peated by a large number of prominent mathematicians.[1] That the graph on page 214 is the graphic representation of the function in question is open to serious doubts, which should not have been passed over in a work on logical foundations. The statement on page 101 that "Diophantus of Alexandria. who lived 300 A.D., seems to have been the first actually to have made use of rational numbers" is apt to mislead the reader even if a footnote helps to ascertain the author's meaning. Taken by itself this statement seems absurd.

These are, however, matters of secondary importance and the book under review seems to be remarkably free from errors if we consider its wide scope. In particular, the proof seems to have been read with unusual care and one can only wish that the book will be very widely read, especially by those who are preparing to teach secondary mathematics. Its style is attractive and many of the questions which it treats are so far reaching that one may reasonably expect that it will find a considerable number of readers outside of the circle of professional mathematicians.

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[1] Cf. "Encyclopédie des Sciences Mathematiques," Tome 2, Vol. 1, p. 3; Cantor's "Geschichte," Vol. 3, 1901, pp. 215, 456–7.

SPECIAL ARTICLES

COLOR DISPERSION IN THE ASTIGMATIC EYE

WHEN an astigmatic eye views a bright point of light in which only the rays near the ends of the visible spectrum are present, the image of the source is blurred by fringes or wings of red and blue. If the eye has a well-defined axis of astigmatism but is otherwise fairly emmetropic, the appearance of such a source is so curious as to compel attention. The purplish image is then crossed by a pronounced red band parallel to that meridian of the eye in which the curvature is least, and by a blue band at right angles to it. In the case of astigmatism with the rule, the red band is approximately horizontal. The experiment is easily tried in a darkened room by allowing the light from a flame or electric lamp to pass through a hole a few millimeters in diameter in a screen, or better by placing the lamp in a box having a small hole in one side. One or two thicknesses of common "pot blue" glass are placed over the opening, which is then viewed from a distance of two meters or more. The blue glass, as is well known, is fairly transparent to red light. Distant blue lights seen at night, such as the "dwarf signals" in railroad yards, show the effect well.[1] Indeed, the appearance can be seen by viewing any bright light through a blue glass held in front of the eye. A person free from astigmatism can see the effect by holding a cylindrical lens in front of the eye.

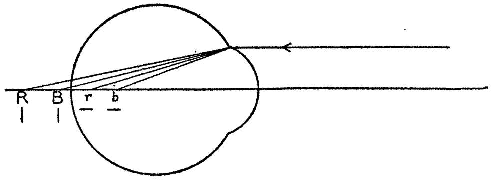
The explanation is simple, and has very likely occurred to many who have noticed the effect. However, the writer has been unable to find any reference to it, either in the classical memoirs of Helmholtz and his predecessors, or in such later writings as he has access to. Astigmatic vision seems to have been considered only on the tacit assumption that dispersion could be neglected—an assumption that is sufficient with ordinary white light, in which the yellow and green rays predominate in determining our visual sensations. It is only when these intermediate rays are excluded that the effects of dispersion become noticeable.[2]

Taking the type of astigmatism most commonly found, let us assume that the radius of curvature of the cornea is less for the vertical than for the horizontal meridian. If the eye observes a distant point-source giving only

[1] Among those whom the writer asked whether they had noticed the crossed red and blue bands was a certain railroad employé, who not only observed the appearance to a marked degree, but also volunteered the explanation that the dwarf signal "had a dirty glass." Remarks of this sort show how unconscious we are of our own defects of vision.

[2] For example, Helmholtz describes some interesting experiments on the effects of chromatic dispersion in the eye; these are also recounted by Lummer in Müller-Pouillet's "Lehrbuch der Physik." In all these cases the eye is assumed to be free from astigmatic defects.

red and blue rays, then the foci for these rays with respect to the curvature of the vertical meridian will fall, say, at r and b, while the foci with respect to the curvature of the horizontal meridian will fall at R and B respectively. The relative distances are exaggerated for the sake of clearness. Helmholtz gives the distance between the focal planes for red and violet (RB or rb in the diagram) as about 0.6 mm. Whether r falls to the right or left of B depends on the degree of astigma-



tism. In an otherwise emmetropic eye focused on infinity the retina will be somewhere between the points B and r. If the accommodation is such that the point R falls on the retina, the eye perceives instead of a point a vertical red band. The direction of the bands is indicated by short lines under the letters in the diagram. A horizontal blue band is perceived if the point b falls on the retina, in each case the band being surrounded by an indistinct halo due to the other images. If B and r fall close enough to the retina, R and b will be out of focus and practically indistinguishable, so that only a horizonal red band crossing a vertical band of blue will be seen. While the writer normally sees these bands crossing, he is able, by changing the accommodation of the eye, to observe the vertical red band (surrounded by a bluish halo) or the horizontal blue band. To see the two bands crossing, the degree of astigmatism must, of course, fall between certain limits, but these limits turn out to be surprisingly wide, partly because the appearance of crossed bands is heightened by the effects of contrast, partly because the bands become longer with increasing astigmatism. As would be expected, the red band *looks* nearer than the blue.

As for the quantitative relations, it is easy to show that there is a degree of astigmatism, well within the limits commonly found, for which the crossed red and blue bands are in focus simultaneously. For Helmholtz, as stated above, shows that the distance RB between the focal lengths for red and blue due to dispersion is about 0.6 mm. On the other hand, in an astigmatic eye the difference between the focal lengths due to the two curvatures at right angles (Rr or Bb) may be anything from zero to 2 mm. or more. Thus an eye need be only slightly astigmatic (correction about 1.5 diopters) in order to bring the points B and r into coincidence.

If the eye has an astigmatism so complex that several astigmatic axes in different planes have to be dealt with, the red and blue bands are correspondingly complex. Indeed, this method might perhaps prove useful in the examination of astigmatic eyes. By placing cylindrical lenses of varying focal lengths before the eye, it is possible to tell by the disappearance of the bands when the optimum correction has been attained. It may be remarked in passing that the phenomenon of color dispersion furnishes an interesting demonstration of the manner in which ametropia is corrected. For, as is well known, far-sighted and near-sighted eyes perceive purple sources as surrounded by a red or a blue halo respectively, and this halo can be increased, or reduced to a well-marked minimum, by the use of suitable spherical lenses.

Even a small source of *white* light looks a little ruddier on two opposite sides if the eye is astigmatic.

Very peculiar effects are produced in an astigmatic eye by repeating the experiments described above with a purple or "blue" light, using in place of a round hole a row or group of pinholes, or a slit, or a maltese cross set at varying angles.

To show the crossed red and blue bands objectively, an astigmatic lens of high dispersion was made in the following manner: the surface of a large incandescent bulb was tested at different points, until a region was found where the ratio of the curvatures in planes parallel and at right angles to the axis of the bulb was of the desired value (a 60-

watt tungsten lamp was the largest available giving the required ratio). This ratio was computed from a consideration of the wave-lengths transmitted by the blue glass used, namely, about 710 and 420 $\mu\mu$, and of the indices of refraction of carbon disulphide for these wave-lengths. This part of the bulb was cut out, forming a sort of shallow bowl into which a quantity of carbon disulphide was poured. I thus had an astigmatic plano-convex lens which gave as an image a minute red-and-blue cross when purple light was passed vertically through it. Later the fragment of the bulb was cemented with shellac on to a flat piece of glass, forming a cell into which the carbon disulphide could be introduced. A diagram was used to screen off all but a small portion of the lens.

In a rudimentary way the appearance can be projected on to a screen by passing light obliquely through a common plano-convex lens.[3]

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[3] I am indebted to Professor Raymond Dodge for this suggestion.

THE IOWA ACADEMY OF SCIENCE

THE sessions of the academy were held in Carnegie Science Hall, Coe College, Cedar Rapids, April 28 and 29.

The public address by Dr. Edward L. Nichols, of Cornell University, on "The Ends of the Spectrum—the Infra-red and the Ultra-violet," was given on Friday at 8:00 P.M.

Sessions of the academy for the reading of papers were open to the public.

Nitrogen in Rain and Snow: NICHOLAS KNIGHT.

Seventeen samples of rain and snow were collected on the college campus at Mt. Vernon, Iowa, during nine months of the year 1909–1910. The nitrogen in the free and albuminoid ammonia and in the nitrates and nitrites was determined. Comparisons were made of the relative amounts of nitrogen precipitated with the rain and snow. According to the experiments, each acre would receive in the nine months between thirteen and fourteen pounds of nitrogen from the rains and snows.

Perchloric Acid in Electro-chemical Analysis: W. S. HENDRIXSON.

Asteroid, 1909, JA: SETH NICHOLSON and ALMA STOTS.

Vaccination against Typhoid Fever: HENRY ALBERT.

The writer briefly reviewed the experimental work on immunization with cultures of typhoid bacilli, then gave the technique and preparation of vaccine as generally employed at present for the preventive vaccination against typhoid fever. The reaction was divided into a local one which disappears in the course of a few days and a general one which he divided into leucocytic, phagocytic, agglutinitic and bacteriolytic. There is an increase in the number of leucocytes, the power of phagocytosis, and of the agglutinins and bacteriolysins in the blood serum. The presence of immune bodies may be demonstrated as long as one year after vaccination and is of both a higher degree and of longer duration than usually occurs following an attack of typhoid fever. Reasoning from analogy from the protection afforded by one attack of the disease, he believes that the immunity conferred by vaccination exerts more or less protective influence throughout the life of an individual. He would extend anti-typhoid vaccination to all liable to exposure to infection with typhoid fever.

Flowers of Story County: J. M. LINDLEY.

The Succession of Floras on the Sand Dunes of Iowa: B. SHIMEK.

A discussion of the changes of the earliest flora of these dunes, consisting largely of leguminose plants, to the typical prairie flora of the older areas.

The Nebraskan Drift: B. SHIMEK.

A discussion of its distribution and correlation, including the results of recent investigations. The conclusion is reached that this drift can not be correlated with the Jerseyan and Albertan, and that the name "Kansan" should not be transferred to it.

Notes on Fungus Diseases: L. H. PAMMEL.

Gives a record of a very destructive *Exoascus* upon the hard maple in the Rocky Mountains, also an account of the destructive *Exoascus* on the oak, the destructive *Fomes iginarius* on the quaking aspen in some parts of the Wasatch Mountains in Utah, as well as the destructive work of *Pleurotus* upon the box elder and other deciduous trees in Iowa.

An Abnormal Carpel in Stenospermatium: J. E. GOW.