

in the totality of functions mathematically conceivable.

Even for functions of Class 0 or 1 the trigonometric series has a limited power of representation. This is manifest from an example given by Paul Du Bois Reymond of a continuous function which can not be represented by a trigonometric series. It remains to determine in the future just what properties are necessary and sufficient to characterize those functions of Classes 0 and 1 which are expressible by means of trigonometric series.

Earlier in my paper I pointed out that the generality of functions representable through Fourier's series was so great that the mathematician was led irresistibly to the Dirichlet definition of a function. If, namely, to every value of x in an interval we have a corresponding value of y , then y is called a function of x , no matter how the correspondence is set up, whether by a graph, a mathematical expression, a law, or any other way. To-day the pendulum has swung back to the old question of Euler. The study of representability in terms of trigonometric series has been succeeded by the broader question of the possibility of analytic expression in general. Now every continuous function, as is well known, can be represented by a uniformly convergent set of polynomials. Starting then from the totality of polynomials as a basis of functions for Class 0, we arrive successively at Baire's and Lebesgue's classes of functions corresponding to or, if you prefer, marked, by the transfinite numbers of the first and second classes.

Do these different classes of functions comprise all which are "*analytically expressible*"? Before answering the question it is necessary first to sharply define the phrase "*analytically expressible*." This is done by Lebesgue. Then, after broadening the content of these classes in a manner

I have not the time to describe, he goes on to show that they do in truth comprise all such functions. The final question then confronts us: Are all possible functions included which are defined in accordance with the general definition of Dirichlet? In other words, are there functions *incapable of being "analytically expressed"*? Lebesgue by an example shows that this is the case. Our study of the Fourier series opened with the question: What is an arbitrary function? Here, at last, apparently, we have discovered the existence of a function of such a height or depth of arbitrariness as to be mathematically inexpressible. Having started with the Fourier series on a voyage of exploration, shall we conclude by saying that there is for us an unattainable pole?

EDWARD B. VAN VLECK

UNIVERSITY OF WISCONSIN

UNIVERSITY REGISTRATION STATISTICS

THE registration returns for November 1, 1913, of thirty of the leading universities of the country will be found tabulated on the following page. Specific attention should be called once again to the fact that these universities are neither the thirty largest universities in the country, nor necessarily the leading institutions. The only universities which show a decrease in the grand total attendance (including the summer sessions) are Harvard, Western Reserve and Yale, the attendance of the two institutions last named having remained practically stationary. The largest gains in terms of student units, including the summer attendance, but making due allowance by deduction for the summer session students who returned for instruction in the fall, were registered by New York University (965), Illinois (944), Columbia (927), Wisconsin (749), Pennsylvania (681), California (614), Iowa (598),

Ohio State (503), Chicago (483), and Michigan (388). Last year there were only five institutions that showed a gain of over 300 students, namely, Columbia, California, New York University, Texas and Harvard. Omitting the summer session attendance, the largest gains this year have been made by Illinois (887), New York University (772), California (640), Pennsylvania (571), Iowa (538), Wisconsin (493), Ohio State (434), Michigan (381), Chicago (353), Syracuse (305), Washington University (267), and Columbia (255). It will thus be seen that this year twelve institutions exhibited an increase of over 200 students in the fall attendance, as against eight in 1912 and four in 1911. Of these institutions eight are in the west and four in the east.

According to the figures for 1913, the thirty institutions, inclusive of the summer session rank as follows: Columbia (9,929), California (7,071), Chicago (6,834), Michigan (6,008), Pennsylvania (5,968), Wisconsin (5,890), Harvard (5,627), Cornell (5,612), New York University (5,508), Illinois (5,259), Ohio State (4,111), Minnesota (3,932), Northwestern (3,877), Syracuse (3,845), Yale (3,263), Missouri (3,135), Texas (3,106), Nebraska (2,850), Kansas (2,610), Iowa (2,542), Tulane (2,298), Indiana (2,271), Pittsburgh (1,906), Cincinnati (1,871), Stanford (1,756), Princeton (1,599), Western Reserve (1,370), Johns Hopkins (1,311), Washington University (1,225), and Virginia (885), whereas last year the order was Columbia, California, Chicago, Harvard, Michigan, Cornell, Wisconsin, Minnesota, Pennsylvania, New York University, Illinois, Northwestern, Ohio State, Syracuse, Yale, Texas, Missouri, Nebraska, Kansas, Tulane, Iowa, Cincinnati, Pittsburgh, Stanford, Princeton, Western Reserve, Washington University, Johns Hop-

kins, Virginia. If the summer-session enrollment be omitted, the universities in the table rank in size as follows: Columbia (6,403), Pennsylvania (5,305), Michigan (5,304), California (5,225), Harvard (4,922), Illinois (4,835) and New York University (4,835), Cornell (4,760), Wisconsin (4,450), Northwestern (3,776), Chicago (3,719), Ohio State (3,708), Syracuse (3,699), Minnesota (3,616), Yale (3,263), Missouri (2,547), Nebraska (2,482), Texas (2,373), Kansas (2,308), Iowa (2,294), Pittsburgh (1,906), Cincinnati (1,871), Stanford (1,743), Princeton (1,599), Indiana (1,417), Western Reserve (1,370), Tulane (1,244), Washington University (1,225), Johns Hopkins (1,012) and Virginia (885), whereas last year the order was: Columbia, Michigan, Harvard, Cornell, California, Pennsylvania, New York University, Wisconsin, Illinois, Northwestern, Minnesota, Syracuse, Chicago, Ohio State, Yale, Nebraska, Missouri, Texas, Kansas, Cincinnati, Pittsburgh, Iowa, Stanford, Princeton, Western Reserve, Tulane, Washington University, Virginia, and Johns Hopkins.

Including the summer-session attendance the largest gains in the decade from 1903 to 1913 were made by Columbia (5,372), California (3,594), New York University (3,331), Pennsylvania (3,324), Chicago (2,688), Wisconsin (2,669), Ohio State (2,423), Cornell (2,174), Michigan (2,082) and Illinois (2,020).

So far as the individual faculties of the various universities are concerned, Harvard with 2,350 men and 564 women (Radcliffe College) leads in the number of college undergraduates, being followed by California, with 1,112 men and 1,626 women; Michigan, with 1,736 men and 784 women; Stanford, with 1,243 men and 500 women; Chicago, with 936 men and 767 women; Kansas, with 942 men and 688

	California	Chicago	Cincinnati	Columbia	Cornell	Harvard	Illinois	Indiana	Iowa	Johns Hopkins	Kansas	Michigan	Minnesota	Missouri	Nebraska	New York	Northwestern	Ohio State	Pennsylvania	Pittsburgh	Princeton	Stanford	Syracuse	Texas	Tulane	Virginia	Washington University	Western Reserve	Wisconsin	Yale
College, Men.....	1112	936	200	841	1120	2350	427	694	610	178	942	1736	639	834	541	486	479	485	409	305	1267	1243	1340	811	166	396	152	459	828	1402
College, Women.....	1626	767	419	623	564	564	402	423	553	221	688	784	882	516	753	221	609	412	87	87	500	500	1340	709	152	152	229	358	776
Scientific Schools*	828	406	665	1343	76	1001	248	221	79	372	1282	638	360	306	233	80	811	657	266	156	308	288	164	101	169	775	1133
Law.....	142	211	450	267	695	106	85	201	174	553	193	103	200	771	294	181	381	143	140	291	339	94	257	80	109	161	130
Medicine.....	119	191	84	341	107	306	445	132	112	368	107	353	172	75	102	432	178	181	283	123	64	96	170	399	102	60	164	79	49
Non-professional gradu-
ate schools.....	408	480	170	1496	315	489	268	83	130	219	99	225	127	123	169	369	85	121	438	46	176	134	75	77	27	43	57	9	290	334
Agriculture.....	530	1354	792	464	501	423	889	70	38	968
Architecture.....	143	143	61	351	23	120	43	1	65	259	50	54	25	46
Art.....	58	42	20	64	46	178	85	305	53
Commerce.....	280	160	97	113	282	22	2013	567	1430	57
Dentistry.....	90	203	94	244	268	566	589	209	111	139	100
Divinity.....	125	56	233
Forestry.....	9	36	65
Journalism.....	106
Music.....	106
Pedagogy.....	16
Pharmacy.....	16
Veterinary Medicine.....	114	441
Other Courses.....
Deduct Double Registra-
tion.....
Total.....	24	246	231	389	11	16	549	49	489	127	68	211	302	121	360	22	60	338	112	676	115	14	32	37	112	55
Summer Session 1913...	5225	3719	1871	6403	4760	4922	4835	1417	2294	1012	2308	5304	3616	2547	2482	4835	3776	3708	5305	1906	1599	1743	3699	2373	1244	885	1225	1370	4450	3263
Deduct Double Registra-
tion.....
Grand Total 1913, Nov. 1	517	656	1013	540	92	289	230	178	48	208	704	215	222	143	260	92	300	202	25	114	248	109	680
Grand Total 1912, Nov. 1	17071	6834	1871	9929	5612	5627	5259	2271	2542	1311	2610	6008	3332	3135	2850	5508	3877	4111	5968	1906	1599	1756	3845	3106	2298	885	1225	1370	5890	3263
Grand Total 1910, Nov. 1	16457	6351	1924	9002	5412	5729	4315	2192	1944	1068	2403	5620	3737	2871	2811	4543	3632	3608	5287	1833	1568	1670	3529	3016	2249	799	958	1378	5141	3265
Grand Total 1908, Nov. 1	14552	5883	1416	7411	5169	5329	4659	2132	1957	890	2246	5339	4972	2678	2733	3947	3543	3181	5187	3248	2597	1985	688	796	1274	4745	3287
Grand Total 1903, Nov. 1	13644	5114	1364	5677	4700	5342	4400	2113	2356	707	2086	5188	4607	2558	3154	3951	3113	2700	4555	3204	2410	1171	757	806	1016	3876	3466
Extension and Similar
Courses.....	638	3182	3644	597	1100	1185	235	1546	152	902	1676	379	411
Officers.....	525	337	261	907	725	731	737	109	275	208	200	537	465	305	354	425	434	291	564	290	217	218	280

Note.—The grand totals of the University of Minnesota prior to 1912 are inclusive of “extension and similar students,” which were listed separately in 1912 and 1913, hence the apparent decrease.

* Includes schools of mines, engineering, chemistry and related subjects.

† Included elsewhere.

‡ 1,220 students in attendance on summer courses.

women; Wisconsin, with 828 men and 776 women; Minnesota, with 639 men and 882 women; Texas, with 811 men and 709 women; Columbia, with 841 men and 623 women; Yale, with 1,402 men; Nebraska, with 541 men and 733 women, and Princeton, with 1,267 men.

In agriculture, Cornell leads with 1,354 students, being followed by Wisconsin with 968, Ohio State with 889, and Illinois with 792. In architecture Illinois with 351 is followed by Pennsylvania with 259, and Columbia and Cornell with 143 each. Washington University, with 305 art students, leads in that field, being followed by Syracuse with 178; while New York University continues to lead in commerce with 2,013 students, being followed by Pennsylvania with 1,430, Northwestern with 567, Wisconsin with 374, Illinois with 282, and California with 280. The largest dental school is at Pennsylvania, where 589 students are enrolled, as compared with 566 at Northwestern, 282 at Michigan, and 268 at Minnesota. Northwestern has the largest divinity school, enrolling 233 students, as against 125 at Chicago, 100 at Yale and 56 at Harvard; these are the only universities in the list that maintain schools of theology.

Syracuse has 220 students of forestry, Ohio State 65, Minnesota 41, Nebraska 36, and Yale 32; at California, Illinois, Michigan and Missouri, the forestry students are counted in with other departments. Columbia has a long lead in the number of non-professional graduate students, there being no less than 1,496 students enrolled in its faculties of political science, philosophy and pure science. Columbia is followed by Harvard with 489 students, Chicago with 480, Pennsylvania with 438, and California with 408. Columbia has the largest school of journalism, enrolling 106 students as compared with Wisconsin's 91,

Indiana's 73, and Missouri's 64. The largest law school is at New York University, where 771 students are registered in this subject; Harvard follows with 695 students, Michigan with 553, and Columbia with 450. In medicine Illinois leads with 445, being followed by New York University with 432, Tulane with 399, Johns Hopkins with 368, Michigan with 353, Columbia with 341, and Harvard with 306. Syracuse has the largest number of music students, namely, 925, there being 407 at Northwestern and 130 at Kansas. The Teachers College of Columbia University is by far the largest school of education connected with any of the institutions in the list. It has an enrollment this fall of no less than 1670 students, as against 699 at Pennsylvania, 443 at Texas, 421 at Pittsburgh and 420 at New York University. Columbia also has by far the largest school of pharmacy, enrolling 441 students, as against 255 at Illinois, 209 at Pittsburgh, and 135 at Northwestern. As for the scientific schools, Cornell continues to maintain its lead in this branch, enrolling 1,343 students, as against Michigan's 1,282, Yale's 1,133, Illinois's 1,001, California's 828, Ohio State's 811, Wisconsin's 775, Columbia's 665, Pennsylvania's 657, and Minnesota's 638. In veterinary medicine Ohio State leads with 163, being followed by Pennsylvania with 125, and Cornell with 122. All of the above figures for individual faculties are exclusive of the summer-session attendance. The largest summer session in 1913 was at Columbia University, where 4,539 students were enrolled, as against 3,771 at Chicago, 2,363 at California, 2,120 at Wisconsin, 1,408 at Michigan, 1,392 at Cornell, 1,163 at Tulane, and 1,084 at Indiana.

The largest number of officers is found at Columbia, where the staff of teaching and administrative officers consists of 907 mem-

bers, as against 737 at Illinois, 731 at Harvard, 725 at Cornell, and 633 at Wisconsin.

The 638 students enrolled at the University of California in extension and similar courses are distributed as follows: San Francisco Institute of Art 201, Wilmerding School of Industrial Arts 155, University Farm School 187, and short courses in agriculture 95.

The 3,182 students listed under extension and similar courses at the University of Chicago are enrolled in correspondence study courses. The 808 students mentioned under other courses are enrolled in regular university courses given primarily for teachers which meet on Saturday mornings and late in the afternoons.

Among the 84 students of medicine enrolled at the University of Cincinnati, 23 are registered in clinics and pathological work. The 486 students mentioned under other courses are enrolled in evening academic courses.

Of the 3,644 students enrolled under extension and similar courses at Columbia University, 1,152 are students in special classes at Teachers College and 2,492 are students in extension courses.

Of the 1,120 students registered at Cornell University under arts and sciences, 950 are candidates for the degree of A.B. and 170 for the degree of B.Chem. Of the total number of 4,760 students enrolled in the fall, 4,273 are men and 487 women. The students mentioned under extension and similar courses were enrolled in the short winter course in agriculture for 1912-13.

The 113 students mentioned under commerce at Harvard University are enrolled in the graduate school of business administration. Of the 61 students in architecture, 23 are enrolled in landscape architecture, while the 76 students mentioned under scientific schools are enrolled in the graduate school of applied science. Only

one department, forestry, which is a part of the graduate school of applied science, shows a marked decrease, the enrollment having dropped from 18 to 9. This may possibly be accounted for by the fact that this year for the first time students were expected to register in July, instead of in September, and this may not have been sufficiently widely known. Also, the course has been changed somewhat, including general work in the first year and special work in the second year. In landscape architecture, which is also in the graduate school of applied science, there has been a marked increase, 23 students being enrolled this year, as opposed to 15 in 1912, and 7 in 1911.

Of the 350 students given under other courses at the University of Illinois, 310 are enrolled in household science and 40 in library economy. In Chicago, the departments of medicine and dentistry have been reopened with higher entrance requirements—hence the material loss in the registration in these schools as compared with the registration of two years ago.

The students mentioned under scientific schools at Indiana University are pursuing courses in chemistry, provision for technical work being made at Purdue University.

The 86 students given under other courses at the State University of Iowa are enrolled in courses for nurses.

At Johns Hopkins, 169 men and 39 women are enrolled in the graduate school, and 15 are taking graduate work in engineering. Of the 368 students given under medicine, 8 are physicians attending special courses.

Of the 353 students given under medicine at the University of Michigan, 75 are enrolled in the homeopathic medical college.

At the University of Minnesota, agricul-

ture, inclusive of the home economic division, shows a steady growth. The apparent increase in the graduate school is explained by regulations concerning registration rather than by an influx of advanced students. However, impetus has been given to this department by the appointment of Professor Guy S. Ford, formerly of the University of Illinois, as dean. The law school is feeling the full effect of the requirement of two years of academic work for admission to regular courses, also the effect of discontinuing night classes. Many of the subjects heretofore offered by the law school in evening courses have been transferred to the extension department. Heretofore little emphasis has been given to music. This year marks the beginning of a regular four years' course in arts and music leading to the degree of bachelor of arts in music, consequently these students are listed separately this year for the first time. The requirements for admission are the same as for the college of science, literature and the arts. The college of engineering has secured F. E. Mann, of the University of Illinois, to take charge of the department of architecture, and the courses in architecture and architectural engineering have now been permanently established. The requirements for admission are the same as the engineering courses and the B.S. and appropriate professional degrees are granted at the close of four and five years, respectively. The reorganization of the extension division and the advent of Professor R. R. Price, formerly of Kansas University, as director of this department, explain the increased enrollment in this division.

The decrease in enrollment in the school of law at the University of Missouri is due to the fact that during this session all of the three classes in that school are based upon an admission requirement of two

years of college work, while the third-year class of last year was admitted under the former requirement of four years of high school work. There is a decrease in the school of engineering due to a similar reason, but as there has been an increase in the school of mines, the total enrollment in technical schools shows a slight increase. It has been anticipated that a similar decrease would be manifested in the school of journalism. Notwithstanding the fact, however, that there is during the present session only one of the classes which entered under the former requirements as compared with two such classes during the session of 1912-13, the total enrollment in the school shows an increase.

At the University of Nebraska, there is but little change in registration this year as against 1912, the most apparent increase being in the school of agriculture, at the expense of the professional schools.

Every school in New York University with the exception of the veterinary college shows a marked increase. The increase in the college of arts and pure science is due largely to the transfer of the medical preparatory class from the collegiate division to the freshman class of the college.

Of the 296 students enrolled in other courses at Northwestern University, 172 are registered in the school of oratory and 124 are taking courses for nurses. There have been noteworthy increases in attendance in several of the schools of the university, especially in the college of engineering, the dental school, and the school of commerce. There is an increase in the college of liberal arts in spite of the increased tuition fees. The same increase in fees was put into effect in the college of engineering, but, nevertheless, the enrollment is the most encouraging in the history of the college, it being due to a consistent and dignified sys-

tem of publicity. In the dental school, the university is increasingly strict in administering entrance requirements, and yet the enrollment has increased at a phenomenal rate. The increase in the school of commerce is an evidence of the need of such work in a large city; these courses are given in the evening, and the registration is in great part made up of men who are in business during the day. In the statistics of November 1, 1912, the report showed 368 students enrolled in the law school; the figure should be 268.

The 280 students given by Ohio State University under other courses are enrolled in home economics. The 227 students in extension and similar courses represent the enrollment of 1912-13. The fall enrollment shows substantial increases in all colleges except the college of law, which is 13 short of the enrollment November 1, 1912. The largest gains in the colleges have been as follows: agriculture 188, engineering 109, arts 94, and education 37. The increase in the fall enrollment is 434. These increases are probably due to a revival of interest among the alumni of the university. Two years ago, under the able leadership of Mr. Ralph D. Mershon, of New York, the alumni and former students of the university were reorganized into a live association. Local organizations have been formed in all parts of the state and in all prominent cities of the country. A permanent secretary, who gives all of his time to alumni interests, has been employed. The annual observance of "Ohio State Day" by alumni and former students throughout the state and country has done much to bring the university into prominence in local communities. A new department of competitive and recreative athletics was created by the board of trustees last June. The director of athletics and the men associated with him have been

given faculty rank. A limited amount of credit will be given for work in courses in the teaching of athletics. A new course in applied entomology, leading to the degree of bachelor of science in entomology, has been added to the curriculum. Two new combination courses have also been arranged, arts—agriculture and arts—home economics, making it possible for a student to receive two degrees in five years. The last General Assembly of Ohio authorized the establishment of an engineering experiment station at the university and the organization of a college of medicine. No definite action has been taken by the board of trustees. The General Assembly also authorized and directed the university to establish and organize a university extension division, for the purpose of carrying on educational extension and correspondence instruction throughout the state.

At the University of Pennsylvania, the 379 students listed under extension and similar courses are enrolled in courses at Wilkes Barre (185) and Scranton (194). The 9 students listed under other courses are enrolled in hygiene. The 699 students listed under pedagogy are enrolled in college courses for teachers. Nearly all departments share in the gains in enrollment, the most noticeable increase being in the Wharton school of finance and commerce, and in the evening school of accounts and finance. The school of architecture continues its advance in registration, the total this year being 259, or an increase of 43. The graduate school, with a total enrollment of 438, is 67 ahead of the preceding year. The law school, with a total enrollment of 381, is just 2 short of the figures of last year. The medical school is still feeling the effects of the raise in entrance requirements put into effect within the past few years. The enrollment of 283 shows a loss of 37. The dental school, with 589

students, has an increase of 80. Other departments show slight gains or losses which are negligible.

At the University of Pittsburgh, the 157 students listed under other courses are male candidates for degrees in economics. Of the 411 students given under extension and similar courses, 127 are taking teachers' courses without credit and 284 are enrolled in the evening school.

The students at Leland Stanford University listed under law and medicine and the graduate school are all included also under the college.

At Syracuse University, of the 233 students listed under separate courses, 50 are taking work in library economy; 275 students from the college of liberal arts are also taking work in education.

At Tulane University, 7 are enrolled in tropical medicine and hygiene and 22 are taking post-graduate medical work. The students mentioned under scientific schools are enrolled for engineering and sugar chemistry. The increase in numbers of the college of technology is due to changes made in the college, amounting practically to reorganization.

The 48 students listed under other courses at Washington University are in the school of social economy. The 319 students listed under extension and similar courses are taking Saturday courses for teachers and others in the college.

The 45 students listed under other courses at Western Reserve University are taking work in library economy. In Adelbert College, the college for women, the graduate school, the school of law and the library school, the annual tuition fee has been advanced from \$100 to \$125, and in the school of medicine from \$125 to \$150, these charges going into effect this year. The loss in enrollment in the school of law is

due to the graduation of the last class received upon the non-graduate basis, all classes now being upon the college graduate basis. The change from the non-graduate to the graduate basis is seen to have taken effect with but a small loss in numbers. The loss in enrollment in the school of medicine is more apparent than real, the difference being caused largely by the recent yearly graduation of two classes,—Ohio Wesleyan University and Western Reserve University,—and the reception of one class on the Western Reserve University college graduate basis only.

Of the 45 students listed under other courses at the University of Wisconsin, 43 are enrolled in library economy and 2 are taking work in public health. The 40 students in pharmacy include 24 students in a two-year pharmacy course, for which four years of high school preparation is not required. A forest rangers' course consisting of 12 students began work at the university January 7, 1913. These students did not complete their field work until November 30.

The most considerable increase in enrollment at Yale University is in the college. The entering classes in the law and medical schools also show noticeable increases and indicate that the requirement of a college degree for admission to the law school and of two years of college work for admission to the medical school, adopted two years ago, is being appreciated by the best grade of students. The only noticeable falling off is in the graduate school. This is due exclusively to the new requirement of two years' work for the master of arts degree instead of one year, as in force at Yale prior to 1912 and still the custom at almost all the American universities.

RUDOLF TOMBO, JR.

COLUMBIA UNIVERSITY