiritual law of cause and effect and its everlasting extinction of individuality in Nirvana—the final blowing out of consciousness and character alike; Ahura Mazda, the holy One, of Zarathustra; Fate, especially in the Greek tragedies and Greek religion—the chain of causes in nature, “the compulsion in the way things grow,” a fine thread running through the whole of existence and binding even the gods; the cosmic matter, or *το απειρον* of Anaximander; the cosmic order, the rhythm of events, the logos or reason or nous, of Heraclitus; the finite, space-filling sphere, or One, of the deep Parmenides; the four material and two psychic, six eternal, elements, of Empedocles; the infinitude of everlasting mind-moved simple substances of Anaxagoras; the infinite multitude and endless variety of invariant “seeds of things” of Leucippus, Democritus, Epicurus and Lucretius, together with their doctrines of absolute void and the conservations of mass and mo-

tion and infinite room or space; Plato's eternal world of pure ideas; the great Cosmic Year of a thousand thinkers, rolling in vast endlessly repeated cycles on the beginningless, endless course of time from eternity to eternity; the changeless thought-forms of Zeno, Gorgias and Aristotle; Leibnitz's indestructible, preestablished harmony; Spinoza's infinite unalterable substance; the Absolute of the Hegelian school; and so on and on far beyond the limits of practicable enumeration. This somewhat random partial list of things will serve to recall and to represent the enormous motley crowd of answers that the ages of philosophic speculation have made to the supreme inquiry of the human spirit: what is there that survives the mutations of time, abiding unchanged despite the whirling flux of life and the world?

And now, in the interest of further representing salient features in a large perspective view, let me next ask what contribution to the solution of the great problem has been made by jurisprudence. Jurisprudence is no doubt at once a branch of philosophy and a branch of science, but it has an interest, a direction and a character of its own. And for the sake of due emphasis it will be well worth while to remind ourselves specifically of the half-forgotten fact that, in its quest for justice and order among men, jurisprudence long ago found an answer to our oft-stated riddle of the world, an answer which, though but a partial one, yet satisfied the greatest thinkers for many centuries, and which, owing to the inborn supernalizing proclivity of the human mind, still exercises sway over the thought of the great majority of mankind. I allude to the conception of *jus naturale* or *lex naturæ*, the doctrine that in the order of Nature there somehow exists a perfect, invariant, uni-

versally and eternally valid system or prototype of law over and above the imperfect laws and changeful politics of men—a conception and doctrine long familiar in the juristic thought of antiquity, dominating, for example, the Antigone of Sophocles, penetrating the Republic and the Laws of Plato, proclaimed by Demosthenes in the Oration on the Crown, becoming, largely through the Republic and the Laws of Cicero, the crowning conception of the imperial jurisprudence of Rome, and still holding sway, as I have said, except in the case of our doubting Thomases of the law, who virtually deny our world the existence of any perfection whatever because they can not, so to speak, feel it with the hand, as if they did not know that to suppose an ideal to be *thus* realized would be a flat contradiction in terms.

If we turn for a moment to art and enquire what has been *her* relation to the poignant riddle, shall we not thus be going too far afield? The answer is certainly no. *In æternitatem pingo*, said Zeuxis, the Greek painter. “The purpose of art,” says John La Farge, “is commemoration.” In these two sayings, one of them ancient, the other modern, we have, I think, the evident clue. They do but tell us that art, like the other great enterprises of man, springs from our spirit’s coveting of worth that abides. Like theology, like philosophy, like jurisprudence, like natural science, too, as I mean to point out further, and like mathematics, art is born of the universal passion for the dignity of things eternal. Her quest, like theirs, has been a search for invariants, for goods that are everlasting. And what has she found? The answer is simple. “The idea of beauty in each species of being,” said Joshua Reynolds, “is perfect, invariable, divine.” We know that by a faculty of

imaginative, mystical, idealizing discernment there is revealed to us, amid the fleeting beauties of Time, the immobile presence of eternal beauty, immutable archetype and source of the grace and loveliness beheld in the shifting scenes of the flowing world of sense. Such, I take it, is art’s contribution to our human release from the tyranny of change and the law of death.

And now what should be said of science? Not so brief and far less simple would be the task of characterizing or even enumerating the many things that in the great drama of modern science have been assigned the rôle of invariant forms of reality or eternal modes of being. It would be necessary to mention first of all, as most imposing of all, our modern form of the ancient doctrine of fate. I mean the reigning conception of our universe as an infinite machine—a powerful conception that more and more fascinates scientific minds even to the point of obsession and according to which it should be possible, were knowledge sufficiently advanced, to formulate, in a system of differential equations, the whole of cosmic history from eternity to eternity in minutest detail, not even excluding a skeptic’s doubt whether such formulation be theoretically possible nor excluding the conviction, which some minds have, that the doctrine, regarded as an *ultimate* creed, is an abominable libel against the character of a world where the felt freedom of the human spirit is not an illusion. It would be necessary to mention—as next perhaps in order of impressiveness—another doctrine that is, curiously enough, vividly reminiscent of old-time fate. I allude this time to the doctrine of heredity, a tremendous conception, in accordance with which—as Professor W. B. Smith has said in his recent powerful address on “Push or Pull?”—

"the remotest past reaches out its skeletal fingers and grapples both present and future in its iron grip." And there is the conservation of energy and that of mass—both of them, again, doctrines prefigured in the thought of ancient Greece—and numerous other so-called natural laws, simple and complex, familiar and unfamiliar, all posing as permanent forms of reality—as natural invariants under the infinite system of cosmic transformations—and thus together constituting the enlarging contribution of natural science towards the slow vindication of a world that has seemed capricious, lawless and impermanent.

Such, then, is a conspectus, suggested rather than portrayed, of the results which the great allies of mathematics, operating through the ages, have achieved in their passionate endeavor to transcend the tragic vicissitudes and limitations of life in an "ever-growing and perishing" universe and to win at length the freedom, the dignity and the peace of a stable world where order and harmony reign and spiritual goods endure. If we are to arrive at a really just or worthy sense of the human significance of mathematics, it is in relation with those great results of her sister enterprises that the achievements of this science must be appraised. Immense indeed and high is the task of criticism as thus conceived. How diverse and manifold the doctrines to be evaluated, what depths to be plumbed, what heights to be scaled, how various the relationships and dignities to be assigned their rightful place in the hierarchy of values. In the presence of such a task what can we think or say in the remaining moments of the hour? If we have succeeded in setting the problem in its proper light and in indicating the sole eminence from which the matter may be rightly viewed, we ought perhaps

to be content with that as the issue of the hour, for it is worth while to sketch a worthy program of criticism even if time fails us to perform fully the task thus set. And yet I can not refrain from inviting you to imagine, before we close, a few at least of the things that one who essayed the great critique would submit to his auditors for mediation. And what do you imagine the guiding lines and major themes of his discourse would be?

I fancy he would say: The question before us, ladies and gentlemen, is not a question of weighing utilities nor of counting applications nor of measuring material gains; it is a question of human ideals together with the various means of pursuing them and the differing degrees of their approximation; we are occupied with a question of appreciation, with the problem of values. I am, he would say, addressing you as representatives of man, and in so doing, I am not regarding man as a mere practician, as a hewer of wood and drawer of water, as an animal content to serve the instincts for shelter and food and reproduction. I am contemplating him as a spiritual being, as a thinker, poet, dreamer, as a lover of knowledge and beauty and wisdom and the joy of harmony and light, responding to the lure of an ideal destiny, troubled by the mystery of a baffling world, conscious subject of tragedy, yearning for stable reality, for infinite freedom, for perpetuity and a thousand perfections of life. As representatives of such a being, you, he would say, and I, even if we be not ourselves producers of theology or philosophy or science or jurisprudence or art or mathematics, are nevertheless rightful inheritors of all this manifold wisdom of man. The question is: What is the inheritance worth? We are the heirs and we are to be the judges of the great responses that time has made to

the spiritual needs of humanity. What are the responses worth? What are their values, joint and several, absolute and relative? And what, especially, is the human worth of the response of mathematics? It is, he would say, not only our privilege, but, as educated individuals and especially as representatives of our race, it is our duty, to ponder the matter and reach, if we can, a right appraisal. For the proper study of mankind is man, and it is essential to remember that "*La vie de la science est la critique.*" I have, he would say, tried to make it clear that mathematics is not an isolated science. I have tried to show that it is not an antagonist, nor a rival, but is the comrade and ally of the other great forms of spiritual activity, all aiming at the same high end. I have reminded you of the principal answers made by these to the spiritual needs of man, and I do not, he would say, desire to underrate or belittle them. They are a precious inheritance. Many of them have not, indeed, stood the test of time; others will doubtless endure for aye; all of them, for a longer or shorter period, have softened the ways of life to millions of men and women. Neither do I desire, he would say, to exaggerate the contributions of mathematics to the spiritual weal of humanity. What I desire is a fair comparative estimate of its claims. "Truth is the beginning of every good thing, both to gods and men." I am asking you to compare, consider and judge for yourselves. The task is arduous and long.

There are, our critic would say, certain paramount considerations that every one in such an enterprise must weigh, and a few of them may, in the moments that remain, be passed in brief review. Consider, for example, our human craving for a world of stable reality. Where is it to be found? We know the answer of theology,

of philosophy, of natural science and the rest. We know, too, the answer of literature and general thought:

The cloud-capped towers, the gorgeous palaces,  
The solemn temples, the great globe itself,  
Yea, all which it inherit, shall dissolve,  
And, like the baseless fabric of this vision,  
Leave not a rock behind.

And now what, he would ask, is the answer of mathematics? The answer, he would have to say, is this: Transcending the flux of the sensuous universe, there exists a stable world of pure thought, a divinely ordered world of ideas, accessible to man, free from the mad dance of time, infinite and eternal.

Consider our human craving for freedom. Of freedom there are many kinds. Is it the freedom of limitless room, where our passion for outward expression, for externalization of thought, may attain its aim? It is to mathematics, our critic would say, that man is indebted for that priceless boon; for it is the cunning of this science that has at length contrived to release our long imprisoned thought from the old confines of our three-fold world of sense and opened to its wing the interminable skies of hyperspace. But if it be a more fundamental freedom that is meant, if it be freedom of thought proper—freedom, that is, for the creative activity of intellect—then again it is to mathematics that our faculties must look for the definition and a right estimate of their prerogatives and power. For, regarding this matter, we may indeed acquire elsewhere a suspicion or an inkling of the truth, but mathematics, and nothing else, is qualified to give us *knowledge* of the fact that our intellectual freedom is absolute save for a single limitation—the law of non-contradiction, the law of logical compatibility, the law of intellectual harmony—sole restriction imposed by "the nature of

things'' or by logic or by the muses upon the creative activity of the human spirit.

Consider next, the critic might say, our human craving for a living sense of rapport and comradeship with a divine Being infinite and eternal. Except through the modern mathematical doctrine of infinity, there is, he would have to say, no rational way by which we may even approximate an understanding of the supernal attributes with which our faculty of idealization has clothed Deity—no way, except this, by which our human reason may gaze understandingly upon the downward-looking aspects of the overworld. But this is not all. I need not, he would say, remind you of the reverent saying attributed to Plato that "God is a geometrician." Who is so unfortunate as not to know something of the religious awe, the solace and the peace that come from cloistral contemplation of the purity and everlastingness of mathematical truth?

Mighty is the charm of those abstractions to a mind beset with images and haunted by himself.

"More frequently," says Wordsworth, speaking of geometry,

More frequently from the same source I drew  
A pleasure quiet and profound, a sense  
Of permanent and universal sway,  
And paramount belief; there, recognized  
A type, for finite natures, of the one  
Supreme Existence, the surpassing life  
Which to the boundaries of space and time,  
Of melancholy space and doleful time,  
Superior and incapable of change,  
Not touched by welterings of passion—is,  
And hath the name of God. Transcendent peace  
And silence did wait upon those thoughts  
That were a frequent comfort to my youth.

And so our spokesman, did time allow, might continue, inviting his auditors to consider the relations of mathematics to yet other great ideals of humanity—our human craving for rectitude of thought, for ideal justice, for dominion over the

energies and ways of the material universe, for imperishable beauty, for the dignity and peace of intellectual harmony. We know that in all such cases the issue of the great critique would be the same, and it is needless to pursue the matter further. The light is clear enough. Mathematics is, in many ways, the most precious response that the human spirit has made to the call of the infinite and eternal. It is man's best revelation of the "Deep Base of the World."

CASSIUS J. KEYSER

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*THE NATIONAL ACADEMY OF SCIENCES*  
PRELIMINARY PROGRAM OF SCIENTIFIC PAPERS FOR  
THE AUTUMN MEETING, NOVEMBER 15-17

THE National Academy of Sciences will hold its stated autumn meeting at the American Museum of Natural History, New York City, on November 15, 16 and 17. The council will meet at 4 P.M. on Monday, November 15. There will be a lecture on "The Problem of Aerial Transmission" by Professor M. I. Pupin, of Columbia University, at 8 P.M., followed by a reception in the museum. On Tuesday and Wednesday morning at 9:30 A.M. there will be business sessions of the academy, followed at 10:30 by public scientific sessions. On the afternoon of November 16, there will be four papers of general interest. On the afternoon of November 17, luncheon will be served at the New York Zoological Park, followed by a visit to the New York Botanical Garden and afternoon tea. There will be a dinner on the evening of November 16 at the Chemists' Club.

The preliminary program of scientific papers is as follows:

*The Nature of Cell Polarity:* EDWIN G. CONKLIN.

*Heredity of Stature:* CHAS. B. DAVENPORT.

*Parental Alcoholism and Mental Ability, a Comparative Study of Habit Formation in the White Rat:* E. C. MACDOWELL. (Introduced by CHARLES B. DAVENPORT.)

The purpose of this investigation is to compare the mental capabilities of rats whose parents were