SCIENCE

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MSS. intended for publication and books, etc., intended for review should be sent to Professor J. McKeen Cattell, Garrison-on-Hudson, N. Y.

THE INFLUENCE OF FOURIER'S SERIES UPON THE DEVELOPMENT OF MATHEMATICS[1]

IN selecting a subject for to-day's address I have had the difficult task of interesting two distinct classes of men, the astronomer and the mathematician. I have therefore chosen a topic which, I trust, will appeal to both—trigonometric series. Though I propose to treat it only in its mathematical aspects, I shall try to do so in a broad way, tracing its *general* influence upon the trend of mathematical thought.

As you know, the theory of the infinite trigonometric series,

$$(\text{I.}) \quad f(x) = \tfrac{1}{2}a_0 + (a_1 \cos x + b_1 \sin x) + (a_2 \cos 2x + b_2 \sin 2x) + \cdots$$

is different *ab initio* from that of the power series,

$$P(x) = c_0 + c_1(x-a) + c_2(x-a)^2 + \cdots.$$

For the latter the fundamental element is x^n, of which the graph is, for positive x, a monotone increasing function, wholly regular, without peculiarities of any sort. It is therefore in no way surprising that the power series obtained by combining terms of form $c_n x^n$ define the most civilized members of mathematical society—the so-called analytic functions—which are most orderly in their behavior, being continuous throughout their "domains," possessing derivatives of all orders and a Taylor's series at every point; and so forth. On the other hand, the graph of $\sin nx$ or $\cos nx$ is a wave curve with crests and troughs, whose number in any x interval increases indefi-

[1] Address of the vice-president of Section A—Mathematics and Astronomy, American Association for the Advancement of Science, Atlanta, 1913.