Never believe that Gauss ever doubted the actual truth of the parallel assumption for our space.

Yet just now, 1912, this space of ours is being proved non-euclidean by the principle of relativity. Says Vladimir Varićak in a wonderful lecture, "Ueber die nichteuklidische Interpretation der Relativtheorie,"[2]

I postulated that the phenomena happened in a Lobachevski space, and reached by very simple geometric deduction the formulas of the relativity theory. Assuming noneuclidean terminology, the formulas of the relativity theory become not only essentially simplified, but capable of a geometric interpretation wholly analogous to the interpretation of the classic theory in the euclidean geometry. And this analogy often goes so far, that the very wording of the theorems of the classic theory may be left unchanged.

To see that this will prove our space Bolyaian, we have only to remember Poincaré's dictum:

These two propositions, the earth turns round, and, it is more convenient to suppose that the earth turns round, have one and the same meaning.[3]

The first man to so bring forth the non-euclidean geometry that it was not stillborn, but lived and grew, was the Frenchman Hoüel, by his translations of Lobachevski in 1866 and John Bolyai in 1867. Thirty years later, in my translator's preface, I said:

No part of Lobachevski's largest work, "New Elements," has ever before been published in any language but the original Russian.

I gave an account of it in 1893 at the Mathematical Congress of the World's Columbian Exposition, and promised then the publication of my translation.[4] This promise was delayed for a personal visit to Kazan, the home of Lobachevski, and Maros-Vásárhely, the home of Bolyai. Only through his little book "Geometrical Researches,"[5] have Lobachevski's ideas been heretofore accessible to the world in general.

But it is preeminently in his "New Elements" that the great Russian allows free expression to his profound philosophic insight, which on the one hand shatters forever Kant's doctrine of our absolute *a priori* knowledge of all fundamental spatial properties, while on the other hand emphasizing the essential relativity of space.

The realities which with the aid of the euclidean space form we understand under motion and position, may, with the coming of more accurate experience, refuse to fit in that form. Our mathematical reason may decide that they would be fitted better by a non-euclidean space form. Space is presupposed in all human notions of motion or position. We may drop out such specifications from our space form as render it specifically euclidean. Euclidean space is a creation of that part of mind which has worked and works yet unconsciously.

It is not the shape of the straight lines which makes the angle-sum of a rectilineal triangle two right angles.

With straight lines of precisely such shape but in a non-euclidean space, this sum may be greater or less. In non-euclidean spaces, if one edge of a flat ruler is a straight line the other edge is a curve, if the ruler be everywhere equally broad. In any sense in which it can be properly said that we live in space, it is probable that we really live in such a space.

And now fifteen years later comes the relativity theory to prove all this, and to make non-euclidean geometry a powerful machine for advance in physics.

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[2] *Jahresber. D. Math. Ver.*, 21, 103–127.

[3] "The Value of Science," Halsted's translation, p. 140.

[4] See "Mathematical Papers of Chicago Congress," pp. 92–95.

[5] Hoüel, 1866; Halsted, 1891.

Allen's Commercial Organic Analysis. Volume V. Tannins, Dyes and Coloring Matters, Inks. Edited by W. A. DAVIS and SAMUEL S. SADTLER. P. Blakiston's Son and Co. Philadelphia, 1911. Price $5.00.

This volume contains the following chapters: Tannins by W. P. Dreaper. Analysis of Leather by W. P. Dreaper. Dyes and Coloring Matters by W. P. Dreaper and E. Feilmann. Dyestuffs of Groups 6 to 12 by J. T. Hewitt. Coloring Matters of Natural Origin by W. M. Gardner. Analysis of Coloring Materials by W. P. Dreaper and E. Feilmann.

Coloring Matters in Foods by Albert F. Seeker. Inks by Percy H. Walker. Index.

Volume VI. Organic Bases, Vegetable Alkaloids. Philadelphia, 1912. Price $5.00. This includes Amines and Ammonia Bases by W. A. Davis. Aniline and its Allies by S. S. Sadtler. Naphthylamines, Pyridine, Quinoline and Acridine Bases by W. H. Glover. Vegetable Alkaloids by Thomas A. Henry. Volatile Bases of Vegetable Origin by Frank O. Taylor. Nicotine and Tobacco by R. W. Tonkin. Aconite Alkaloids by Francis H. Carr. Cocaine by Samuel P. Sadtler. Opium Alkaloids by Frank O. Taylor. Strychnos Alkaloids by Charles E. Vanderkleed. Cinchona Alkaloids by Oliver Chick. Berberine and its Associates by Edward Horton. Caffeine, Tea and Coffee by J. J. Fox and P. J. Sageman. Cocoa and Chocolate by R. Whymper.

The extensive scope of the work and the pains taken to secure the assistance of experts in the various fields is well indicated by the list of chapters given.

W. A. Noyes

The Flight of Birds. By F. W. Headley, M.B.O.U., with sixteen plates and many text figures. Witherby and Co., 326 High Holborn, London. 1912. 12mo. Pp. xii + 163. Price 5 shillings.

In this little book Mr. Headley has endeavored to describe briefly and clearly the flight of birds, keeping in view the methods and difficulties of those who are striving to rival them. In ten chapters he deals with methods, modes and apparatus of flight, an eleventh chapter being devoted to some accessories, circulation, breathing, etc., that are connected with, or modified by flight.

The first chapter deals with gliding, the resistance of the air, the curve of the wings and the area of supporting surface, matters which lie at the base of all flight. We then pass to stability, including voluntary adjustment, the latter a point wherein the feathered biped has the great advantage over his featherless rival of many thousands of years' experience. What man has to think about, the bird does instinctively. It is this instinct that enables a bird to fly successfully at the first trial of his wings, although he can not handle them so dexterously as he will later on and they may not carry him so far nor so fast as they will subsequently. And just here may we say that a bird does not fly stupidly into a telegraph wire; he simply does not see it until too late to evade it. The *Titanic* did not run stupidly into an iceberg; by the time it could be seen she was upon it. Also horizontal wires are not within the province of the bird's instincts, and in the grouse districts of Scotland, bits of wood are hung on telegraph and telephone lines to catch the bird's eye.

In connection with the relation of the shape of the wings to stability, it may be said that Mr. Huffaker, one of Professor Langley's assistants, reached the conclusion that the curved secondaries and more or less flattened primaries of the bird's wing were the great factors in stability and that the flattened wing tips also served as horizontal rudders, points wherein man has advantageously patterned after the bird.

Then come starting and steering, and the machinery of flight, muscles and bones. A little more space, perhaps, might have been devoted to the framework and to some of the rather perplexing problems it suggests—why is it that while sailing birds, the albatross and frigate bird, for example, have very small muscles their shoulder girdles are most rigidly constructed, the coracoid, clavicle and breastbone of the latter being immovably soldered together. In the great pterodactyl, *Pteranodon,* most marvelous of all flying creatures, we have in the massive collar bone special provision for bracing the wings. Perhaps in all these cases this strength is necessary, because the muscles themselves can not be relied upon to stand the strain. But in the almost flightless hoatzin we find the apparent anomaly of a rigid shoulder girdle. We think that, as is usually done, too much value is put upon the clavicle as a brace to the coracoids. Among the birds of prey it is of importance, but in the ducks and pheasants, birds of powerful flight, it is a negligible quantity; so it is in