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THE ORBITS OF FREELY FALLING BODIES

THE path described by a body falling freely from a considerable height above the surface of the earth presents a problem of interest alike to the mathematical and to the experimental physicist. The former sees in it a capital application of the principles of "relative motion"[1] and the latter sees in it a promising way of demonstrating the rotation of the earth. It has attracted perennial attention for more than a century and has been frequently referred to in this journal during the past decade.

The mechanical aspects of this problem were first carefully considered by Gauss and Laplace one hundred and ten years ago. Gauss's equations of motion for a falling body were furnished in a letter to Benzenberg, who was interested especially in the proper interpretation of experimental results. Gauss's solution of the problem is now accessible in the fifth volume of his collected works. He concluded that in addition to the obvious easterly deviation there should be a small meridional deviation towards the equator from the plumb line defined by a bob suspended from the initial position of the body and normal to some plane of reference below. It seems probable that this latter conclusion prompted Laplace to reinvestigate the subject, for he published a very remarkable paper in May, 1803, in the *Bulletin de la*

[1] This means only that account must be taken of the variations in position of some of the axes or planes of reference with the lapse of time. Why such motion should have been called "relative" and the less complex motion called "absolute" is a question worthy of investigation in the history of mechanics.

Société Philomatique, in which he invites special attention to his conclusion that there is no meridional deviation towards the equator. In view of this discrepancy between these preeminent authors it is a surprising circumstance that nearly all subsequent writers on the subject should have followed Gauss; and it is still more surprising that the more comprehensive and more suggestive, though more difficult, treatment of the problem by Laplace should have been little noticed and less followed by recent authors. Since the appearance of the papers just referred to by Gauss and Laplace only one author, until quite recently, appears to have considered the subject worthy of an independent investigation. This author is Poisson, who published in 1838 an important memoir on the theory of gunnery (in the *Journal de l'École Polytechnique,* Tome XVI.) of which a freely falling body presents a special case. As regards the meridional deviation in question Poisson goes one step further than Gauss and Laplace and leads us to infer (correctly) that his investigation shows no deviation either towards or away from the equator.

My attention was called to this subject about ten years ago, chiefly through the communications concerning it published in this journal by Professor Cajori and Professor E. H. Hall. A casual reading of the papers of Gauss, Laplace and Poisson indicated that they ought all to agree essentially, since they all limit themselves to terms of the first order of approximation of the small quantities involved, especially the angular velocity of the earth, which is obviously a fundamental factor in any solution of the problem. In the meantime, other occupations have led me to neglect this branch of geophysics until my attention was reattracted to it by the suggestive papers of Professor William H. Roever published recently in the *Transactions of the American Mathematical Society.*[2] A preliminary survey of the subject indicated that the obscurities and the discrepancies presented by it could be removed only by an independent investigation founded on present-day knowledge of geodesy. Such an investigation has been made and is now available to the mathematical physicist in Nos. 651–652 of the *Astronomical Journal* (August 4, 1913) under the title "The Orbits of Freely Falling Bodies." The object of this communication is to explain briefly for the information of the general reader the salient features of the subject, the sources of its obscurities, the requirements of a precise and correct determination of the orbits in question, the new results reached, and the reasons why they differ in certain important respects from those hitherto considered valid.

The motion of a falling body depends on three elements, namely: (1) the rotation of the earth; (2) the attraction of the earth; and (3) the difference between geocentric and geographic latitude. The effect of rotation is expressed in the equations of motion of a falling body by terms involving both the first and the second powers of the earth's angular velocity. In general, following Gauss, Laplace and Poisson, terms in the second power of this velocity have been neglected. It turns out that the meridional deviation is a term of the second order in this velocity and other quantities of the same order. Hence it failed to appear in the investigations of the above-named authors, or appeared only as a mathematical fiction and with the wrong

[2] "The Southerly Deviation of Falling Bodies," Vol. XII., No. 3, July, 1911; and "The Southerly and Easterly Deviations of Falling Bodies in an Unsymmetrical Gravitational Field," Vol. XIII., No. 4, October, 1912.

sign in the case of Gauss. The effect of the attraction of the earth presents difficulty, for the earth is not centrobaric, though many authors have assumed it to be such. Gauss and Laplace undoubtedly understood the nature of this difficulty: Laplace's paper (referred to above), is, indeed, entirely satisfactory even now so far as its generalities are concerned. But the necessary observational knowledge, since accumulated, was not available to these pioneers. Each of them was justified, perhaps, in assuming that the effect of the square of the angular velocity would be negligible and that the attraction would be sensibly what has been generally, but now quite vaguely and inappropriately, called "gravity" or "acceleration of gravity," and expressed by the letter g. But this attraction varies certainly with the latitude of the position of the falling body and possibly also with its longitude, and it is not identical with the resultant acceleration due to the attraction and to the rotation of the earth. In respect to both of these points the details of the papers of Gauss, Laplace and Poisson along with the papers of their followers, are all, so far as I am aware, not only obscure, but inadequate. Closely related to the question of the earth's attraction of a falling body is the distinction between its varying geocentric latitude and the constant geographical latitude of the plumb line to which the orbit of the body is referred. This distinction is essential to a correct determination of the meridional deviation, but its fundamental importance does not appear to have been recognized hitherto.

Failure on the part of the earlier authors to perceive the essential rôles of these elements and a tendency to avoid the complications they entail in dealing with the differential equations of motion, account completely for the obscurities and the confusion which initially beset the modern reader who attempts to understand the present extensive literature of this subject. The admirably conceived investigation of Laplace, since published as Chapitre V., Tome IV., of his Mécanique Céléste, presents additional difficulties by reason of his autocratic and unnecessary neglect of terms, without assigning their relative magnitudes, and by reason of his ready suppression, after the fashion of his day, of the identity of any quantity by calling it unity. Following Gauss, many recent authors also after neglecting terms of the second order in their equations of motion, have proceeded to get such terms by a purely mathematical process which has no warrant in the physical circumstances of the case. It has been necessary, therefore, in order to remove the prevailing uncertainties of the subject, to reinvestigate it, avoiding precedent and visualizing the conditions of the problem in the light of the more recent developments of physical geodesy rather than in the light of the foundations of this science laid so largely and so effectively by Gauss, Laplace and Poisson a century ago.

Accordingly, the equations of motion of the falling body are established without neglect of any terms which belong to them, and no terms in the integration of these equations are neglected without precise specification of their relative magnitudes. The energy method of Lagrange is followed in establishing the equations of motion, partly because it is specially adapted to the case and partly because it does not appear to have been used for this purpose hitherto. The position of the body is defined by reference to four sets of axes, and the equations of motion for each of three of these sets are derived and integrated so as to include all terms of the second order. These latter depend not only on the square of the angular velocity of the earth, but on its attraction and on the difference between the geocentric and the geographic latitudes of the point in which a line drawn through the initial position of the body and normal to some plane of reference below pierces this plane. The three sets of equations of motion just referred to are expressed in terms (1) of the polar coordinates of the body (r, ψ, λ), r denoting radius vector from the center of the earth, ψ geocentric latitude and λ longitude from a principal equatorial axis of inertia of the earth; (2) of the rectangular coordinates (ξ, η, ζ), with origin at the point of intersection of that plumb line through the initial position of the body which is perpendicular to the horizontal plane of reference below, with distance ξ measured in this horizontal plane and parallel to the meridian plane through the initial position of the body, positively towards the equator, with distance η positive towards the east and normal to the initial meridian plane, and with distance ζ positive upwards and parallel to the normal at the origin; (3) of the orthogonal coordinates (η, ρ, σ), giving the distance η of the body east of the initial meridian plane, the distance ρ of η from the earth's axis of figure and the distance σ of the body from the plane of the earth's equator. It is thus practicable not only to approach the problem by different routes and to check all steps in the processes of solution, but also to see at once wherein the results reached differ from the conflicting results hitherto published.

Of the three sets of equations of motion, that for the last, or that for the coordinates η, ρ, σ, is the simplest. The integrals of this set (new to the subject, so far as I am aware) give the distance σ to a high order of approximation as a simple harmonic function whose amplitude is the initial value of σ; while the distances η and ρ are given with equal precision by sums respectively of two simple harmonic functions of two different angles. It is remarkable also that the diminution of the radius vector r and the easterly deviation η are each expressed with precision by a single hyperbolic term. In general, the system of coordinates r, ψ, λ is most convenient for the purposes of computation. But the equations for interconversion of all of the sets of coordinates are given in detail in the mathematical paper referred to.

It is shown that the meridional deviation specified by the ordinate ξ is always negative, or that this deviation is always towards the adjacent pole in either hemisphere instead of towards the equator as hitherto supposed. For a fall of 10 seconds, or 490.24 meters (in vacuo), in latitude 45° the meridional deviation would be 3.03 centimeters, and the easterly deviation 16.85 centimeters. These two deviations are proportional approximately to the square and to the cube, respectively, of the time of fall.

My investigation is subject to two voluntary restrictions and to one limitation dependent on our present lack of observational information in geodesy. The first restriction lies in the neglect of the effect of atmospheric resistance on the orbit of the falling body. This effect is known from the work of Laplace, Poisson and others to be very small, since the path of the body throughout its fall is everywhere very nearly normal to the stratification of the air. For such falls as may be practicable for observation this effect is negligible, especially in comparison with the effects of currents of air and of lateral displacement due to the rolling of the smoothest spheres.[3] The other restriction lies in solving the

[3] I consider it quite impracticable to make any conclusive experiments on the deviation of spheres falling in air.

problem of fall for the case in which the orbit is wholly external to the earth. The more complex case of a body falling down a bore-hole, or mine shaft, or the case in which the orbit lies partly without and partly within the earth's crust, is not considered. In view of the difficulties in the way of experimental applications it has not seemed to me worth while to extend the paper so as to include the additions and the modifications essential to these more complex cases.

The limitation referred to arises from insufficient knowledge as to the distribution of the earth's mass in respect to the plane of the equator. For nearly a century it has been generally assumed that this distribution is such as to make the two principal equatorial moments of inertia of the earth equal. In the absence of adequate information on this point I have followed the current assumption, the effect of which in the case of a falling body is to make its orbit independent of longitude. But I do not believe this assumption is justified, and I would take this occasion to urge upon astronomers and geodesists the great need for the settlement of this and other questions in geophysics of a systematic gravimetric survey of the earth. Any inequality in these moments of inertia produces also a necessary prolongation of the Eulerian cycle which figures so prominently in the theory of latitude variations, and it appears to me highly probable that this prolongation is due quite as much to that inequality as to an elastic yielding of the mass of the earth.

R. S. WOODWARD

FUNCTIONS AND LIMITATIONS OF THE GOVERNING BOARD[1]

THE development of higher education in America during the past quarter of a century has no parallel in history. In no other country have private citizens lavished upon universities so many millions for equipment and endowment. In no other country have universities received from state or national governments so many millions for maintenance. The annual income of Columbia University is greater than the combined incomes of Oxford with her score of colleges—Oxford with a thousand years behind her, the great national university of England. The University of Illinois, which twenty-five years ago was scarcely the equal in income or equipment of a first class agricultural high school of the present day, has an annual income far greater than that of the great national university of Germany, at Berlin, an income greater than that of the Sorbonne—in short, an income far greater than is claimed for any of the ancient and famous universities of the Old World. More money—one may venture to assert, the figures are not at hand—has been spent upon buildings and equipment for the University of Chicago during the past fifteen years than has been spent upon the buildings and equipment for the University of Bologne throughout its thousand years of history.

But after all, vast endowments and stately halls of granite or marble do not make a university. A real university is the creation of great men. Only in an inspiring environment which lures to it real scholars and thinkers may a great university be created or maintained. The finer spirits of the republic of letters will shun and hate the stifling atmosphere of a university, no matter how vast its endowment or how splendid its buildings, that does not give its professors a feeling of security and of freedom.

Does the American university offer to its teachers such an environment? Some doubtless do, the vast majority unquestion-

[1] Speech delivered (July 9) before the National Educational Association, at Salt Lake City, by Edwin Boone Craighead, LL.D., D.C.L., president of the University of Montana.