

Jepson's "Silva of California,"² the same to be reprinted under the above name in University of California Publications, Agricultural Science Series, Vol. II., No. 1 (now in press).

The chief reason for describing this form as a variety rather than a species is that *it does not breed true*. Several tests of seeds from different trees of this form have been made by the writer and in all but one test a number of the seedlings (never the same proportion) are typical *J. californica* in leaf characters. Obviously this is sufficient proof of a relationship which it is highly desirable to indicate by the name employed.

The reason for rejecting the name *quercifolia* is that the leaves are *not* oak-like. They resemble leaves of certain species of *Rhus* more than oaks. For this reason the writer had considered *anacardifolia* as a name, but the leaves are very unlike those of some species of the Anacardiaceæ. On the other hand, in general appearance of the trees this walnut does resemble a small-leaved oak. This is largely due to the habit of growth, the small size of the leaves and the dark green color of the foliage. Hence the name *quercina* is deemed proper, especially when used in varietal rank.

E. B. BABCOCK

SCIENTIFIC BOOKS

Principia Mathematica. By ALFRED NORTH WHITEHEAD, Sc.D., F.R.S., Fellow and late Lecturer of Trinity College, Cambridge, and BERTRAND RUSSELL, M.A., F.R.S., Lecturer and late Fellow of Trinity College, Cambridge. Cambridge University Press. 1912. Vol. II. Pp. xviii + 772.

Differential and Integral Calculus. An Introductory Course for Colleges and Engineering Schools. By LORRAIN S. HULBURT, Collegiate Professor of Mathematics in the Johns Hopkins University. New York, Longmans, Green and Co. 1912. Pp. xviii + 481.

An Elementary Treatise on Calculus. A Text-book for Colleges and Technical Schools. By WILLIAM S. FRANKLIN, BARRY MACNUTT

² *Ibid*.

and ROLLIN L. CHARLES, of Lehigh University. Published by the authors. South Bethlehem, Pa. 1913. Pp. vi + 292.

The Calculus. By ELLERY W. DAVIS, Professor of Mathematics, the University of Nebraska, assisted by WILLIAM C. BRENKE, Associate Professor of Mathematics, the University of Nebraska. Edited by EARL RAYMOND HEDRICK. New York, The Macmillan Company. 1912. Pp. xx + 446.

Readers who desire to gain with a minimum of effort a fair knowledge of the nature, magnitude, method and spirit of Messrs. Whitehead and Russell's great undertaking and achievement may be referred to the *Bulletin of the American Mathematical Society*, Vol. XVIII., and to *SCIENCE* for January 19, 1912, where will be found somewhat extensive reviews of Vol. I. of the "Principia." The immensity of Vol. II., together with its exceedingly technical content and method, make it undesirable to review this volume minutely in this journal, and the purpose of this notice is merely to signalize the appearance of the work and to indicate roughly the character and scope of its content.

Owing to the vast number, the great variety and the mechanical delicacy of the symbols employed, errors of type are not entirely avoidable and the volume opens with a rather long list of "errata to Volume I." The volume in hand is composed of three grand divisions: Part III., which deals with cardinal arithmetic; Part IV., which is devoted to what is called relation-arithmetic; and Part V., which treats of series. The theory of types, which is presented in Vol. I., is very important in the arithmetic of cardinals, especially in the matter of existence-theorems, and for the convenience of the reader Part III. is prefaced with explanations of how this theory applies to the matter in hand. In the initial section of this part we find the definition and logical properties of cardinal numbers, the definition of cardinal number being the one that is due to Frege, namely, the cardinal number of a class *C* is the class of all classes similar to *C*, where by "similar" is meant that two classes are similar when and only when the elements

of either can be associated in a one-to-one way with the elements of the other. This section consists of seven chapters dealing respectively with elementary properties of cardinals; 0 and 1 and 2; cardinals of assigned types; homogeneous cardinals; ascending cardinals; descending cardinals; and cardinals of relational types. Then follows a section treating of addition, multiplication and exponentiation, where the logical muse handles such themes as the arithmetical sum of two classes and of two cardinals; double similarity; the arithmetical sum of a class of classes; the arithmetical product of two classes and of two cardinals; next, of a class of classes; multiplicative classes and arithmetical classes; exponentiation; greater and less. Thus no less than 186 large symbolically compacted pages deal with properties *common* to finite and infinite classes and to the corresponding numbers. Nevertheless finites and infinities do differ in many important respects, and as many as 116 pages are required to present such differences under such captions as arithmetical substitution and uniform formal numbers; subtraction; inductive cardinals; intervals; progressions; Aleph null, $\aleph_0$; reflexive classes and cardinals; the axiom of infinity; and typically indefinite inductive cardinals.

As indicating the fundamental character of the "Principia" it is noteworthy that the arithmetic of relations is not begun earlier than page 301 of the second huge volume. In this division the subject of thought is relations including relations between relations. If R_1 and R_2 are two relations and if F_1 and F_2 are their respective fields (composed of the things between which the relations subsist), it may happen that F_1 and F_2 can be so correlated that, if any two terms of F_1 have the relation R_1 , their correlates in F_2 have the relation R_2 , and *vice versa*. If such is the case, R_1 and R_2 are said to be *like* or to be *ordinally similar*. Likeness of relations is analogous to similarity of classes, and, as cardinal number of classes is defined by means of class similarity, so relation-number of relations is defined by means of relation likeness. And 209 pages are devoted to the fundamentals of relation-

arithmetic, the chief headings of the treatment being ordinal similarity and relation-numbers; internal transformation of a relation; ordinal similarity; definition and elementary properties of relation-numbers; the relation-numbers, 0_r , 2_r and 1_r ; relation-numbers of assigned types; homogeneous relation-numbers; addition of relations and the product of two relations; the sum of two relations; addition of a term to a relation; the sum of the relations of a field; relations of mutually exclusive relations; double likeness; relations of relations of couples; the product of two relations; the multiplication and exponentiation of relations; and so on.

The last 259 pages of the volume deal with series. A large initial section is concerned with such properties as are common to all series whatsoever. From this exceedingly high and tenuous atmosphere, the reader is conducted to the level of sections, segments, stretches and derivatives of series. The volume closes with 58 pages devoted to convergence, and the limits of functions.

To judge the "Principia," as some are wont to do, as an attempt to furnish methods for developing existing branches of mathematics, is manifestly unfair; for it is no such attempt. It is an attempt to show that the entire body of mathematical doctrine is deducible from a small number of assumed ideas and propositions. As such it is a most important contribution to the theory of the unity of mathematics and of the compendence of knowledge in general. As a work of constructive criticism it has never been surpassed. To every one and especially to philosophers and men of natural science, it is an amazing revelation of how the familiar terms with which they deal plunge their roots far into the darkness beneath the surface of common sense. It is a noble monument to the critical spirit of science and to the idealism of our time.

Of the making of many text-books of the calculus there is no end. The phenomenon is doubtless due to a variety of causes, literary, economical, scientific and educational. Chief among the causes is the felt desirability of producing text-books of mathematics that will

work the miracle of pleasing at once mathematicians who are not engineers and engineers who are not mathematicians.

Perhaps the most notable feature of Professor Hulburt's book is the excellence of its English. No doubt mathematical truth is like other scientific truth in the characteristic respect that its significance does not depend primarily upon the form in which it is expressed. It ought not to be forgotten, however, that its accessibility does depend upon its form. A loose definition of a mathematical term is not a mathematical definition. A vague statement of a proposition is not a statement of a mathematical proposition. Discourse that is not precise, cogent and concatenative is not mathematical discourse. For some unexplained cause departments of English fail to give their pupils such facility in English expression as is available for mathematical purposes. And those whose fortune it is to teach undergraduate mathematics find it necessary in classroom to devote half their time and energy to trying to secure on the part of their pupils decent, I do not say elegant or imposing or fine, but merely decent expression of ideas. In this important matter, an excellent model is of very great assistance, and such a model Professor Hulbert has furnished. Most excellence is excellence of emphasis. In this respect, too, the book is a model; doctrines are presented in perspective. The nature of the differential and the utility of the differential notation are made perfectly, unmistakably, intelligible—something that unfortunately can not be said of some current presentations. As to the order of themes, there may be difference of judgment. Integration is introduced on page 175. Practise in integrating is recommended and afforded before the use of tables, given at page 190. Teachers will value the introduction to analytical geometry of three dimensions, page 265. Taylor's series is presented as late as page 349. The work closes with an excellent account of simple differential equations, and a list of answers to exercises distributed throughout the volume. Printing and binding are well done and the page pleases the eye.

In the composition of their interesting work, Messrs. Franklin, MacNutt and Charles have been guided by certain convictions. For example, they believe that "to break the thread of the textual discussion by unnecessary algebraic developments and by large and frequent groups of purely formal problems," as is commonly done, is a "really hideous feature"; and they have sought to avoid such a blemish by relegating the majority of the formal problems to an appendix. This plan has not prevented them, however, from introducing a plenty of exercises into the body of the text. Again, they are convinced that, very unfortunately, nearly all scientific text-books carry the "false suggestion of completeness and finality," and, accordingly, in order to guard the reader against gaining such an impression from their book, the authors have very laudably given in an appendix "a carefully selected list of treatises on mathematics and on mathematical physics." The book is notable for the pains the writers have taken to keep the science of the calculus attached to reality, and everywhere throughout the work one detects the odor of physical science. On this account, perhaps, theoretical developments seem to have suffered in comparison, sometimes even consciously, as in case of the notions of infinitesimal, differential, divergence and curl. Indeed the authors characterize the articles dealing with these ideas as "fallacious," "mere plausibilities," and as being such that "the harder one tries to understand them the more vague and unintelligible they become." We are disposed to think that the authors, if not too modest and frank, have overrated the difficulty of presenting the matters in question soundly and clearly. The final chapter, 43 pages, is devoted to an elementary exposition of vector analysis, an element of the book that many will gladly welcome.

Professors Davis, Brenke and Hedrick have produced a very teachable book. It would be more pleasing if the print were larger and the pages less crowded. In an unusual degree one finds here the spirit of the calculus. Designed equally for the college and the engineering

school, the volume is rich alike in fine theoretical considerations and in varied applications. Theory, however, is not overdone and the applications are chosen with unusual regard to their intelligibility.

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COLUMBIA UNIVERSITY

Instinct and Experience. By C. LLOYD MORGAN, Professor in the University of Bristol. New York, The Macmillan Company. 1912. Pp. xvii + 299.

"Once more I urge that the more clearly we distinguish the scientific problems from the metaphysical problems the better it will be both for science and for metaphysics" (p. 292). This, the concluding sentence of Professor Morgan's book, suggests the tenor of his discussion.

The volume is the direct outcome of a symposium on instinct and intelligence which was held in London in the summer of 1910. The several papers contributed to the symposium were published in the *British Journal of Psychology*, Vol. 3, 1909-10. Professor Morgan's views concerning instinct and intelligence differed in many respects from those of certain of the other speakers, and in the present work he has, at some length, presented and defended them in contrast with those of Messrs. Myers, McDougall and Stout.

Although the author would doubtless resent the suggestion, the reviewer looks upon this work as philosophical rather than purely scientific in nature. It deals largely with definitions, relations, speculations and presuppositions, and with attempts to draw a line between the naturalistic and the metaphysical disciplines. This is undoubtedly a profitable task from Professor Morgan's standpoint, but from the reviewer's it is decidedly less profitable than attempts to supply the deficiencies in our knowledge of instinct and intelligence.

And yet Professor Morgan insists, even in his opening paragraph, "My aim is to treat the phenomena of conscious existence as a naturalist treats the phenomena of organic life. I shall therefore begin with instinctive behavior and shall endeavor to give some ac-

count of the nature of the instinctive experience which, as I believe, accompanies it. In this way we shall get some idea of what I conceive to be the beginnings of experience in the individual organism" (p. 1). From this statement, one might suppose that the book would be devoted chiefly to the phenomena of instinctive and intelligent behavior, rather than to a consideration of the relations of instinct and experience or of the necessity of avoiding metaphysical problems.

Resting his contention upon the physiological discoveries of Sherrington and his co-workers, Professor Morgan insists that we must, in the end, distinguish instinctive from intelligent activities by describing the changes which occur in the central nervous system. The instinctive is dependent upon subcortical processes; and the intelligent, by contrast, is dependent upon cortical processes.

Throughout the book, but especially in Chapters II., The Relation of Instinct to Experience, III., Reflex Action and Instinct, and IV., Hereditary Dispositions and Innate Mental Tendencies, the importance of studying the functions of the central nervous system in their relations to different forms of activity is emphasized.

Effective consciousness, by which the author means consciousness that has something to do with the form of behavior, is supposed to be "connected with the process of profiting by experience" and to be "correlated with" the functions of the cerebral cortex. There is every reason, the author contends, to attempt to write a natural history of effective consciousness, a natural history of experience "as it somehow actually runs its course."

Concerning the doctrine of epiphenomenalism, the author observes that we have no proof whatever that the same brain processes which occur in connection with intelligent activity, accompanied by consciousness, ever occur in precisely the same way when these accompaniments are lacking. Professor Morgan does not believe that behavior would remain the same if the cerebral processes occurred without "correlated intelligence" (p. 262).