

# SCIENCE

FRIDAY, AUGUST 25, 1911

## THE CALCULUS IN TECHNICAL LITERATURE

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It would be difficult to get a majority report on either the quantity or quality of the calculus used by practising engineers in any country even if their individual opinions on the matter could be obtained. Evidences of the many conflicting views likely to be held are presented in the testimony along this line contributed by former students of ours who, after a brief experience in the technical world, give us their impressions of how much they have been called upon in actual practise to use the various mathematical principles with which they wrestled so laboriously in freshman and sophomore college days.

Instructors of mathematics in universities where sections of engineering majors are formed have, no doubt, heard recited, as I have, the many and varied experiences of these young engineers with the problems involving mathematics which arose at various times in their brief experience. Perhaps some cub engineer, who already had done a piece or two of engineering work worth while, has told you of how he has never yet had to use his calculus and that he wonders why we keep on teaching it. Perhaps you have been told, as I have, that if the prospective engineers are thoroughly grounded in the differentiation and integration of  $u^n$ , and know what they mean and how to use them, they will then have as much calculus as they are likely to use in the problems which may arise. Again, you may have heard another say, as I have, that he is already using all the mathematics he ever learned—and then some—and that he wished he had taken various

other advanced courses in mathematics while in college.

The problems sent on by the young men in practise, and referred to you acting as a sort of consulting engineer, may vary, as I have experienced it, from those in which in the solutions sent an error was made in using the common logarithm of a number instead of the natural to those where the principles of the calculus involved were beyond what they had had time to study while in college. The problems were always live ones, definitely stated, and the solutions simply must be obtained—if not exactly then at least approximately correct. When they needed their calculus they needed it right away—of course they were going to get the result some way.

On the other hand, I have heard a professor of a technical subject in an engineering school of merit say in substance that perhaps, after all, calculus ought to be regarded as a culture subject; that it afforded good mental discipline, but that he doubted its value as a tool in engineering practise.

However much the opinions expressed may have varied, it has always seemed to me that the further a practising engineer advanced in his profession the more respect did he show for the elegant processes of not only the calculus but also of mathematics in general. These men may not themselves be called upon to work out the details of a design involving perhaps principles of the calculus, yet they will be competent in checking to pass on designs executed by others. They will also have a wide acquaintance with technical literature, especially that bearing on their special field.

When the mathematicians and engineers met in a joint conference at Chicago to investigate further the subject of mathematics for engineering students there seemed to be no question whatever as to

the desirability of a thorough knowledge of the calculus, with the ability to use it, on the part of the engineer; the question simply was one of quantity, quality and efficiency in mastering the same.

The practising engineers, and I am speaking of those who have attained a position of at least average merit, keep in touch with modern developments not only in this country but in others as well, just as far as their knowledge of foreign languages will allow them to follow the literature. Assistants in large libraries will tell you that files of the current foreign technical papers, which may have been neglected entirely in undergraduate days, are later eagerly read by men of affairs who seemed to know what they were after.

While not every advance in technical lines is reported in the journals, yet it would be safe to say that the most of those possessing merit receive recognition in publication, and that the files of a journal for a period of years are apt to reflect quite accurately the thoughts and deeds of engineers in the particular field covered by the journal in question. The aims and interests of engineers are clearly reflected—perhaps we may even get their attitude toward the calculus. And I would rather judge by what is done than what is said—an engineer early learns the value of results.

Different engineers will use and be interested in different fields of mathematics. Some editors of technical journals wouldn't care to publish an article heavy because of the mathematics used, and yet all use mathematics more or less—because they must. The question then arises as to the journal in any country chosen as the reflector of the opinions of engineers on the mathematics used and read during a period of years.

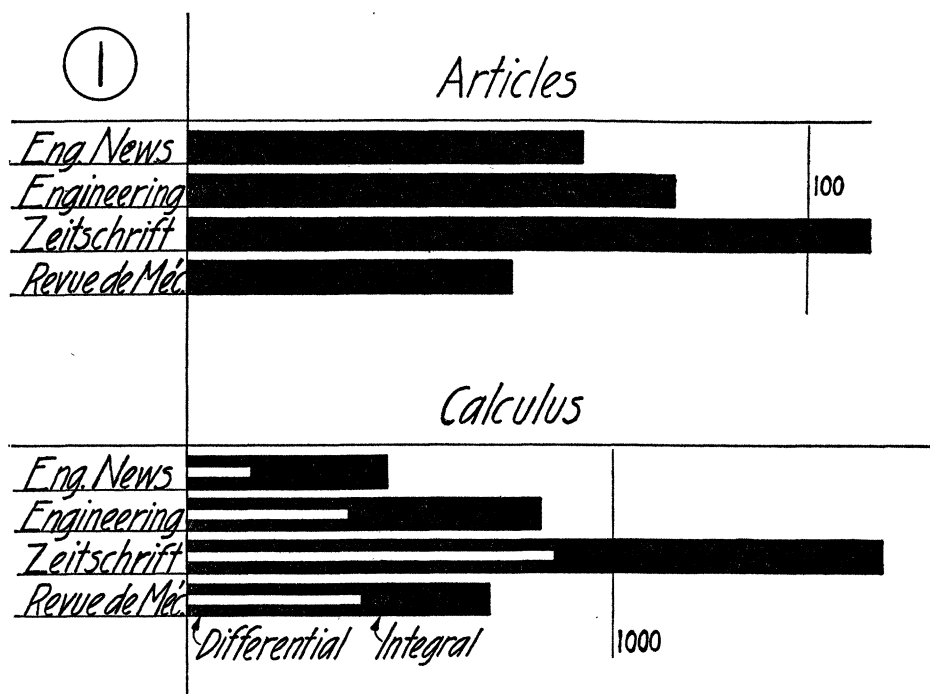
I have chosen the following journals as:

representative in their respective countries: America, *Engineering News*; England, *Engineering*; Germany, *Zeitschrift des Vereins Ingenieur*; France, *Revue de Mécanique*. The first three papers are published weekly, the last monthly.

To get at the principles of the calculus used, and to what extent, I read the articles in the above journals making use of the principles of the calculus, published dur-

gently a knowledge of the fundamental principles of the calculus was necessary—on this basis were the articles and principles listed.

The contributors of these articles included all classes of practising engineers, army officers, government officials, consulting engineers, whether connected with technical institutions or not, and professors in technical schools and universities. That



ing the five years 1905–1909. The articles in which the calculus was used were listed and the results shown in Fig. 1. Many articles contained a species of “near calculus,” thus making the question of including it or not doubtful; final disposition of the case was made on the basis of the article including at least one principle of the calculus and employing its nomenclature. The authors of the articles made use of the principles as needed, and understandingly. To read the articles intelli-

the articles were read by engineers is evidenced by the numerous comments on the same sent in to the editors, as well as by the records of assistants in libraries specializing in files of technical papers.

I do not use as a basis the opinions of men as expressed in the journals but rather investigated the principles actually used. The opinions of engineers, as expressed in the journals, were found to vary as much as the oral reports made by the young engineers formerly mentioned, and along this

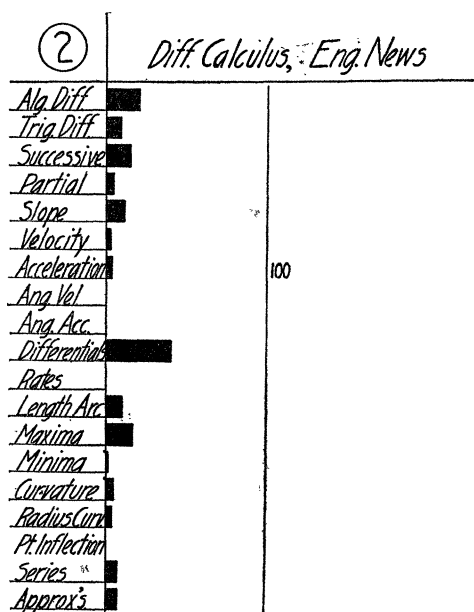
line we find, quoting from the four journals chosen, such expressions as "formidable mathematics," "mathematical rubbish"; one writer apologizes for his "mathematical fireworks"; another says of a writer that he "enveloped elementary principles of engineering theory in such a haze as to render pursuit hopeless to any but a confirmed mathematician." Again, speaking of the work of young college graduates, one writer said: "All of them did calculating without checking as they proceeded," and "Accuracy is one of the keys to success."

From another paper we quote: "It is the pride of mathematicians to compress a great deal into a single formula. But a diet of tabloids, however full of nourishment, is not adapted to all digestions; and the present paper goes to the other extreme—namely, spoon-feeding." Again we quote: "... great respect for mathematical proofs—if experimental results don't support the theory so much the worse for experimental results."

In a third journal we find a writer openly stating that he is writing his article so that the engineer knowing very little calculus, especially the integral, may yet read his article. Many other opinions expressed could be cited, and much discussion back and forth concerning proper methods of instruction in engineering mathematics abstracted with profit; yet it seems to me that, after all is said and the smoke of battle has cleared away, the engineers would, or should, rather be judged by what they do, and hence I present in the following table, I., a summary of the number of times which it seemed to me the calculus was used in each of the papers mentioned above during the period 1905–1909. The figure will explain itself when it is suggested that as far as a quantitative result is concerned I listed each principle

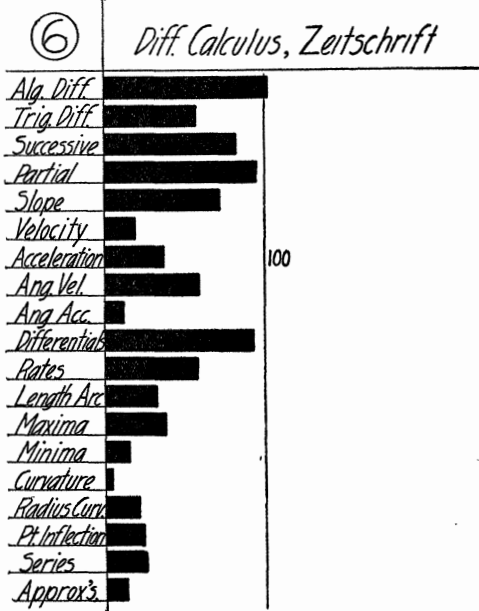
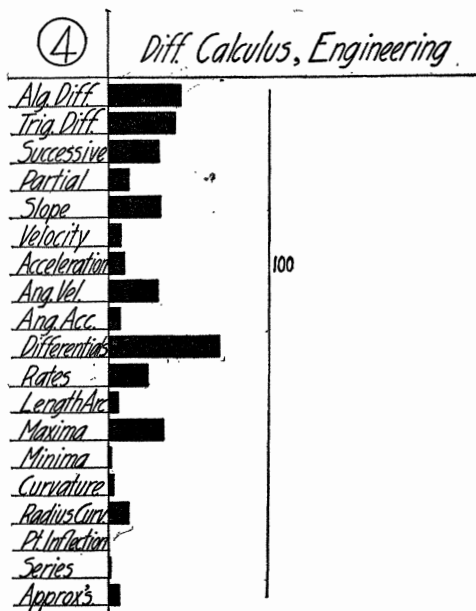
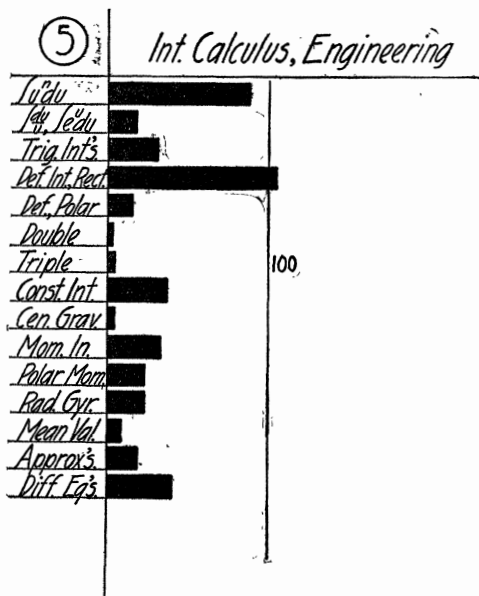
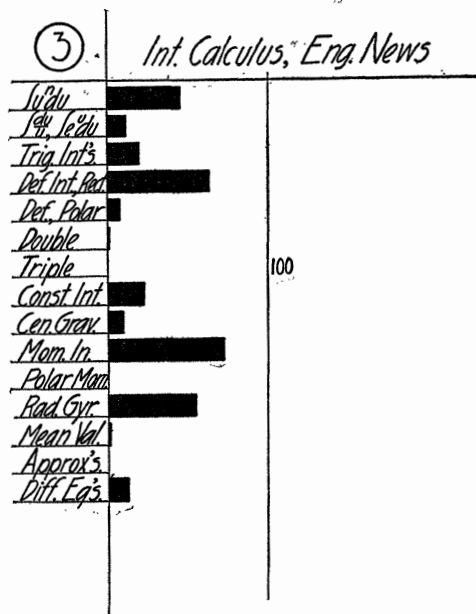
of the differential and integral calculus as it was used in an article. It might easily be that, with a viewpoint different from the one used, that the table as made out by another might look entirely different from a quantitative standpoint, yet relatively the results could not differ materially. Different principles, as applied, might be listed numerically differently by two men working out the same problem. In listing these principles I counted a single one only once during a discussion, even though the same expression may have been used many times. However, when the same principle was used in a new form, or a new application made, it was again counted.

Regardless of opinions expressed in the journals and on the basis of the use made of the principles of the calculus in the years 1905–1909 we present Figs. 2 to 9 as giving the relative importance placed on these separate principles by the journals named. It may be said that each was used with the idea in mind that the reader was acquainted with all of them, and even

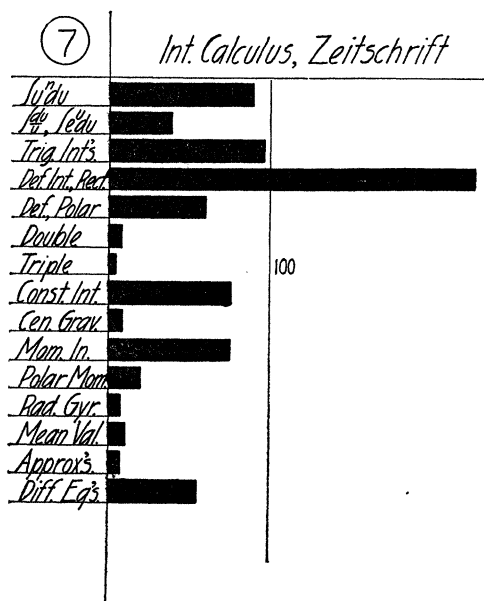


when an elliptic integral, a differential equation, or a Fourier series was used it was assumed that the reader was familiar with such.

The results as shown in the figures will, on comparison with similar figures accompanying an article in SCIENCE, October 22, 1909, in which a study of the calculus in



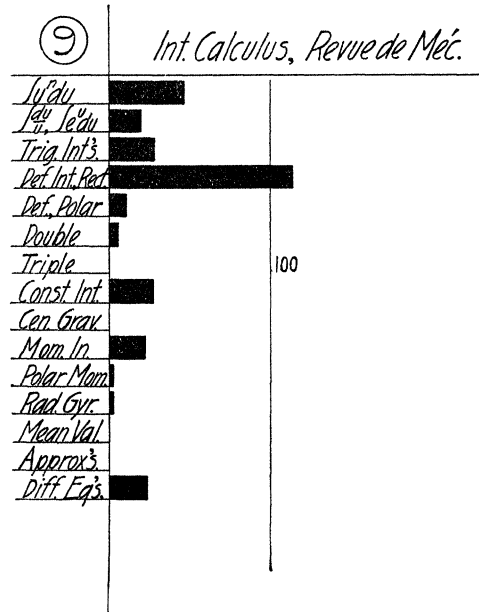
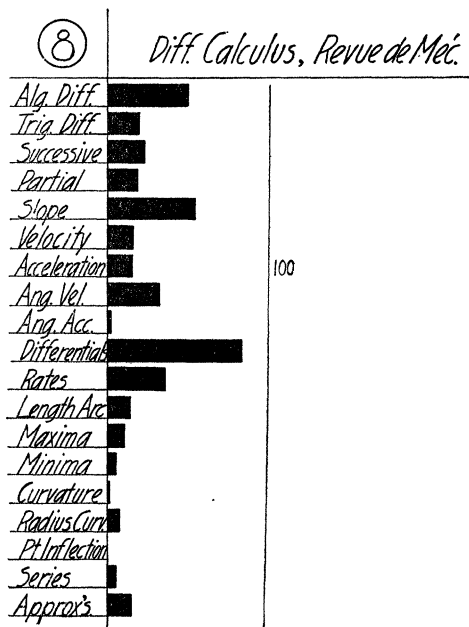
undergraduate technical courses was made, be found to agree closely with those published formerly in the relative emphasis placed on the various principles. Again we find that, for instance, the formulation of the definite integral and its interpretation by means of an area drawn to scale, stands out most prominently. This fundamental principle is just beginning to receive the recognition due it in the teaching of the calculus to engineering students, and deserves all the emphasis it receives in the Preliminary Report of the Committee



on the Teaching of Mathematics to Students of Engineering recently issued, and in several of the elementary texts on calculus recently published.

On the other hand, the results of the present investigation confirm the writer in his former belief that the subject of indeterminate forms and their evaluation has in it from a practical standpoint little of value for the engineer. There were found just two distinct cases where such forms arose and where methods of the calculus

were used to evaluate the same. In view of the fact that such forms occur rarely in



engineering practise and that the required limit of each can generally be obtained

after transformation it is doubtful from the standpoint of the engineer if the subject is worth over a page of discussion in the report referred to above, or the many pages—in one case as many as 18—of recent elementary texts on the calculus. The time given to them in the course on calculus could be used to much better advantage elsewhere.

The relatively few times that the double and triple integrals are used agrees with the results in the undergraduate discussion. Double and triple integrals could be used more, but they aren't; engineers seem to prefer the single integral. From this we would not argue that they should receive attention in a course in the same ratio as they are found used in practise, for both have greater merit and occasions do arise in which they are fundamentally necessary and important.

The differentiations and integrations, as shown, are, for the most part, limited to a few types and are generally readily executed. The algebraic integrations do not at all resemble the heavy forms involving radicals and reduction formulas which were so painfully evident in the college course in calculus. The trigonometric forms, both in differentiation and integration, are limited almost entirely to sines and cosines and their combinations. The heavier integrations, both algebraic and trigonometric, are apt to occur in connection with the solution of the differential equations arising in the discussions.

Partial differentiation comes in for considerable attention partly because of the fact that the journals listed, especially the foreign, always seemed to welcome an article making use of the principles of thermodynamics. The relative importance of this portion of the calculus to the engineer seems to be greater than would be indicated by the amount of time given the

subject in most of the courses in calculus for engineers.

The principles of angular velocity and acceleration are used with considerable frequency in the foreign journals, thus indicating clearly the strong influence of the mechanical engineering side of the technical field.

The attitude of all the journals in the matter of symbols is the same and agrees with the almost universal custom among mathematicians in this country. The somewhat forced efforts made for some time by certain mathematicians in this country to introduce capital letters in connection with derivatives seems to have received no recognition whatever among engineers.

Whenever a differential is used it has a strong resemblance to the infinitesimal of the mathematician, or perhaps a rate, where the time element comes in; but it will be noticed early that the processes of the engineer, by nature correct to, say, so many decimal places are not always the processes of the mathematician confirmed in his use of the limit. Increments sometimes take the place of these differentials and very neat bits of theory are carried out by their use—a "near calculus" as it were. It is at times difficult to distinguish between the use of  $\Delta x$  and  $dx$  and yet the idea of the limit is there all the time. Rigor within the limit of allowable error is the key-note throughout. As an illustration I quote: "Let the radii of curvature of the surfaces where the oil film is infinitely thin be  $r_1$  and  $r_2$ ; then, since we are dealing with films of capillary thickness, we may treat the dimensions as infinitesimals as compared with  $r_1$  and  $r_2$ ." With which many a pure mathematician will find it difficult to agree. And yet the proofs do not break down at any point.

More maxima and minima are found directly from the study of algebraic and

trigonometric expressions than by the methods of the calculus, and but little attention is paid to the sign of the second derivative in determining the nature of the same. The conditions of the problem are, in general, sufficient to determine the nature of the result on solving the equation obtained by putting  $dy/dx=0$ .

Series where used are assumed to be convergent, or at least their convergence is not questioned. They are generally simple types.

Many approximations occur in engineering practise, while those listed seem to be few in number. However, none was counted except the approximations of the calculus. Among such we might mention  $1/r=d^2y/dx^2$ , which is used in a case where, as stated, " $dy/dx$  is small."

The symbol of summation  $\Sigma$  is used often and we find many a case of "near integration." The great importance placed on the formulation and evaluation of the definite integral is everywhere evident and many areas are found where no definite integral is expressed and where it is absolutely essential to keep in mind the relation between the two. In this connection we wish to mention the universal use of indicator diagrams, and the frequent mention of the planimeter used in determining areas approximately—a point of view which should be kept in mind when the subject of definite integrals is being considered in the class-room.

In connection with the integrations found it seemed that at times the constants multiplying the integral were by far the most important part of the expression. Instructors of calculus might with profit at times allow their students to make their own choice of such constants, which should be placed on the outside of the integral sign before evaluating the definite integral. The term moment of inertia seems to mean

two things to two different classes of engineers. The engineer dealing with static problems will have almost exclusively to do with moments of inertia of sections, while one working with problems bringing in dynamics will think of what in one case is called the "equatorial moment." The two points of view should receive equal attention in any course on the calculus.

Concerning the differential equations used and their solutions it may be said that those used were of the simpler types usually included in an elementary treatise on ordinary differential equations. However, it seems to me that their solutions must at times have been far above the head of the average engineer, unless he had given the subject special attention after completing his university course in engineering. The recommendation of the committee on engineering mathematics is to the point, and should be carefully considered by the instructor of calculus. It agrees with results as found in practise.

A further study of the mathematics used by the practising engineer will reveal other conditions in every way similar to those existing in the undergraduate technical course. The algebra and trigonometry used are heavy as compared with the calculus; naturally they are used much oftener.

If we look for things characteristic of the engineer we easily find that numerical results, correct to a certain decimal place, are common and that much stress is placed on accurate computation. Much care is bestowed on the drawings and illustrations, and constant attention is given to the scale of the same. This is necessary in checking up. Much use is made of indicator diagrams and the planimeter is used to obtain or check up on areas. At least one of the journals makes a considerable use of the first and second derivative curves and



their interpretation. The policy of the *Zeitschrift* in using such curves is to be recommended to the instructor when the subjects of velocities and accelerations are under consideration.

A common practise in approximating is that of using small angles, their sines, and tangents synonymously.

The checking up process stands out prominently; not only do the engineers say they believe in it but they also practise it as well. Instructors of mathematics may easily learn a lesson here.

The definite character of the results is evident; the authors get down to fundamental principles, remain clear throughout a discussion, and finish with concrete results.

Here and there is found an article on a special subject which will tax the mathematical capacity of most engineers, perhaps be far above their heads. Such are generally contributed by professors in universities and mark the limit of the mathematical field for engineers. We find a rare use of an elliptic integral, a Fourier series, homogeneous coordinates, partial differential equations, and the fundamental principles of the calculus of variation. However, these are rare and the articles using such will be read by but a very limited number of engineers.

A comparison of the articles in the different journals will show for the American the strong preponderance of the civil engineering, while the foreign journals lean more to the mechanical engineering side. In none of them do the articles go into the details of the projects in electrical engineering. The articles in this latter field are mostly of a descriptive nature, in which electric power installations, machines and appliances are discussed. Whenever mathematics is used in the electrical engineering field it verges on the more "formidable"

mathematics of mathematical physics, combined with a liberal sprinkling of the complex variable and differential equations. The list examined can hardly be said to contain a journal specializing in the field of electrical engineering. However, if the stronger journals in the electrical field be examined they will be found to strongly emphasize the descriptive features of the field; and a conclusion which may be drawn from this fact is that even the rudiments of research and design in that field would immediately involve mathematics in the principles of which the average engineering major has had but little training. Articles going into these details would not be read as the more general articles in the other fields of engineering are read.

The technical literature also reflects the highly developed scientific spirit of Germany, which has permeated into all the branches of its technology. The continental journals, especially the German, start with fundamental engineering principles and make a liberal use of the calculus and other branches of mathematics; so that when a discussion is completed it is evident that a piece of work worth while has been thoroughly done. On the other hand, the American attitude of wanting to get things done in short order is also plainly evident; the American engineer will generally not take the time to work out a bit of theory in the details of which the German engineer will revel. He will use—and with a full significance of their purpose—the results laboriously obtained by others, thus specializing on the applications. A formula developed with much care from fundamental principles by his foreign brother will appeal to an American engineer as something which should immediately be put to practical use.

The English journals take as much pride in the design of their battleships and ves-

sels of commerce as the Germans and French in their air-ships and the Americans in their sky-scrapers. And all are interested in turbines. The *Zeitschrift* seems to be by far the greatest source of scientific advances in technology, and the engineering journals of other nations look to it as the dean of them all.

A study of this sort would not be complete unless it took into consideration the far-reaching effect which the failure of the Quebec bridge, on August 29, 1907, had on technical literature, especially in America. In the many discussions of column formulas resulting, with special reference to the value of  $l/r$  found in most of them, we can easily see the strong inclination of the American engineer toward a plausible formula. Many discussions followed the disaster, most of them making use of  $l/r$  and suggesting modifications of the column formulas in existence. It may be questioned, in view of the results of recent tests made on built-up columns, whether the old formulas, even with modifications, will not be superseded by some entirely new rules for the design of such columns.

It may be stated, in conclusion, that the attitude of the engineers toward the efficient teaching of the principles of mathematics, as gathered from their discussions, is sane and their interest great. Naturally, they call for results and are apt to be impatient if a college graduate violates fundamental principles which should have been thoroughly mastered long before. They are aware of the difficulties encountered in the efficient teaching of mathematics and of the different viewpoints of instructors of mathematics. On the other hand, instructors of mathematics for students of engineering should maintain an attitude of sympathy with the problems of the engineer, or at least recognize and become acquainted with them. That both

engineers and mathematicians are working more and more toward a common end, and with a better understanding of the problems involved, is evidenced by the results of the many joint conferences held recently for the purpose of securing that greater efficiency, which is the watchword of the age.

ERNEST W. PONZER

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RICHARD KLEBS

PROFESSOR DR. RICHARD KLEBS, geologist and knight of high degree, connected with the Royal Geological Survey, and scientific adviser to the Royal Amber Works, died in Königsberg, Prussia, on June 20, 1911, in his sixty-seventh year.

Dr. Klebs was well known throughout the world for many papers on the subject of amber and its industry, the inclusions and the study of the coleoptera, and plant and insect inclusions in amber masses, he himself gathering and owning the great collection which was exhibited under the auspices of the Imperial German government at the St. Louis Exposition in 1904. This great collection consists of 10,000 inclusions in amber, including beetles, fleas, spiders, wood, leaves and many other interesting objects associated with the history of amber. It is valued at \$40,000 and will only be sold as an entirety.

The last paper he wrote, and of which he sent me a reprint, is entitled: "Ueber Bernsteineinschlüsse im allgemeinen und die Coleopteren meiner Bernsteinsammlung," with text illustrations, which appeared in the "Schriften der Physik-ökonom. Gesellschaft zu Königsberg i Pr." Jahrg. LI., pp. 217-242, III., 1910. Dr. Alfons Dampf, assistant in the Königl. Zoologischen Museum, Königsberg, described a fossil flea occurring in Baltic amber and named it "*Palæopsylla klebsiana*," in honor of his friend, Dr. Klebs (pp. 248-259, pl. 2, 1910-11).

Dr. Klebs possessed an earnest, cheerful personality; was an indefatigable worker, published many papers on his subject, and suc-