intellectual distinction." Make him drop his conferences on "punctuation, grammar and the split infinitive" (I suppose the last two words refer to the latitude of Boston; the rest of the country does not bother itself about the split infinitive) and let his students "criticize their own compositions and those of one another" on these points, as the "educator" does. Let him "read themes less and play golf more," and let him, like the "educator," disseminate the culture of sweetness and light. Give him the right kind of a text-book, with some logic in it, even with the "Barbara, Celarent"—which is a good thing to have in the text-book for reference, although it need not be memorized.

Why should not such a man be found? A teacher, or an educator, like every other man, is the product of heredity and environment, also of habit and of the kind of boss he has—which last may be considered part of his environment. The heredity of the teacher, in Boston at least, is all right; his environment is fairly good, but his teaching habits are bad and he has not been properly bossed; therefore he is unhappy. He is supposed to be teaching English composition, but he is not; he is reading "themes" and correcting errors of grammar and punctuation; he is doing the work that should have been done in the grammar school. "This man of solid thoughtful mind is the only real teacher." Yes, but he is unhappy, and he needs a boss to direct him how and what to teach, and how to "educate," and how to be happy, though a teacher.

Can a boss be found? Why not? Is there not in Harvard some authority that can get the "teachers" and the "educators" together around a table and say to them: "Show us the results of your teaching and educating. Do your graduates have 'mechanical perfection in technique' and there stick; have they style, or do they 'write with the mechanical regularity of one pumping into a bucket'? What proportion of them write even passable English? If the results are not what they should be, get together, you teachers and educators, and plan a better method. If you can not plan one, do as the football players do, hire a coach to plan the method, and let him be your boss until you can show results with it."

"Some day there will be a shaking among these dry bones." Why not now?

WILLIAM KENT

MONTCLAIR, N. J.

### UNIVERSITY LIFE IN IDAHO

TO THE EDITOR OF SCIENCE: Permit me to state, in reference to the question of veracity between President James A. MacLean, of the University of Manitoba, and Professor V. L. Kellogg, of Stanford University, that at the request of Professor Kellogg I furnished him with a rather full statement of the facts concerning my recent separation from the University of Idaho. From what I wrote him he prepared his article published by you under the caption "University Life in Idaho." It contains no material statement not furnished by me, and none which I do not at the present time fully believe to be true, notwithstanding President MacLean's denial. In fact, most of the details are matters of common knowledge, which no one could deny in Moscow, Idaho, though it might be done in Winnipeg.

As it is obviously impossible to try the case in your columns, I must be content to assume full responsibility for the essential correctness of Professor Kellogg's article.

J. M. ALDRICH

BUREAU OF ENTOMOLOGY,
WASHINGTON, D. C.,
June 8, 1913

## SCIENTIFIC BOOKS

*The Space-Time Manifold of Relativity, the Non-Euclidean Geometry of Mechanics and Electromagnetics.* By EDWIN B. WILSON and GILBERT N. LEWIS. Proc. Amer. Acad. Arts and Sci., Vol. 48, No. 11, November, 1912. Pp. 120.

Probably the most startling scientific conclusion of the past was the assertion that the earth moved. Even yet, while every one would probably assent passively to this statement, it is doubtful if many persons actually construct their idea of the universe so as to make this motion of the earth a reality. It is not many years since the assertion of Poincaré that the rotation of the earth is a hypothesis, met with as much glee in some quarters as it did astonishment in others. When the average man says that so far as he is concerned, the earth is stationary, he means practically that none of the experiences he has had lead him to think of the earth as in motion. When, however, he becomes a physicist, and tries to harmonize the motion of Foucault's pendulum with his ideas of mechanics, or the deviation of a falling body from the vertical, or the motion of storms across the surface of the earth, he is led to assert as the simplest explanation that the earth must rotate. When he becomes an astronomer and endeavors to reduce the varying configurations of the heavens to some kind of simplicity, he is ultimately led to assert that the earth is moving in space. He is thus brought to consider the question, is the whole universe in motion? and how can its motion be detected? At once he remembers Archimedes's remark: Give me a fulcrum and I will move the world. What is the fulcrum? Here is the trouble. If the earth may be thought of as moving around the sun, the sun may be thought of as moving around the earth. If the mountains and oceans of the earth can turn smoothly around once every twenty-four hours why can not the heavens turn? The problem comes home and must be phrased thus: How can the absolute motion of the earth be proved? Was Galileo right when he said: *E pur si muove.*

We turn first to dynamics for help, but we find that its laws will not avail. The fundamental law is that the rate of change of the momentum of a moving body is proportional to the force acting, where force and momentum are taken as directed quantities. But if we locate the moving particle with respect to an origin which itself is moving uniformly in a straight line we can not detect the fact that the origin is in motion, for the law holds equally well in either case; that is to say, change in momentum is measured by a difference of velocities and can never give us the absolute value of velocity itself. This is the relativity principle of ordinary mechanics. Conversely, if we agree that all we know of the kinetic energy of a particle is that the increment of the energy is measured by the work done by the external force in moving the particle an infinitesimal distance, and that this is invariant for a system whether it is at rest or in motion, with a uniform rectilinear velocity, then all the equations of dynamics are deducible from this basis.

Since dynamics gives us no help, we turn to electrodynamics. There is here a constant which seems to be an absolute constant, the velocity of light. It would seem that if light is a movement or a disturbance in a stationary ether, then we should be able to detect the motion of the earth against this ether. Aberration indeed seems to indicate that we have found our fulcrum. But other experiments seem to show that if there is an ether, it moves with the earth. And the only apparent way to reconcile all the experiments seems to be the assumption of certain laws which make the hypothesis of an ether superfluous. The physicist is here hard pressed for a satisfactory substitute. When the fundamental equations of electrodynamics are examined mathematically, it is found that certain changes can be made in the variables of these equations without affecting the form of the equations. In the new variables the equations read just the same as in the old. That is to say, for certain moving systems the equations are of the same form as for a stationary system. Consequently the quantities involved can only be determined relatively.

The specific statement of the case is this: Let one end of our laboratory table be our origin, and we will suppose that with respect to an omnipresent stationary observer who appreciates distance and time directly, the origin is in motion in a straight line with uniform velocity, $v$. The velocity of light we will represent by $c$, and we will suppose that any velocity can be measured absolutely. This is our first assumption. Then if the following things happen to be facts, we can not ever hope to detect the absolute motion of the table by experiments on light or electricity.

1. If there is a bar $AB$ on the table pointing in the line of the motion, to the stationary observer its length would be only $AB\sqrt{(1-\beta^2)}$ where $\beta = v/c$.

2. The time on a clock at $A$ on the table would read less in the same ratio, that is, if the motion began at noon, when the stationary observer knew that the time was really $t$ the clock would read $t\sqrt{(1-\beta^2)}$.

3. A clock at $B$ could not be made to read the same as a clock at $A$ at the same instant but would be behind that at $A$ by

$$\frac{AB}{v}\frac{\beta^2}{\sqrt{(1-\beta^2)}}.$$

If a clock were instantly moved from $A$ to $B$ the hands would instantly shift through that amount. This is the principle of *local time*.

The stationary observer would deduce at once some very startling conclusions, such as these. If the table could move with the velocity of light, $\beta = 1$, and the length of $AB$ would be nothing at all. The clock at $A$ would cease to register time at all. The obvious conclusion would be that the velocity of light is a maximum that no velocity could ever reach. But even for velocities below that of light we have to give up the idea of incompressible bodies. Energy and mass become confused and physics has to be remade. And the difficulty of time being attached to the place at which we are, so that no time meter could be devised which could be moved around and retain its correct reading, is disturbing. If two clock faces are at the ends of a long axis, and read together when across the line of motion, why should there be a twist in the axis when it is turned into the line of motion?

To enable one to understand these proposed relations of distance and time, Minkowski conceived the notion of giving them a geometrical setting. This is nothing new in physics, for many models have been made to represent various laws and hypotheses. They enable us to look at the relations in a much more direct way; to be able, as it were, to look over a map of the ground. It must be borne in mind, however, that such representations are not substitutions for the thing itself. A temperature-entropy diagram is not steam in a boiler, of course, but only shows certain relations as to the steam in the boiler. So too, Minkowski's geometric setting of relativity is not a picture of the world, but a representation of the relations that are set forth in the theory of relativity.

His suggestion was that if we use a four-dimensional space, measuring $x$, $y$, $z$ (which give us the position of the laboratory table) along three axes, and measure on the fourth axis the distance $ct$, then the fourth axis can be spoken of as a time axis, since $c$ is a constant. In this way we can speak of the situation of the real world at time $t$ as a section in the four-dimensional world by moving a space of three dimensions. The idea is easily illustrated by imagining a wave on a pond made by a stone dropped into the water. The wave spreads out with a given velocity. If now we construct a cone of the proper angle, immerse the point at the center of the wave and let the cone sink at the right speed, the expanding wave will always remain in contact with the cone. Or, so far as geometry is concerned, we can keep the cone stationary and let a cutting plane move upward. The circular section on the plane will then appear to expand like a wave. In an analogous manner we can at least get a phraseology that will describe the ideas underlying relativity of the electrodynamic kind. It turns out that if we represent these in a four-dimensional space the whole statement of the relativity property can be summed up in one simple statement, that is: In the four-dimensional space the choice of our axes of reference is fairly arbitrary. We may take axes inclined at the proper angle to our original axes, as new axes of reference, and the equations for the new $x'$, $y'$, $z'$, $t'$, are just like the original equations. Indeed if we suppose the table mentioned above to move along the $x$ axis, as viewed by a stationary observer, with a uniform velocity $v$, which we may set equal to $c \tanh \phi$, where $\tanh \phi = \beta$, $\tanh^1$ being the symbol for hyperbolic tangent, we may write the equations of transformation in the form

$$x = x' \cosh \phi + ct' \sinh \phi,$$
$$ct = x' \sinh \phi + ct' \cosh \phi.$$

That is, if one end of the table, say $B$, is apparently to the moving observer at a distance $x'$ ahead of the other end, which is the moving origin, $A$, then the stationary observer knows that the real stationary distance is $x' \cosh \phi$. If the clock at $B$ reads $t'$ to the moving observer, then the stationary observer knows that the time which has elapsed from the beginning of the motion is really $t' \cosh \phi$; and at velocity $v$, this means that the origin $A$ has moved away from the stationary origin a real distance $vt' \cosh \phi$ or $ct' \sinh \phi$. Hence the real distance of $B$ from the stationary origin is

$$x = x' \cosh \phi + ct' \sinh \phi.$$

Also the stationary observer knows that the clock at $B$ is off from two causes, one its position, at a distance apparently $x'$ from $A$, which sets it back really by

$$\frac{x'\beta^2}{v\sqrt{(1-\beta^2)}},$$

that is,

$$\frac{x' \sinh \phi}{c}.$$

The other cause is that the time read on the clock since the motion began is $t'$, but the real time as seen by the stationary observer, is $t' \cosh \phi$. Hence we have the equation

$$ct = x' \sinh \phi + ct' \cosh \phi.$$

From these equations the stationary observer could compute $x'$, which the moving observer would think was the distance of $B$ from his moving origin, and the time $t'$ on his moving clock. We have

$$x' = x \cosh \phi - ct \sinh \phi,$$
$$ct' = -x \sinh \phi + ct \cosh \phi.$$

These equations evidently are much like the first pair, and indeed we see that if we change the sign of $\phi$, that is, of $\beta$, or finally of $v$—which means that we imagine the moving observer to be at rest and the stationary observer to be relatively in motion—we have the second set. We would therefore expect that if we have two moving observers, with different velocities $v$ and $v'$, we would find similar equations for their respective interpretations of each other's data as to distance and time. Thus indeed if

$$x = x'' \cosh \psi + ct'' \sinh \psi,$$
$$ct = -x'' \sinh \psi + ct'' \cosh \psi,$$

we find $x'$ and $ct'$ to be in terms of $x''$ and $ct''$,

$$x' = x'' \cosh(\phi - \psi) - ct'' \sinh(\phi - \psi),$$
$$ct' = -x'' \sinh(\phi - \psi) + ct'' \cosh(\phi - \psi).$$

We see at once from this that the relative velocity is not found by getting the difference of the velocities $v$ and $v'$, but by getting the difference of $\phi$ and $\psi$, that is, the relative velocity is

$$\tanh(\tanh^{-1} v - \tanh^{-1} v').$$

After this long preliminary we come to the paper before us which presents a full study of the geometrical representation of these facts, in a most elegant manner. The formulæ above are interpreted as representing a rotation in a four-dimensional space, but not a common space. The rotation in a common space would involve the $\sqrt{-1}$, and to preserve the real numbers as reals, the space chosen is a non-Euclidean space. After all, the difference is really this, that certain terms like *rotation, perpendicular,* etc., do not mean what they ordinarily do, but have meanings related to a given hyperbola, rather than to a given circle. Thus really *perpendicular lines* through the origin are conjugate diameters of a circle whose center is the origin. In the paper "*perpendicular*" still means *conjugate*, but as to a hyperbola and not a circle. This illustrates sufficiently the way in which the terms appear. Only a careful study of the paper itself can give a clear idea of the character of the presentation. The reader simply needs to be on the alert as to the geometrical meaning assigned here to familiar terms whose meaning has been altered.

[1] $\sinh \phi = \beta/\sqrt{(1-\beta^2)}$, $\cosh \phi = 1/\sqrt{(1-\beta^2)}$, $\operatorname{sech} \phi = \sqrt{(1-\beta^2)}$.

The algebraical character of the paper needs a word. Instead of using a coördinate system and ordinary algebra, the authors develop a vector-algebra whose expressions represent directly the geometrical entities under discussion, and which in itself is unchanged by the changes in the axes of reference. This algebra is based upon the notions of Gibbs, and is the same as was developed by Lewis.[1] A rather complete development is given, including the analysis, or differential calculus of these vectors. In terms of the constancy of one of the vectors defined, the vector of extended momentum, the laws of conservation of mass, energy and momentum, are deduced, as well as fields of gravitational force and potential. It is not possible to enter into detail, as the technical character of the developments would demand a large amount of space to do them justice. However, any one desiring a complete and elegant account of the relativity theory, as it is seen in a geometric setting, will find it here. The laws of electromagnetics and mechanics are seen to be theorems in this geometry, which means of course that the representation as a non-Euclidean geometry of four dimensions is not only a fair representation, but is a complete representation of all the facts. It is not to be concluded, however, that it is the only representation; others have been suggested, which do not introduce the notion of a four-dimensional space in the sense it has above.[2] It should be pointed out, however, that the electrodynamic equations remain unaltered if we substitute a distance $X$ for $ct$ and at a time for $x$ given by $cT$. So that if the universe is four-dimensional and we are moving with the velocity of light in one of the four directions of the fundamental axes, we can not tell which one it is, and indeed it makes no difference. Which means in the end (does it not?) that as we assumed in the beginning that the only thing we could measure absolutely is velocity, therefore, all distances must be expressed as velocities, that is, as times, or conversely, that time as we view it is a distance. Indeed this is the fundamental assumption of the whole theory, that we may never know correctly absolute distance (if there be such a thing) nor absolute time, but we do know correctly absolute velocity.

[1] *Proc. Amer. Acad. Arts and Sci.*, 46: 163–182.

[2] Timerding, *Jahresb. d. Math. Ver.*, 21: 274–285, 1913.

The memoir is interesting also to mathematicians as a study of a particular non-Euclidean space and the corresponding vector algebra. It illustrates in a very happy way the great simplification introduced into a problem when we apply the proper symbolic analysis.

JAMES BYRNIE SHAW

*Introduction into Higher Mathematics for Scientists and Physicians.* By DR. J. SALPETER. Jena, Verlag von Gustav Fischer. Pp. 336.

This book has the advantage—as compared with similar previous works—of being written in a very elementary and yet thoughtful fashion. The author has succeeded very well in explaining the principles of higher mathematics in an exceedingly plain way, yet so that he gives all the essential points. For instance, the first three chapters of the book (32 pages or about one tenth of the whole book) are exclusively devoted to a most detailed and elaborate explanation of the three fundamental conceptions upon which higher mathematics are based. These are: (1) the conception of the limiting value of an infinite series of figures; (2) the conception of a function; and (3) the conception of the derivation of a function. To explain the importance and real meaning of these fundamentals the author uses much space, and especially cites a great number of examples from different domains of natural science. In view of the purpose of this work, however, this explanation is not too long. After this introduction only, the technique of differentiating is discussed, also very clearly. Maxima and minima of functions, differential equations, integration, etc., are then explained thoroughly and clearly. At the end of each chapter numerical examples are given, as well as applications to scientific problems. The graphic method is extensively used. As a whole, the book can be recommended to such experimental investigators