Life Histories; Environmental Change; Metabolism, Growth, etc.; Adjustment between the Environment and Animal. Its scientific value lies in the author's outline for the organization of the science.

In the first chapter, the subject is divided into "individual ecology" or the ecology of individuals and species, and "aggregate ecology," or the ecology of taxonomic groups of species, genera, families, etc. These two divisions have usually not been recognized separately. The distinction is good, but the two divisions taken together are coordinate with his third division, "associational ecology," or the ecology of communities. Chapters II.–IV. are devoted to ecological surveys and methods of conducting them. The author rightly deplores the tendency of museums to rate the work of collectors and expeditions on the basis of number of specimens brought back, as this discourages the recording of ecological facts. The methods of collecting, preserving and arranging notes and specimens, and securing proper identification of the latter are given. These chapters will be of material aid to those undertaking field ecological study.

Although divided under several chapter headings, the remainder of the book consists essentially of about 90 pages of classified bibliography. The references are of a diversified type and are intended to guide the worker to needed information, ranging from the making up of the sometimes necessary camping outfit, to the preparation of his results for the printer. They are classified under general headings and many are followed by statements as to contents. Representative papers on the environment, animal communities, struggle for existence, physiology, behavior and many other topics are included. The references are complete enough to give valuable suggestions to workers from almost any point of view in ecology. The comments on the more important older ecological papers make it clear that a considerable number of incomplete attempts at organization of knowledge of animal communities have been recorded. Thus by means of the references, the book gives the history of the development of the science.

V. E. SHELFORD
UNIVERSITY OF CHICAGO

SPECIAL ARTICLES

THE "GOLDEN MEAN" IN THE INHERITANCE OF SIZE

IN bulletin 242 of the New Jersey State College Experiment Station, page 39, appears the following statement:

> The size and shape of the F_1 (tomato) fruits are the geometric means between the size and shape corresponding to those factors of the parents, which were active in crossing.

This statement was based upon the measurements of many thousands of tomato fruits. A mode of inheritance in which $x(F_1) = \sqrt{ab}$ assumes that in the union of gametes representing factors for size a multiplication and the extraction of a square root take place. I was unable at the time to explain how that might occur, and so far no attempt by any one else to interpret the significance of my results has come to my notice. In a forthcoming bulletin I shall present details of F_2 and F_3, and this paper is published solely to set forth the nature of the principle of size inheritance by the "golden mean," to indicate its bearing upon vital questions on subjects in heredity, and to establish the priority of its discovery.

When two factors for different sizes of homologous parts meet in a cross, the resulting F_1 size is commonly intermediate in such parts as are not greatly subject to fluctuation. If a and b represent the parental size characters, it has been tacitly assumed by most investigators that the F_1 size is $(a+b)/2$. In tomato fruits I found it to be $\sqrt{ab}$. On the face of it the average (algebraic mean) seems the more probable, but when used as the basis for the comparable inheritance of lines, surfaces, and volumes, it becomes impossible. For example, let plants be crossed, which have spherical fruits of diameters a and b, and let us assume, for the sake of simplicity, that all cells constituting the volumes of both fruits are of equal dimensions. Then the ratio between parents and F_1 of the number of cells forming any diameter one cell in thickness would be as $a:(a+b)/2:b$; the ratio between the cells making up the surface (epidermis) of the respective fruits would be as $a^2:[(a+b)/2]^2:b^2$; while the ratio between the number of cells constituting the entire volume would be as $a^3:[(a+b)/2]^3:b^3$, all of which ratios are different. It is evident, however, that any cell might be cut by a diameter and that the surface cells are part of the volume. The size inheritance of an epidermal cell might fall under either of these three ratios, depending on which aliquot part of the fruit was under study. If there is inheritance of the nature $x=(a+b)/2$, it is not clear why the diameters and not the surfaces or volumes should fall under that principle. If one does, the other two can not.

If it be assumed, under like conditions, that $x=\sqrt{ab}$, the ratio between parents and F_1 of the number of cells forming a diameter one cell in thickness becomes $a:\sqrt{ab}:b$; that between the surface cells $a^2:ab:b^2$; and that between the total cells in the volume of the sphere $a^3:ab\sqrt{ab}:b^3$. These three ratios are all equal to $a:\sqrt{ab}:b$, if a and b mean the respective parents, so that no matter what aliquot part of the fruit is under study, the ratio of size inheritance of lines, surfaces, and volumes would be the same. Surely this appears the more probable.

But how can a plant extract the square root of a product? For the sake of convenience in demonstration let us assume that the fruits are cubes instead of spheres, that all cells are equal and of size 1, and that parental characters of size are 4 and 9. Then the length factor of the F_1 would be made up of two forces, one tending to build strings of four cells in the direction of the longitudinal axis, the other tending to build strings of nine cells. Similarly, the breadth factor would be made up of two such forces building at right angles, and in the same way the factor for thickness at right angles to both. Constituent 4 of the length factor, coming from one parent and constituent 9 of the breadth factor coming from the other parent may then be imagined as building at right angles to each other, in strings of nine in one direction, in strings of four in the other. The respective partners of the two factors would be similarly engaged, and likewise the two length factors with those for thickness and the two breadth factors with those for thickness. Each set of two would tend to build rectangles of the dimensions $9\times 4=36$. If now there were some force stronger than the tendency of the size factors, which would prevent the formation of rectangles and permit only squares, while having no influence on absolute size, area, or volume, the sets of size factors would be forced to modify the shape of their structures, making squares instead of rectangles. Since the modifying force did not influence area, the resulting squares would also be of the area 36, their sides 6, and the cubical fruits $6\times 6\times 6=216$. This modifier of size is the factor for shape. When both parents carry only the factor for cubical shape, the F_1 fruits are cubical, no matter what the tendencies of the size factors.

In spheres the diameters are directly proportional to the sides of equal cubes; so that what applies to cubes in this respect, applies to spheres as well. The factor for spherical shape is the modifier of the interaction of the factors for absolute size.

At first sight it may seem as if the fact that the size of the F_1 is the golden mean between the parental sizes can be of little value beyond furnishing an explanation of partial dominance in the F_1. However, the recurrence of the action of the modifier (shape) upon the various size combinations in the F_2 interferes greatly with the chances for the appearance of certain visible size characters.

We know that size characters do segregate in the F_2, but we admit that with them the simple Mendelian ratio of 1:2:1 is never realized, though in large populations the parental sizes may reappear. Mendelians commonly try to account for the complicated ratios by assuming the presence of multiple factors; non-Mendelians point to the same ratios as quasi-evidence against Mendelian inheritance. I here offer a different explanation.

In the F_1 fruit $6\times 6\times 6$ the size character 6 is not the result of a factor for size 6, but of the three forces exerted by two size factors and one modifier. It has not been assumed in the above explanation that the partners in an F_1 "factor" fuse to make a new factor. Thus we have to deal in both ♂ and ♀ gametes produced by the F_1 with factors 4 and 9 for each of the three size factors, length, breadth and thickness, occurring in equal numbers, and mating by chance (in self-fertilization). This is the same assumption made by Mendelians, even if they should not admit the F_1 inheritance of $x = \sqrt{ab}$. There would be these eight possible combinations uniting with each other by chance:

Combination	Length	Breadth	Thickness
1	4	4	4
2	4	4	9
3	4	9	4
4	9	4	4
5	9	9	4
6	9	4	9
7	4	9	9
8	9	9	9

These would result in the following 64 matings:

Mating	Resultant	Volume
1 × 1	4 × 4 × 4	64
1 × 2	4 × 4 × 6	96
1 × 3	4 × 6 × 4	96
1 × 4	6 × 4 × 4	96
1 × 5	6 × 6 × 4	144
1 × 6	6 × 4 × 6	144
1 × 7	4 × 6 × 6	144
1 × 8	6 × 6 × 6	216
2 × 1	4 × 4 × 6	96
2 × 2	4 × 4 × 9	144
2 × 3	4 × 6 × 6	144
2 × 4	6 × 4 × 6	144
2 × 5	6 × 6 × 6	216
2 × 6	6 × 4 × 9	216
2 × 7	4 × 6 × 9	216
2 × 8	6 × 6 × 9	324
3 × 1	4 × 6 × 4	96
3 × 2	4 × 6 × 6	144
3 × 3	4 × 9 × 4	144
3 × 4	6 × 6 × 4	144
3 × 5	6 × 9 × 4	216
3 × 6	6 × 6 × 6	216
3 × 7	4 × 9 × 6	216
3 × 8	6 × 9 × 6	324

Mating	Resultant	Volume
4 × 1	6 × 4 × 4	96
4 × 2	6 × 4 × 6	144
4 × 3	6 × 6 × 4	144
4 × 4	9 × 4 × 4	144
4 × 5	9 × 6 × 4	216
4 × 6	9 × 4 × 6	216
4 × 7	6 × 6 × 6	216
4 × 8	9 × 6 × 6	324
5 × 1	6 × 6 × 4	144
5 × 2	6 × 6 × 6	216
5 × 3	6 × 9 × 4	216
5 × 4	9 × 6 × 4	216
5 × 5	9 × 9 × 4	324
5 × 6	9 × 6 × 6	324
5 × 7	6 × 9 × 6	324
5 × 8	9 × 9 × 6	486
6 × 1	6 × 4 × 6	144
6 × 2	6 × 4 × 9	216
6 × 3	6 × 6 × 6	216
6 × 4	9 × 4 × 6	216
6 × 5	9 × 6 × 6	324
6 × 6	9 × 4 × 9	324
6 × 7	6 × 6 × 9	324
6 × 8	9 × 6 × 9	486
7 × 1	4 × 6 × 6	144
7 × 2	4 × 6 × 9	216
7 × 3	4 × 9 × 6	216
7 × 4	6 × 6 × 6	216
7 × 5	6 × 9 × 6	324
7 × 6	6 × 6 × 9	324
7 × 7	4 × 9 × 9	324
7 × 8	6 × 9 × 9	486
8 × 1	6 × 6 × 6	216
8 × 2	6 × 6 × 9	324
8 × 3	6 × 9 × 6	324
8 × 4	9 × 6 × 6	324
8 × 5	9 × 9 × 6	486
8 × 6	9 × 6 × 9	486
8 × 7	6 × 9 × 9	486
8 × 8	9 × 9 × 9	729

Among the possible combinations there are 16 in which both ♂ and ♀ have furnished the factor for length 4, and it has been commonly assumed that therefore one quarter of the F_2

population should show the character of length 4. The column of volumes, however, shows that there is only one out of the 64 with a volume 4^3. All others with factors for length four have larger volumes, because their factors for breadth and thickness are greater than four. Here again the law of the golden mean is followed as all combinations bearing unequal size factors are forced to build cubes by the modifying factor shape. Evidently all matings resulting in equal volumes will make equal cubes and therefore show equal characters of length, breadth and thickness, though not necessarily possessing equal size factors nor even the factor for the size which they exhibit. If we group the results by volume and length of side, we have:

Volume	Instances	Side of Cube
64	1	$\sqrt[3]{64} = 4.$
96	6	$\sqrt[3]{96} = 4.57 -.$
144	15	$\sqrt[3]{144} = 5.25 +.$
216	20	$\sqrt[3]{216} = 6.$
324	15	$\sqrt[3]{324} = 6.87 +.$
486	6	$\sqrt[3]{486} = 7.86 -.$
729	1	$\sqrt[3]{729} = 9.$

That means that the chance for a parental size (whether line or surface or volume) to reappear is only 1:64 instead of 1:4. Moreover it is clear that each of the 8 possible combinations given on page 7, when mating with its like, will breed true to all three size characters, and continue to breed true thereafter if selfed. That means that these matings will form constant races, viz.:

Mating	Resultant	Volume	Size
1 × 1	4 × 4 × 4	64	4
2 × 2	4 × 4 × 9	144	5.25 }
3 × 3	4 × 9 × 4	144	5.25 }
4 × 4	9 × 4 × 4	144	5.25 }
5 × 5	9 × 9 × 4	324	6.87 }
6 × 6	9 × 4 × 9	324	6.87 }
7 × 7	4 × 9 × 9	324	6.87 }
8 × 8	9 × 9 × 9	729	9

Besides the parent-like strains then, we shall seemingly have two other races, one of volume 144, size 5.25, the other of volume 324, size 6.87, which will continue to breed true if selfed. Each of these strains consists of three gametically different, though visibly indistinguishable lines, which when crossed will give an F_1 equal to both parents, but segregating to some extent in the F_2. The finding in the F_2 or later generations of lines which breed true to size characters is thus not proof of the presence of multiple size factors in the original parents, etc.

In the bulletin in preparation I intend to discuss the bearing of the law of the golden mean upon the interpretation of inheritance of shape and number, mutants, latent factors, inhibitory factors, coupling and repulsion, factors other than those of size, shape, and number, and other points as they may come up, but for the sake of science I invite investigation into these relationships on the basis I here offer, even before I am able to publish the bulletin, which may not appear for several months.

B. H. A. GROTH

"THE LOWEST TEMPERATURE OBTAINABLE WITH ICE AND SALT"

FAHRENHEIT placed the 0° mark on his arbitrary thermometer scale at "the lowest temperature obtainable with ice and salt" or 32° below the freezing point of water, believing that water did not have a constant freezing point because of the undercooling which precedes solidification.[1]

While discussing freezing mixtures with a friend recently I stated that a temperature of — 19° C. could be *easily* obtained and maintained for some hours with an ice and salt mixture. My friend questioned the accuracy of the thermometer inasmuch as — 19° C. is below 0° F. (0° F. = — 17.78° C.). I have, therefore made a careful test to ascertain whether an ice and salt mixture may not show a lower temperature than 0° F.

About a gallon of finely chopped, hard, ice was mixed with a quart or more of coarse salt in a water-tight wooden box, the wooden box being used because of the insulation which it

[1] See Encyclopedia Britannica, 11 ed., "Heat," article 2.