Professor Gordon H. True has been appointed director of the Nevada Experiment Station, at Reno.

Dr. Arthur B. Lamb, professor of chemistry and director of the Havemeyer Chemical Laboratory of New York University, has been appointed assistant professor of chemistry at Harvard University. Professor Solon I. Bailey has been promoted to the Philips chair of astronomy vacant by the retirement of Professor Arthur Searle, and Dr. Charles Palache has been promoted to a full professorship of mineralogy.

Dr. Ira W. Howerth, of the University of Chicago, has been appointed professor of education and director of university extension in the University of California. Dr. J. C. Merriam has been promoted to a full professorship of paleontology.

Dr. W. M. Conger Morgan, assistant professor of chemistry at the University of California, has been appointed professor of chemistry in Reed College.

Dr. Franz Doflein, associate professor of zoology at Munich, has been called to the chair of zoology at Freiburg.

DISCUSSION AND CORRESPONDENCE

NON-EUCLIDEAN GEOMETRY IN THE ENCYCLOPÆDIA BRITANNICA

The sixth heading under the word Geometry is Non-Euclidean Geometry. The article is by Whitehead and Bertrand Russell, the best men in England to have written it, and is worthy this one of the three greatest works of reference in the English tongue, the others being Murray's dictionary and the Century.

It begins:

A short historical sketch will . . . describe the famous and interesting progress of thought on the subject.

But first it gives characteristic properties, beginning with Bolyai's space.

The sum of the three angles of a triangle is always less than two right angles. The area of the triangle ABC is $\lambda^2(\pi-A-B-C)$. If the base BC of a triangle is kept fixed and the vertex A moves in the fixed plane ABC, so that the area ABC is constant, then the locus of A is a line of equal distance from BC. This locus is not a straight line.

I have called it an *equidistantial.*

The angle A [which a perpendicular to one of two parallels makes with the other] is called by N. I. Lobatchewsky the "angle of parallelism."

Here as everywhere else in the spelling of Lobachevski's name, the authors have made a very regrettable slip. Lobachevski transliterated his own name into French as Lobatcheffsky, and so it stands in the "édition de Kasan," 1886.

In 1869 Potocki transliterates the name into French as Lobatchefsky, and this spelling is used in the French prospectus issued at Kazan to found the great Lobachevski prize; and the volume "In Memoriam N. I. Lobatschevskii," bears as subtitle, Collection des mémoires présentés à la Société Physico-mathématique de Kasan pour la fête de l'inauguration du monument de Lobatchefsky (1/13 Septembre, 1896) par Mm. Hermite, Halsted, Girardville, Laisant, Lemoine, Neuberg, Ocagne.

My contribution I wrote while sojourning in Kazan, where I had abundant opportunities to learn the name. Gino Loria adopts in Italian the spelling Lobatscheffsky. Now Lobachevski himself also transliterated his name into German, and it stands on the title page of the original edition of his Geometrische Untersuchungen as Lobatschewsky. But Stäckel and Engel Germanize it as Lobatschefskij, the abomination ij being an attempt to represent the i, as in Italian, and the ĭ, very short, with which the name ends in Russian. My friend Sommerville falls into this pit, and spells the name Lobačevskij. Had he dropped that j and replaced his fifth letter by its exact equivalent, our ch as in church, he would have had the proper English transliteration, Lobachevski.

If we be willing to permit in the Encyclopædia the final y, still as English its t is superfluous and its w is indefensible, so that, as the name occurs 25 times, there are 50 places where the quicker the stereotype plates are corrected the better. Let not the name of a world hero be bungled in the world language, English.

The theory of parallels as it exists in hyperbolic space has no application in elliptic geometry. But another property of Euclidean parallel lines holds in elliptic geometry, and by the use of it parallel lines are defined. Thus throughout every point of space two lines can be drawn which are lines of equal distance from a given line l.

This property was discovered by W. K. Clifford. The two lines are called Clifford's right and left parallels to l through the point.

In both elliptic and hyperbolic geometry the spherical geometry is the same as the "spherical trigonometry" in Euclidean geometry.[1]

The historical sketch is blemished by the unwarranted prominence it gives to Gauss. It says:

We find him in 1804 still hoping to prove the postulate of parallels. In 1830 he announces his conviction that geometry is not an a priori science; in the following year he explains that non-Euclidean geometry is free from contradictions, and that, in this system, the angles of a triangle diminish without limit when all the sides are increased. He also gives for the circumference of a circle of radius r the formula $\pi k(e^{r/k} - e^{-r/k})$.

[In this formula the Encyclopædia has a misprint.]

But all that and immensely more had been given by John Bolyai in 1823 and by Lobachevski in 1826, and published in 1829, while as our authors themselves say, "Gauss published nothing on the theory of parallels."

Then comes the most offensive clause:

It is not known with certainty whether he influenced Lobachevski and Bolyai, but the evidence we possess is against such a view.

But it *is* known that he did *not*, and the evidence we possess against any such influencing is absolute and final. The very next sentence is the opening one of my Translator's Preface, 1891:

Lobachevski was the first man ever to publish a non-Euclidean geometry.

Of Bolyai's work is said:

[1] See chapter XVI., Pure Spherics, in my "Rational Geometry."

Its conception dates from 1823. It reveals a profounder appreciation of the importance of the new ideas, but otherwise differs little from Lobachevski's. Both men point out that Euclidean geometry is a limiting case of their own more general system.

[The Encyclopædia, by a misprint, has *as* for *is*.]

The works of Lobachevski and Bolyai, though known and valued by Gauss, remained obscure and ineffective until, in 1866, they were translated into French by J. Hoüel.

Bolyai was not translated until 1868. Not only were these known to Gauss, but I called attention to the very significant fact that the striking work of Saccheri, truly a non-euclidean geometry, was in the Göttingen library and freely accessible to Gauss during the years 1790–1800. See Gino Loria,[2] who says of Gauss:

Ignoto fino a qual punto egli siasi spinto nella nuova via, come è ignoto se egli abbia ricevuto qualche ispirazione dall' opera del Saccheri che esisteva a Gottinga negli anni 1790–1800 (essendo segnata con un asterisco nella *Bibliotheca mathematica* del Murhard).[3]

If figures are to be freely movable, it is necessary and sufficient that the measure of curvature should be the same for all points and all directions at each point. Where this is the case, it a be the measure of curvature. . . .

This *it* should be *if*.

If a be positive, space is finite, though still unbounded, and every straight line is closed—a possibility first recognized by Riemann.

This, as it stands, is a mistake. On page 24 of von Staudt's "Geometrie der Lage" (1847) we read:

Eine Gerade erscheint hiernach . . . als eine geschlossene Linie.

The possibility first recognized by Riemann is that straight lines may be finite.

On page 729 occurs the long dead phrase "anharmonic ratio," now happily superseded everywhere by Clifford's "cross ratio."

[2] Il passato ed il presente delle principali teorie geometriche. Terza edizione, 1907, pp. 286–287.

[3] Osservazione fatta dall' Halsted nell' articolo "The Non-Euclidean Geometry Inevitable" inserto in *The Monist,* July, 1894.

It is explained in section VII. in what sense the metrical geometry of the material world can be considered to be determinate and not a matter of arbitrary choice. The scientific question as to the best available evidence concerning the nature of this geometry is one beset with difficulties of a peculiar kind. We are obstructed by the fact that all existing physical science assumes the Euclidean hypothesis. This hypothesis has been involved in all actual measurements of large distances, and in all the laws of astronomy and physics. The principle of simplicity would therefore lead us in general, where an observation conflicted with one or more of those laws, to ascribe this anomaly, not to the falsity of Euclidean geometry, but to the falsity of the laws in question. This applies especially to astronomy. . . . But astronomical distances and triangles can only be measured by means of the received laws of astronomy and optics, all of which have been established by assuming the truth of the Euclidean hypothesis. It therefore remains possible that a large but finite space constant, with different laws of astronomy and optics, would have equally explained the phenomena. We can not, therefore, accept the measurements of stellar parallaxes, etc., as conclusive evidence that the space constant is large as compared with stellar distances.

Finally, it is of interest to note that, though it is theoretically possible to prove, by scientific methods, that our geometry is non-Euclidean, it is wholly impossible to prove by such methods that it is accurately Euclidean. For the unavoidable errors of observation must always leave a slight margin in our measurements. A triangle might be found whose angles were certainly greater, or certainly less, than two right angles; but to prove them *exactly* equal to two right angles must always be beyond our powers.

This I have been publishing for the past 35 years in articles some 77 of which, not counting translations, Sommerville has registered in his Bibliography of non-euclidean geometry, 1911. But just here a former pupil of mine, Dr. R. L. Moore, has gone beyond his teacher. His results seem to be unknown to the Encyclopædia, though I called attention to them in SCIENCE, October 25, 1907, under the "scare" heading, "Even Perfect Measuring Impotent."

In the brief bibliography appended to this section VI., I notice a number of errors. In the title of Engel's book the *y* should be *ij*. In the title of Dehn's article, the word Legendarischen should be Legendre'schen. In the title of Barbarin's book the capital G and capital E should be lower case letters, and the hyphen should be omitted.

In the title of Bonola's book the capital E should be lower case.

In the title of the article by E. Study the nicht-Euklidische should be Nicht-Euklidische. This title upon a pamphlet of 97 pages [Greifswald, 1900] is Über Nicht-Euklidische und Linien-Geometrie.

In the title of Beltrami's article given on page 728, note 3, the g should be a capital in Geometria and the E lower case in non-euclidea. In note 4, page 725, nicht-Euklidischen should be nichteuklidischen. In note 1, page 727, nicht-Euklidische should be nicht-euklidische.

The final heading, VII., is Axioms of Geometry, under which it is said:

The second controversy is that between the view that the axioms applicable to space are known only from experience, and the view that in some sense these axioms are given *a priori*.

Both these alternatives are wrong. These axioms are assumptions, belonging to what I have treated under the title "The Unverifiable Hypotheses of Science," in *The Monist*, October, 1910.

The cruder forms of the *a priori* view have been made quite untenable by the modern mathematical discoveries. Geometers now profess ignorance in many respects of the exact axioms which apply to existent space, and it seems unlikely that a profound study of the question should thus obliterate *a priori* intuitions. . . . The enumeration of the axioms is simply the enumeration of the hypotheses of which some at least occur in each of the subsequent propositions.

On page 732, line 14, the comma after the word "however" is a misprint, and should be deleted.

Geometry with the assumption: Of any three points of a straight there is always one and only one which lies between the other two, Whitehead calls "descriptive geometry," a horrible piece of nomenclature, which no one should adopt, since this name belongs to the system of Monge, 1794, for representing solids in a plane, though also used by Sylvester for a geometry excluding all notions of quantity, such as my "Synthetic Projective Geometry."

The article proceeds to

the simplest statement of all. Descriptive Geometry is then conceived as the investigation of an undefined fundamental relation between three terms (points); and when the relation holds between three points *A*, *B*, *C*, the points are said to be "in the [linear] order *ABC*."

O. Veblen's axioms and definitions, slightly modified, are as follows:

1. If the points *A*, *B*, *C* are in the order *ABC*, they are in the order *CBA*.

Dr. R. L. Moore (October 26, 1907) says this may be divided into parts, 1_1 inserting "distinct" before "points"; and 1_2 inserting "not all distinct," after "points."

2. If the points *A*, *B*, *C* are in the order *ABC*, they are not in the order *BCA*.

3. If the points *A*, *B*, *C* are in the order *ABC*, then *A* is distinct from *C*.

4. If *A* and *B* are any two distinct points, there exists a point *C* such that *A*, *B*, *C* are in the order *ABC*.

Dr. R. L. Moore modifies this to 4′ by inserting "different from *A* and from *B*," before "such." Then follow a definition, Def. 1, and axioms 5, 6, 7. Both in this definition, and in axiom 5 the shocking misprint occurs of using the symbol $\pm$, "plus or minus," for the symbol $\neq$, "is not equal to."

Dr. R. L. Moore had already in 1907 surprisingly simplified this set of assumptions by proving that 1_1 is a consequence of 2 and 5 and Def. 1, while 1_2 and 3 are both consequences of 2, 4′, 5, 6, 7 and Def. 1.[4]

Lobachevski [or Bolyai] constructed the first explicit coherent theory of non-Euclidean geometry, and thus created a revolution in the philosophy of the subject. For many centuries the speculations of mathematicians on the foundations of geometry were almost confined to hopeless attempts to prove the "parallel axiom" without the introduction of some equivalent axiom.

[4] *Trans. Amer. Math. Soc.*, Vol. XIII., No. 1, pp. 74–76.

In the Bibliography, Whitehead says of Lobachevski:

His first publication was at Kazan in 1826.

This is a mistake. In 1836 in his "Introduction to New Elements of Geometry," of which I was the first to publish a translation,[5] he says:

Believing myself to have completely solved the difficult question, I wrote a paper on it in the year 1826: "Exposition succincte des principes de la Géométrie, avec une démonstration rigoureuse du théorème des parallèles," read February 12, 1826, in the séance of the physico-mathematic faculty of the University of Kazan, but nowhere printed.

No part of this French manuscript has ever been found. The latter half of the title is ominous. For centuries the world had been deluged with rigorous (!) demonstrations of the theorem of parallels.

Saccheri's book of 1733, containing a coherent treatise on non-euclidean geometry, of which I published the first translation, ended with another "demonstration rigoureuse du théorème des parallèles." If Saccheri had realized (as Father Hagen writes me he did) the pearl in his net, he could, with the new meaning, have retained his old title, Euclides ab omni naevo vindicatus, since the non-euclidean geometry is a perfect vindication and explanation of Euclid.

But Lobachevski's title is made wholly indefensible. A new geometry, founded on the contradictory opposite of the theorem of parallels, and so proving every demonstration of that theorem fallacious, could not very well pose under Lobachevski's old title. He himself never tells what he meant by it, never tries to explain it.

The title of Engel's book already given erroneously in the Bibliography under VI., is now, under VII., given again with the former and two additional errors.

After Riemann we see *Gesamte Werke* instead of gesammelte Werke.

In the title of Poncelet's work, on page 736, an accent is omitted which is given in the

[5] "Neomonic Series," Vol. V., 1897.

same title on page 676, where on the other hand the main word of the title is omitted.

The Beiträge of von Staudt appeared in two parts, the first in 1856, the second in 1860. How could Whitehead have made the mistake of calling this second part a "3rd ed." ?

George Bruce Halsted

Greeley, Colo.

PEARL AND JENNINGS ON ASSORTATIVE CONJUGATION IN THE PROTOZOA

In general, the scientist's investigations receive the recognition they deserve from his fellow workers. This is true because the bulk of research consists in the working out of details in a scheme already stamped with authority. It is the unexpected, fundamentally new or truly brilliant result upon which the doctors disagree.

One of the best illustrations is a paper in *Biometrika* for February, 1907. In the demonstration of the existence of an assortative conjugation or homogamy in *Paramecium* analogous to the assortative mating previously found by Pearson in man, Pearl seemed to some of us to have struck a rich vein hitherto passed over by all prospectors. Others thought differently. Pearl's assays were discredited. In America, at least one review was declined. In England, J. J. Lister illustrated[1] by Pearl's paper his warning to biometricians to be sure they have a problem which is "sound from the standpoint of biology before bringing a formidable mathematical apparatus into action for its investigation."

Open criticism like that of Lister was more easily met[2] than the general indifference largely attributable to the *odium mathematicum*. This is now in a fair way to be overcome by the results being announced by Jennings. If these, in their turn, are being received by zoologists with but lukewarm enthusiasm, the fact indicates merely that the work is in advance of its time.

His recent study of conjugation in *Paramecium*[3] must be considered in comparison with Pearl's pioneer paper.[4]

a. Differentiation of Conjugants in Type and Variability.—The general belief that conjugants are on the average smaller than non-conjugants is quantitatively substantiated. In eleven "pure lines"[5] Jennings found conjugants to be from about 4 to nearly 14 per cent. smaller than the non-conjugants. In "wild" cultures, or in a mixture of differentiated pure lines, the mean for conjugants may be higher because only the large pure line is in conjugation. On the other hand, the conjugants may be abnormally small, 30 per cent. less than the non-conjugants, because only the smaller of the lines in the mixture is in conjugation.

Both absolutely and relatively, the conjugants are less variable than the non-conjugants. The difference in variability may be slight but generally it is large, for the conjugants are on an average about 33 per cent. less variable (relatively) than the non-conjugants.

The possible causes of this reduced variability are discussed. Lister's "Gametic Differentiation" is dismissed. Pearl's conclusion that equalization of individuals (undifferentiated or proconjugants) during the process of conjugation can not account for the lessened variability is confirmed. Jennings's conclusion, supported by abundant evidence, is that the low variability of conjugants is fully accounted for by the fact that conjugation does not occur till a certain growth stage has been reached, and does not occur in the largest individuals—the measurable variability of *Paramecium* being largely a growth phenomenon. Thus, the conjugants represent a definite and rather limited growth stage, the exclusion of both the larger and

[1] Lister, J. J., *Nature*, Vol. 74, pp. 584–585.

[2] Pearson, K., *Nature*, Vol. 74, pp. 465–466, 608–610, 635, 1907.

[3] Jennings, H. S., "Assortative Mating, Variability and Inheritance of Size in the Conjugation of *Paramecium*," *Journ. Exp. Zool.*, Vol. 11, pp. 1–134, July, 1911.

[4] Pearl, R., "A Biometrical Study of Conjugation in *Paramecium*," *Biometrika*, Vol. 5, pp. 213–297, 1907.

[5] The offspring of a single individual reproducing by fission has been called by Jennings a "pure line." In retaining the term here nothing more is implied than guaranteed purity of descent.