

Also notes and errata to volumes 7 and 13.

THE opening (October) number of volume 19 of the *Bulletin of the American Mathematical Society* contains: "Surfaces of revolution of minimum resistance," by E. J. Miles; "Shorter Notices": Riquier's *Les Systèmes d'Equations aux Dérivées partielles*, by Edward Kasner; Study's *Ebene analytische Kurven und zu ihnen gehörige Abbildungen*, by Arnold Emch; Coffin's *Vector Analysis*, by J. B. Shaw; *Berichte und Mitteilungen der Internationalen mathematischen Unterrichtskommission und Auerbach und Rothe's Taschenbuch für Mathematiker und Physiker*, by E. W. Ponzer; Bonola-Carslaw's *Non-Euclidean Geometry*, by Arthur Ranum; Barbin-Halsted's *Géométrie rationelle*, by R. C. Archibald; Smith and Granville's *Elementary Analysis*, by Jacob Westlund; Hawkes, Luby and Touton's *Second Course in Algebra*, by J. V. McKelvey; Jacob's *Calcul mécanique*, by C. C. Grove; Schwahn's *Mathematische Theorie der astronomischen Finsternisse* and Haret's *Mécanique sociale*, by Kurt Laves; "Notes," and "New Publications."

THE November number of the *Bulletin* contains: Report of the nineteenth summer meeting of the society, by F. N. Cole; "A few theorems relating to Sylow subgroups," by G. A. Miller; "Theorems on functional equations," by A. R. Schweitzer; "Double curves of surfaces projected from space of four dimensions," by S. Lefschetz; Review of Southall's *Geometrical Optics*, by E. B. Wilson; "Shorter Notices": Rogers-Salmon's *Analytic Geometry of Three Dimensions*, by Virgil Snyder; Volume 3 of Picard's edition of the *Works of Charles Hermite*, by James Pierpont; Heiberg's *Naturwissenschaften und Mathematik im klassischen Altertum* and Mannoury's *Methodologisches und Philosophisches zur Elementar-Mathematik*, by D. E. Smith; Weber und Wellstein's *Encyklopädie der Elementar-Mathematik*, volume 3, part 1, and Korn's *Freie und erzwungene Schwingungen*, by J. B. Shaw; Richard's *Assurance complémentaire de l'Assurance sur la Vie*, by C. C. Grove; Vahlen's *Konstruktionen und*

Approximationen, by E. W. Ponzer; "Notes," and "New Publications."

THE December number of the *Bulletin* contains: General report of the fifth international congress of mathematicians at Cambridge, by Virgil Snyder; Report of Section I of the Congress (arithmetic, algebra, analysis), by A. B. Frizell; "Shorter Notices": Boehm's *Elliptische Funktionen*, Part 2, by L. W. Dowling; Darboux's *Eloges académiques et Discours*, by G. A. Miller; Hedrick and Kellogg's *Applications of the Calculus to Mechanics*, by D. C. Gillespie; "Notes," and "New Publications."

CONDITION OF THE EARTH'S CRUST

THE results of measurements of the force of gravity at various points on the earth, as well as the results of triangulation operations, were early recognized as indicating that the earth's crust is in a condition of approximate equilibrium, to which the name "isostasy" has since been given.

The development by Mr. Hayford of a new method of reduction of gravity observations, in which for the first time the effect of the topography of the whole earth has been taken into account, has furnished strong additional proof of the general fact that the condition of isostasy exists, that elevated regions, whether plains or mountains, are, so to speak, floated on the earth's surface by reason of the lesser density of the underlying materials, and that ocean bottoms are depressed because of the greater density of the materials beneath.

The question of how close is this adjustment, of how local is the compensation of surface irregularities, is of considerable interest. If the compensation is quite complete for each small topographic feature, so that a single mountain or hill or canyon is exactly compensated by a less dense or a more dense material beneath, the surface of the earth would in detail be in a condition of nearly perfect equilibrium, and would largely be free from stresses due to the supporting of topographic features; on the other hand if the compensation is more general such features of moderate

extent would be supported by the partial rigidity of the earth's outer materials.

In a recent report¹ Hayford and Bowie study the question of local versus general compensation by a comparison of the residuals, observed minus theoretical gravity, for 41 stations in the United States, and 4 outside. The observations are reduced according to the new method in four different ways: first, with complete local compensation; second, regional compensation for a zone 19 kilometers (12 miles) in radius; third, the same for radius of 59 kilometers (37 miles), and fourth, the same for radius of 167 kilometers (104 miles), this study having been made along these lines at the suggestion of the writer. In the last three reductions the surface layers are taken as rigid to the respective distances from the station, and it is assumed that there is a uniform compensation of the area as a whole. In the last, for instance, an area of the earth's surface of 167 kilometers radius is taken as being in general equilibrium with a uniform compensation beneath this area, but local irregularities as mountains or valleys within this area are assumed to be not locally compensated, but supported rigidly.

The authors state that the resulting evidence "is necessarily slight and possibly inconclusive." In a number of comments, however, it is indicated that the authors favor the idea that the results point to fairly close but not complete local compensation, and finally that "the evidence, slight as it necessarily is, indicates that the assumption of local compensation is nearer the truth than the assumption of regional compensation uniformly distributed to zone 18.8 kilometers."

It is believed that a close scrutiny of the figures does not support this conclusion so far as evidence from these results is concerned. The differences in the mean of the residuals with the four different reductions are insignificant. The suggested advantage for local compensation is based (page 101) on small differences in the relative number of larger

¹"The Effect of Topography and Isostatic Compensation upon the Intensity of Gravity," Coast and Geodetic Survey, 1912.

and smaller residuals with the respective methods. Along this line, however, it may be noted that of the 41 stations there are only 7 where the local compensation anomaly is lower than any of the three regional, while there are 24 where one of the three regional compensation anomalies is lower than the local. Of the four outside stations three show anomalies in favor of regional compensation. In a later paper by Mr. Bowie under the same title (second paper) a general result is given of an extension of this study to 124 stations in the United States, and it is stated (page 22) that for all these stations "these mean anomalies give only negative evidence," though from a portion of the data he draws a conclusion unfavorable to regional compensation to zone 167 kilometers.

It would seem that the best evidence as to local completeness of compensation from present available observations will be afforded by comparing the residuals at pairs of stations in the same general locality, but differing considerably in elevation, as in such a comparison distant effects and various uncertainties will be largely eliminated, and furthermore it is in such regions that lack of local compensation might be most likely to occur. It is on somewhat the same principle that the most accurate latitude determinations are obtained from observations of pairs of stars. In the 45 stations reduced by the four methods as outlined above there are only five such pairs of stations, that is, stations horizontally not remote, and yet having considerable differences of elevation. The following table gives the facts for these ten stations, and also the differences between the anomalies, subtracting that of the lower from that of the higher station in each instance.

In the second paper there is one more such pair, Cloudland, Tennessee, 1,890 meters, and Hughes, Tennessee, 994 meters, with difference in anomaly for local compensation $+.033$, subtracting the lower from the higher station; for the three regional compensation reductions (not yet published) the differences are, 19 km. $+.031$, 59 km. $+.031$, 167 km. $+.031$.

Station	Elevation, Meters	Difference in Eleva- tion, Meters	Difference in Anomaly, Subtracting Lower from Higher Station			
			Local Compens- ation, Dynes	Regional Compensation		
				19 Km., Dynes	59 Km., Dynes	167 Km., Dynes
Mauna Kea, Hawaii.. .. .	3,981					
Honolulu, Hawaii.. .. .	6	3,975	+ .131	+ .118	+ .105	+ .068
Gornergrat, Switzerland.. .. .	3,016					
St. Maurice, Switzerland.. .. .	419	2,597	+ .046	+ .041	+ .023	+ .016
Pikes Peak, Colorado	4,293					
Colorado Springs, Colorado.. .. .	1,841	2,452	+ .028	+ .020	+ .016	+ .012
Yavapai, Arizona.. .. .	2,179					
Grand Canyon, Arizona.. .. .	849	1,330	+ .011	+ .010	+ .010	+ .012
Mt. Hamilton, California.. .. .	1,282					
San Francisco, California.. .. .	114	1,168	+ .020	+ .020	+ .011	+ .029
Mean anomaly difference..047	.042	.033	.027
Range in anomaly difference..120	.108	.095	.056

This is too small a number of pairs to warrant a conclusion, but so far as they go, the results show an advantage for regional compensation. Also in every instance the difference is plus on subtracting the anomaly for the lower station from that for the higher, and there is slight indication of a relation to the difference in elevation. The plus difference, if real, indicates an apparent excess of gravity at the high station as compared with the low station. This may be actual and due to some condition of materials beneath the surface, or it may result from a compensation correction relatively too large being applied to the high station, or from some other feature of the reduction. The evidence given by these pairs of stations is slight, but points to the possibility of further interesting investigation, which might be extended along similar lines to a study of differences of deflections at neighboring astronomic stations.

The conclusions from the above are:

1. From the general mass of results there is practically no evidence showing whether there is nearly complete local compensation or only general regional compensation within the areas considered, that is, within zones up to 167 kilometers (104 miles) radius.

2. The comparison of pairs of neighboring stations differing considerably in elevation shows an advantage for regional compensation, but the number of results is too small for a definite conclusion.

3. The comparison by pairs shows in each case gravity at the higher station in excess as

compared with gravity at the lower, which if real, may be due to materials beneath the surface, or to some conditions of the reduction. The number of results, five, is, however, too small for safe general conclusions.

4. If practically identical results are obtained with regional and with local compensation or if limited regional compensation is nearer the truth, it may be possible to lessen the labor of reduction of gravity observations by computing the direct topographic effect and the general compensation for a larger zone about the station.

The above discussion bears on one feature only of the interesting reports to which reference is made.

GEORGE R. PUTNAM

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SPECIAL ARTICLES

THE VALUE OF THE CILIATE, DIDINIUM, IN THE STUDY OF BIOLOGY

Didinium appears only occasionally in ordinary cultures for the protozoa usually studied in the laboratory. Owing to this fact it is not widely known and consequently its exceptional possibilities as laboratory material for study in courses in biology have been quite generally overlooked. I have had this animal under almost continuous observation during the past four years and have become fairly well acquainted with it. The following notes are based upon these observations.

Didinia feed largely on *paramecia*. They