stack-room at any distance; farther apart or nearer together, as required, irrespective of their relation to daylight through the windows.

5. As Mr. Green has pointed out, daylight is injurious to books.

6. The temperature of the room will be more equable, the internal heat being retained in the winter, and the external heat being excluded in the summer.

I hope, if this commends itself to architects and librarians, that some day the board of directors of a library may act upon it. The only drawback that occurs to me is that architecturally it would not be attractive in appearance, but as the book-stack is usually in the rear of the building and more or less hidden from view, I think this would not be a very serious objection.

W. W. Keen

Philadelphia,
April 30, 1909

## SCIENTIFIC BOOKS

*The Thirteen Books of Euclid's Elements Translated from the Text of Heiberg with Introduction and Commentary.* By T. L. Heath, C.B., Sc.D. Cambridge, University Press. 1908. Three vols.

*Differential and Integral Calculus.* By Daniel A. Murray, Ph. D., Professor of Applied Mathematics in McGill University. New York, Longmans, Green and Co. 1908. Pp. xviii + 491.

*An Elementary Treatise on the Differential Calculus Founded on the Method of Rates.* By William Woolsey Johnson, Professor of Mathematics at the United States Naval Academy, Annapolis, Maryland. New York, John Wiley and Sons. 1908. Pp. x + 191.

In these days of prolific writing when even the worst books (if indeed such a lower limit exists) must be "noticed" and the best, owing to consequent lack of space, may not be really reviewed, one is at a loss to know how properly to signalize the appearance of so important and excellent a work as this latest production from the pen of Dr. Heath. Of this work it is safe to say—and that is much—that no other better illustrates the great service of British scholarship in rendering the ancient classics accessible and attractive to our time, and no other better illustrates the truth of Cousin's *mot: La critique est la vie de la science.* No account of the work can be bad if it has the effect of inducing the reader to procure a copy for himself, and no account can be good if it have the opposite effect. For students and teachers of mathematics or of philosophy, this edition of the "Elements" may be said to be indispensable.

It is only after examination of the volumes that one can realize how utterly impossible it is to convey in a few lines anything like an adequate conception of the riches that Dr. Heath has given us. Nevertheless, a few hints—the meagerest of indications—must be given. The introduction, which occupies more than a third of the first volume, is composed of nine chapters, entitled Euclid and the Traditions About Him, Euclid's Other Works, Greek Commentators other than Proclus, Proclus and his Sources, The Text, The Scholia, Euclid in Arabia, Principal Translations and Editions, and (ninth chapter) On the Nature of Elements, Elements Anterior to Euclid's, First Principles, etc. Of the man Euclid, as of many another great determinator of the world's career, but little is known, and the chapter on Traditions, though it furnishes nothing new, is valuable as collecting, sifting and citing the literature of the old. One of the traditions, whether or not it be true in fact, is at all events true in spirit, and will be relished by that variety of practician engineer who views mathematics as he views a wheelbarrow or a spade. "But what shall I get by learning these things?" said a pupil to Euclid after learning the first theorem. Thereupon Euclid called his slave and said, "Give him threepence, since he must make gain out of what he learns." The late Sylvestre, it is remembered, on being asked to state the "use" of the theory of substitution groups, on which he had been lecturing, replied, "I thank God that, so far as I know, it hasn't any." It would be interesting to know what proportion of scientific men are aware of the fact that Euclid wrote several scientific works other than the "Elements," as the hopelessly lost "Pseudaria," said by Proclus to be "by way

of cathartic and exercise," designed, that is, to aid the beginner in the discovery of paralogisms and in the discrimination of true principles from the specious and false; the still extant "Data"; the book "On Divisions (of Figures)," which though "lost in Greek," "has been discovered in the Arabic"; the three lost books of "Porisms"; the two works entitled "Surface-loci" and "Conics," both lost; the still extant "Phænomena," an astronomical work; and the "Optics," edited by Heiberg in 1895. In the two chapters (27 pages) devoted to Greek commentators upon Euclid and especially to Proclus and his Sources, Dr. Heath has given not only detailed citations of the immense body of literature bearing upon these matters, but—what is far more—a most luminous and valuable digest of it all. Then follows the chapter of 18 pages dealing with the text. Here we have an interesting account of the several celebrated manuscripts from which knowledge of the content of the "Elements" is mainly derived, a statement of the critical principles followed by Heiberg in the comparison and evaluation of the sources, and a good indication of the ingenuity, of the prodigious scholarship, labor and devotion that enabled Heiberg to produce his monumental work on Euclid. This work was published between 1883 and 1888. It is the "definitive text" contained in it that Dr. Heath has translated into English and that his scholarship has enabled him to set in the light of modern researches into the foundations of geometry. In view of the empire that Euclid has exercised in British education, it is especially interesting to learn, in the chapter on Translations and Editions, page 95, that the great Greek classic, or some portion of it at all events, found itself in English dress as early as the first half of the tenth century. In support of this contention, Dr. Heath cites the following quaint lines from "Rara Mathematica":

> The clerk Euclide on this wyse hit fonde
> Thys craft of gemetry yn Egypte londe
> Yn Egypte he tawghte hyt ful wide,
> In dyvers londe on every syde.
> Mony erys afterwarde y understonde
> Yer that the craft com ynto thys londe.
> Thys craft com into England, as y yow say,
> In tyme of good kyng Adelstone's day.

If among the many students of recent American and English, Italian and German work in the foundations of geometry, there be any one who imagines that nearly all of the fine things presented in such work are new, there await him in the closing chapter of Dr. Heath's introduction a pleasant surprise and a happy emancipation. Indeed this chapter of 38 large octavo pages is an exceedingly valuable contribution to the historico-critical literature that pertains to the common ground of logic and mathematics. In it is set forth in clear and orderly fashion the best thought —and how fine and penetrating much of it is! —of pre-Euclidean and post-Euclidean philosophers and geometricians from Plato and Aristotle down to Proclus—a period of nine centuries—the best thought, I say, of the best intellects of antiquity regarding such eternally interesting matters as the nature of scientific (and especially) geometric *elements,* of the nature and significance of *axioms* (or common notions) and *definitions,* of *hypotheses* and *postulates,* of *existence assumptions* and *existence proofs,* of *theorems* and *problems, lemmas, cases* and *porisms,* of methods of *argument* and *demonstration, objection, reduction, analysis* and *synthesis.* Much that has been recently written about these things is new and much of it is but repetition—repetition that is sometimes inferior in point of form and doubtless often unconscious. It is no small service to give the reader, as Dr. Heath has here done, a lively sense of the scientific atmosphere in which Euclid wrote and of his indebtedness and through him that of all subsequent time to his predecessors and contemporaries. Aristotle's statement, quoted by Heath from the "Posterior Analytics," that, "other things being equal, that proof is the better which proceeds from the fewer postulates or hypotheses or propositions," reminds one of the famous dictum enunciated sixteen hundred years later by the "Doctor invincibilis," William Occam: *Entia non sunt multiplicanda prœter necessitatem.* Nothing conveys better the animating spirit of the

modern critical movements in logic and mathematics.

The remaining 273 pages of the first volume are devoted to books I. and II. of the "Elements." The entire discourse is conducted in the most admirable manner. First are given in clear, bold, beautiful type the definitions, axioms and postulates of book I. Each of these is then taken up in order, restated in Greek followed by the English equivalent, and made the subject of an elaborate and lucid historical and critical commentary. The extent of Dr. Heath's commentaries may be inferred from the fact that no less than eighty-five large pages are occupied by discussion of the thirty-three definitions, axioms and postulates of the first book, the immense wealth of the material presented being gathered from all the principal sources from pre-Euclidean times to the present. Then follows the statements and demonstrations of the propositions of book I. with critical discussion of each. The remaining twelve books are treated in similar fashion. The third volume closes with the so-called "Book XIV." by Hypsicles, a note on the so-called "Book XV.," a carefully prepared list of addenda et corrigenda, an index of Greek words and forms, and an excellent general index to the entire work.

Dr. Heath has been animated by no narrow or partisan spirit. Like every other well-informed student of mathematics, he is well aware of the fact that the time is long since gone when the reading of Euclid was indispensable to one who would become a learned or a productive geometrician. And in so far as this new edition may be regarded as a plea for the "Elements," it is an enlightened plea, one entirely worthy of its great theme, the noblest plea in English speech for the Alexandrine classic.

The content of Professor Murray's book has been taken chiefly from his "Infinitesimal Calculus," though some matter relating to indeterminate forms, solid geometry and applications to motion has been added, and the treatment of several topics has been revised. Among the noteworthy features of the work may be mentioned that it contains more than can be properly read in the time usually given in the best schools to a first course in the calculus; that in view of this fact certain articles have been indicated as constituting a course suitable for "students having a minimum of time"; that the book begins with a discussion of certain problems designed to foreshadow the nature of the calculus and a presentation of such algebraic notions as are of most frequent use in the calculus; that the subject is presented with modern regard—rather too much than too little, if it errs in either respect—for precision and rigor; that, though the subject is divided into differential calculus and integral calculus, the notion of anti-differentiation is presented in connection with that of the direct process; that, with a view to facility of applications, the view of integration as summation is put in advance of the other view; that numerous examples including simple applications to geometry, physics and mechanics, with illustrative solutions, are inserted in connection with the development of cardinal ideas and processes, a table of answers being given at the end of the volume; that unusual care has been given to accentuation of important matters; that especial attention has been accorded to the concepts of speed, velocity and acceleration; that, for alternative or fuller treatment of numerous ideas, the reader is referred to a good deal of the better and more accessible literature; and that there is given a brief introduction to differential equations and a table of integrals. The work is adequate to the needs of the college student, to the technological student and to the rarer spirit preparing for higher flights.

Professor Johnson's book is an abridgment of his "Differential Calculus" and is based on the method of rates. It is distinguished by the absence of the histological methods of the rigorists. Things are presented pretty much at their face values, being shown graphically rather than laboriously proved by help of the refined logical machinery brought in from across the sea. The English is excellent, everything moves along with the calmness, dignity and facility of an elder day, and the reader, while acquiring much useful knowledge, will acquire also a degree of confidence that in these critical times is apt to be rare

and is apt also to suffer mitigation in the course of subsequent study.

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*Hopi Songs.* By BENJAMIN IVES GILMAN, Secretary of the Museum of Fine Arts, Boston, Mass. Hemenway Southwestern Expedition. A Journal of American Ethnology and Archæology. Fifth and Concluding Volume. Pp. xi + 235. Boston and New York, Houghton, Mifflin Company. 1908.

The text of the volume is divided into three sections: I., The Rote Song of the Hopi; II., The Phonographic Method; III., Notation, Diagrams and Comments. Seventeen Hopi songs are included in section III. A brief account of The Hemenway Southwest Expedition closes the volume.

The author opens his treatise by saying:

The study of Hopi, or Moqui, singing, to which this volume is devoted, completes an inquiry into Pueblo music begun in 1891 with a study of Zuni melodies. The records upon which both investigations have been based were obtained in Arizona by Dr. J. Walter Fewkes, now of the Bureau of Ethnology, Washington, at the time in charge of the Hemenway Southwestern Expedition, who first applied the phonograph to the preservation and study of aboriginal folk-lore.

Of his previous study the author writes (p. 11):

The major thesis of the "Zuni Melodies"—that Pueblo music is without scale—is strongly confirmed by this cumulative evidence. The diatonic form of the Hopi songs is (*a*) harmonic necessity or (*b*) apperceptive illusion. In large measure their adiatonic features are at once (*c*) intentional and (*d*) inexplicable by interpolation and transposition. The minor thesis of the "Zuni Melodies"—that "In this archaic stage of the art the scales are not formed but forming"—is rather weakened than corroborated by a closer study of Pueblo music. Its bent toward change inspires a doubt whether, unless by outward compulsion, it would ever submit to the trammels of a system. It appears an unhistoric rather than a prehistoric art.

Under the head Scales an Instrumental Product; the Voice Determining their General Form, the Ear, the Hand and the Eye their Varieties, the author skillfully proceeded to show that "Although the voice provides the raw material for scale building," the instruments have rendered service, so that

It would appear that while still disembodied music tends to remain adiatonic, though always of necessity diatonoid. Only when incarnate by instrumental constraint does it chose, because it must, the best of all possible yokes.

Other factors have influenced scale development so that

Scales may result with which the voice has had little to do, giving back to music, at the convenience and pleasure of ear and hand and eye, a semblance of the liberty of its vocal stage.

Under the head of Freedom, a characteristic of Pueblo music, the author writes:

Apart from the tendency to consonant intervals no metes and bounds to invention manifest themselves in these melodies, and they may apparently be altered by every performer.

In this connection a footnote calls attention to a fact presented at Berlin in 1888 before the International Congress of Americanists that

The anatomists of the Hemenway Southwestern Expedition found the hyoid bone of the ancient skeletons exhumed on the Rio Salado exceptionally elastic in structure. The position of this bone at the base of the tongue makes it an important factor both in speech and song.

This fact should not be forgotten when considering the data presented in this volume as of wide application. Nor can the statement that songs "may apparently be altered by every performer" be accepted as true of Indian songs in general. Accuracy in the rendition of a song, particularly of one that was a part of a religious ceremony, was insisted upon. In some of the tribes a mistake, or variation, in singing a song, constituted so grave a matter that it put a stop to the ceremony, until after a rite of contrition had been performed; that being finished the ceremony had to begin afresh. That there were slight variations in pitch and intonations was true, but they were such as occur among ordinary singers and did not affect the movement and flow of the melody, which the