pharmacology and Dr. William Darrach has been appointed assistant professor of surgery.

Dr. Ross A. Gortner, since 1909 resident investigator in biological chemistry at the station for experimental evolution of the Carnegie Institution of Washington, has been appointed associate professor of soil chemistry in the University of Minnesota.

Dr. Karl F. Meyer, whose special field is the tropical diseases, has been promoted to be professor of bacteriology and protozoology in the University of California.

Dr. J. Howard Agnew, formerly first assistant in the department of medicine, University of Michigan, has accepted the full time professorship in medicine in the University of Alabama, School of Medicine, at Mobile.

At Dartmouth College, Drs. E. J. Rowe and E. S. Allen have resigned as instructors in mathematics, the latter to accept an instructorship at Brown University. Dr. R. D. Beetle, of Princeton University, and Dr. L. C. Mathewson, of the University of Illinois, have been appointed instructors in mathematics.

D. K. Picken, professor of mathematics in Victoria College, University of New Zealand, has been appointed master of Ormond College, Melbourne University.

## DISCUSSION AND CORRESPONDENCE

### DADOURIAN'S ANALYTICAL MECHANICS

In the issue of Science of April 3, Dr. Dadourian replies to my criticism of his "Analytical Mechanics." His reply was read with interest. It was hoped that he would clear up several points in this reply that seemed to the reviewer as unsatisfactory. I do not wish to get into a controversy, but it seems to me that his standpoint is untenable. He says in his reply:

> It is a fact that I have applied vector addition to forces *without hesitation*, but I have shown as little hesitation in treating velocities, accelerations, torques, linear momenta and angular momenta as vectors. Why did not Professor Rettger accuse me of having assumed the "parallelograms" of these magnitudes? Is the "parallelogram of forces" more of a dynamical law than the parallelogram of torques, for instance? The parallelogram law applies to any vector and is not at all a characteristic of forces, therefore, it is not a dynamical law. It does not even deserve being called a "law" when applied to a special type of vectors. In its most general form the "parallelogram law" is the principle of the independence of mutually perpendicular directions in space, a purely geometrical principle. . . . After devoting an entire chapter to vector addition and after defining force as a vector, to introduce the "parallelogram of forces" as a new law, as Professor Rettger would have it, could serve only to show that the man who did it could not have a clear conception of the meanings of the terms he was using.

Let us assume that a body, originally in the position $O$, moves first through a distance, $a$, in a given direction and then through a distance, $b$, in another direction. Assume the body finally to be in the position $C$. The resultant displacement then is $OC = c$. The body would be in the same position, $C$, if it had moved first through the distance, $b$, and then through the distance, $a$, that is, its final position, or its final displacement is independent of the order in which the two displacements take place. They may take place, therefore, simultaneously, and the final or resultant displacement is still equal to $c$. If then we recognize that the two displacements have no mutual effect on each other, or, what amounts to the same thing, that the displacements are independent of each other, then the resultant displacement may be represented by the diagonal of a parallelogram of which the two displacements are adjacent sides. As soon as this "Principle of Independence" is once recognized, then the "parallelogram law" can be proved to hold also for velocities, accelerations and other conceptions of kinematics. The parallelogram law as applied to these quantities is then equivalent to the "principle of the independence of motions" and as such is a purely "geometric principle." These quantities, displacements, velocities and accelerations are therefore vectors in accordance with the definitions of a vector, and the principles of vector analysis may be applied advantageously.

Vector analysis may be called an algebra that rests on certain (arbitrary) assumptions, and the "parallelogram of vectors" is one of these fundamental assumptions. To define a quantity as a vector, and then conclude that the parallelogram law holds begs the whole question. The logical way to proceed would be to first *prove* that the quantity is a vector, that is, that the parallelogram law holds and then (advantageously) apply the principles of vector analysis. We can not prove, however, that a *force* is a vector. We must depend upon experience for our justification in assuming a force to be a vector.

We do not know what a force is. To say that "force is an action" explains nothing, and to define it as a vector begs the whole question. Experience and experience alone can justify us in dealing with forces as vectors of a certain kind. In other words, the "parallelogram law of forces" is nothing more than an assumption and is *not* a purely "geometric principle." If we *assume* that a force can be measured by the motion it produces, and if we *assume* that the effect of each force is independent of the effect of the other forces acting, then it follows that the parallelogram law holds also for forces, since we know that this law, as a consequence of the principle of independence, does hold for the motions (accelerations) produced. This argument, however, makes two assumptions. First, it assumes that a force can be measured by the acceleration it produces (in its own line of action), and, secondly, it assumes "the principle of independence" for forces. Now these two assumptions are involved in Newton's Second Law of Motion. In other words, the parallelogram law of forces is a consequence of Newton's Second Law of Motion, and, therefore, in its last analysis is an assumption. If, however, the parallelogram law is once assumed for forces, then it can be proved for moments and other (vector) qualities involving force. It is, therefore, sufficient to *assume* the law to hold for *forces*.

It is a question whether we have a right to assume the parallelogram law even for velocities and accelerations without proving it, and to assume it for forces is equivalent, as we have seen, to assuming Newton's Second Law of Motion.

In my criticism it was stated:

> On page 102 he assumes that a force is proportional to the accelerations produced. This assumes Newton's Second Law.

In reply he says:

> This statement is not quite right. The relation between force and acceleration which I have called *force-equation* is derived on page 106 from the fundamental principle which I have postulated. In this derivation I have made use of the definition of kinetic reaction which is stated and illustrated on pages 102 to 105, but this is not equivalent to assuming a new principle.

This is true as far as it goes, but he fails to add that the form of this "force-equation" depends upon the actual value of this "kinetic reaction" which he finds as the result of experiments to be equal to the mass times the acceleration produced, that is,

$$\text{Kinetic reaction} = mf.$$

He seems to me to be making a "distinction without a difference." At least he is making an assumption here that is equivalent to assuming Newton's Second Law of Motion.

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### ACCESSORY CHROMOSOMES OF MAN

IN reply to Professor T. H. Morgan's statement in SCIENCE, June 5, 1914, I wish merely to request the reader who may be interested to read my note of May 15[1] and my paper, "Accessory Chromosomes in Man,"[2] and then Professor Montgomery's paper,[3] that he may decide for himself whether Montgomery and I have not agreed in the main regarding the accessory chromosomes of man. This was the only point at issue in my former communication, which was meant not as a "complaint," but as a correction to a misleading inference.

As to the material on which Montgomery and I came to different conclusions regarding a second pairing of the ordinary chromosomes, Professor Morgan is mistaken in stating that

1 SCIENCE.

2 *Biol. Bull.*, XIX., 4; September, 1910.

3 *Jour. Acad. Nat. Sci. Phila.*, XV., second series, 1912.