would have to be done before we could state the general value to the organism of the various methods of training.

In determining the dancer's power of retaining discrimination habits, the author found that a white-black habit may persist during a period of from two to eight weeks of disuse, but that such habits are rarely perfect after an interval of four weeks. The retention of the color discrimination rarely persisted in perfect form for more than two weeks.

Having determined the periods of persistence of such habits, the author next undertook to find out whether training, the results of which have wholly disappeared so far as memory tests are concerned, influences the reacquisition of the same habit. It was found that the ten dancers tested had so lost the habit of the white-black discrimination at the end of a rest interval of eight weeks that memory tests furnished no evidence of the influence of previous training; retraining brought about the establishment of a perfect habit far more quickly than did the original training. Indices of modifiability are given both for the males and for the females, for the learning and for the relearning. The general conclusion issuing from this study is: that the effect of training is of two kinds, the one constitutes the basis of a definite form of motor activity, the other the basis or disposition for the acquirement of a certain type of behavior.

A chapter each is devoted to individual, age and sex differences, and to the inheritance of forms of behavior. Yerkes obtained satisfactory evidence from individuals of one line of descent pointing to the fact that, in their case, a probable tendency to whirl to the left is inherited. In regard to the inheritance of individually acquired forms of behavior, the author states that descent from individuals which had thoroughly learned to avoid the black box gives the dancer no advantage in the formation of a white-black discrimination habit.

In conclusion, we may say that aside from its general usefulness as a reference book for the research student, the book forms a valuable guide to the technique of experimentation upon animals. There is one defect in the book which certainly makes it lose in value for this latter purpose. This defect lies in the over-favorable emphasis given to the method which employs punishment rather than some form of reward (food, etc.) as an incentive. The reviewer feels that Yerkes has not fully justified its claims to priority even for use with the dancer, much less its value as a substitute for other forms of incentive in experiments upon higher mammals.

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SPECIAL ARTICLES

THE ESSENTIAL MEANING OF D'ALEMBERT'S PRINCIPLE

NEWTON'S second law of motion is expressed in the fundamental form, using C.G.S. units,

$$\sum_{0}^{X}(\Delta X)_E = \sum_{0}^{m}(\Delta m\ddot{x}). \qquad (1)$$

The necessary range of the two summations is determined without ambiguity, by the conditions of the problem selected for discussion. The first sum must include every element of external force parallel to a fixed line brought to bear upon any portion of mass within the system, either by a process equivalent to surface distribution at the boundary, or by volume distribution. The second sum covers every part of the system's mass, and no mass external to the system. Equation (1) presents Newton's thought that the physical agencies active (forces) are measurable in terms of one particular result—accelerations produced in masses—other effects, if any, being ignored in the equation. What d'Alembert put into clear relief, when he announced his principle covering "lost forces," is the unimpaired validity of the equality, after eliminating all self-canceling elements from the force-sum. This removes from consideration all inner forces always, and items of external force in certain cases. The second member of the equation then measures the remainder of effective force only, and exhibits the necessary magnitude of the equilibrant that would change the conditions of the problem from those of acceleration to those of equilibrium, or zero acceleration. The "reversed effective force," if superposed upon the forces actually operative, says d'Alembert, would prevent the actual accelerations, and bring about equilibrium that did not in fact occur. This conception of equivalence between the differing modes of statement in the two members of such equations is prominent with d'Alembert and Lagrange, and entirely in accord with out every-day use of equations of motion to evaluate any one of the three quantities force, or mass, or acceleration, when the corresponding values of the two others are known.[1] The advance made by d'Alembert, therefore, is in the direction of devising a static measure for unbalanced forces by generalizing the procedure when we determine weight active by hanging a body from a spring balance. It is parallel to the zero method of the laboratory, that seeks the measure of any unknown quantity in terms of independent conditions adjusted to compensation of its effects. This point of view sets in a proper light the limited sense in which d'Alembert's principle brought dynamics within the scope of statical equations, and disposes effectually of the obscurity or confusion involved in "forces of inertia," or the recently substituted term "kinetic reaction." The extension of d'Alembert's principle to modern generalized dynamics does not modify essentially this conception of the method; we are still dealing with relations between force and inertia—the doing of work, and the quality of storing energy in a particular way. Clear thought in a new field is not furthered by meeting a paradox at its threshold; for nobody accepts literally the dictum that finite acceleration is, as a general statement, consistent with zero values of force, and force-moment, applied to a given system that has inertia.

[1] D'Alembert's "force of inertia" is merely a loose expression for (m); it does not denote $(-m\ddot{x})$. Lagrange uses the phrase "force resulting from inertia" as describing $(m\ddot{x})$, with unchanged sign. See d'Alembert, "Traité de dynamique," ed. 1758, p. x; Lagrange, "Mécanique analytique," ed. 1853, Vol. 1, p. 282.

Equation (1) may be recast mathematically in several ways; and some of its equivalents, being adapted more closely to certain aspects of physical thought, are obviously helpful as well as legitimate. But for clearness the name "equation of motion" shall be confined here to the above primary mode of formulating the idea. This was adopted by the old masters as segregating causes from results, terms of each class appearing by themselves in one member of the equation. We may describe these as "force terms" and "mass-terms" respectively. So soon as homogenousness in this sense is disturbed, the equation is altered in *prima facie* physical meaning. Even removing terms from one member to the other; so that a force-term is now interpretable as a mass-term, or *vice versa;* may be regarded as passing to a new problem, concerned with different masses, or modified forces, or a new classification of the effects of force. Some typical instances are the following, purposely taken on familiar and elementary ground:

1. Denoting by (P) and (R) the aggregates of positive and negative external force, respectively, thought of as acting on a single mass (m), for simplicity, we have the type

$$P = R + m\ddot{x}. \qquad (2)$$

Here the negative forces have been transferred to the second member, and the equation now expresses directly the fact that the forces (P) overcome the resistances (R), and produce acceleration as well. (R) may represent dissipative or conservative agencies. If the latter, equation (2) is preliminary to expressing storage of energy in both forms.

2. Subtracting (R) from both members of equation (2) gives

$$P - R = (R - R) + m\ddot{x}. \qquad (3)$$

This puts to the front the idea that the total force $(P - R)$ sets up static stress $(\pm R)$ to an extent determined by the resistances, the remainder becoming effective as a volume distribution of force producing local acceleration. The connection of equation (3) with the lost forces of d'Alembert is visible at once.

3. Separate the forces to which magnitude may be assigned arbitrarily from those whose magnitudes are fixed by conditions of the system like displacement, velocity, acceleration. Call the former group (A) and the latter (S). Then the form of equation

$$A = S + m\ddot{x} \tag{4}$$

makes the second member a function of elements specified for the system, while the first member is independent of such elements. Such a segregation is convenient for mathematical handling of the differential equation, but (A) and (S) are both external forces, in the original sense of that term. We need, perhaps, to remind ourselves of this fact, when we find (A) alone described as external, in opposition to "forces exerted upon the system by itself," or inner forces.[2]

4. The effects of a force-aggregate (X) being in general to bring about changes of magnitude in some momenta, and of direction in others, that separation of results may be indicated by the notation in both members of the equation of motion, giving

$$X = M + D = m\ddot{x}_M + m\ddot{x}_D. \tag{5}$$

According to that supposition, then,

$$X - D = X - m\ddot{x}_D = m\ddot{x}_M = M. \tag{6}$$

One reading of equation (6) carries out the separation referred to; it measures explicitly the force devoted to producing change of magnitude in momentum. Another legitimate interpretation connects the change in force from (X) to $(X - D)$ with a definite change of reference system. But alongside of these we find surviving still a third, to the effect that (M) is the real force-total in this case (retaining the reference system and mass unchanged), resulting from the combination of (X) with centrifugal force. A similar unclearness allows the "centrifugal couple" of Euler's equations to masquerade as an external force-moment. These forms of confusion are reasonably looked upon as survivals from the days when the process of vector addition to momentum by force was grasped less completely. The changes in direction seemed almost a side issue, to be deducted before proceeding to the serious measurement of force. We still find the thought followed without flinching to the case where (M) happens to be zero, and leaves "equilibrium" between (X) and (D).[3]

The significance of such current forms, which may justify citing them in the present connection, lies in the mingling of force-terms and mass-terms common to them all. This encourages an undiscriminating attitude transferred from the field of mathematics, toward the terms included in equated expressions, which may easily obliterate certain phases of physical thought. To inquire whether a particular distinction of this sort is profitable is one way of exercising discrimination. It is proposed to raise this question presently, as regards mass-term and force-term, especially where those conceptions are employed with the wider meaning of recent usage. We may advance toward that end by considering first the form into which d'Alembert's principle is thrown, in preparation for the equation of virtual moments,

$$\sum_0^X (\Delta X)_E - \sum_0^m (\Delta m\ddot{x}) = 0. \tag{7}$$

How is this to be understood from the physical point of view? If their original meaning is attributed to the summations, and equation (7) is nothing but a transposition of equation (1), the second sum can not represent forces actually applied to (m), since by supposition these are accounted for completely in the first sum. Neither can this be an equilibrium equation for the mass (m), so long as the second sum does not vanish. D'Alembert, however, detected in

$$-\sum_0^m (\Delta m\ddot{x})$$

a new sense, by associating it with the other force-terms as their equilibrant. Or, follow-

[2] See, for instance, Abraham and Föppl, "Elektrizität," Vol. 1, p. 195.

[3] Goodman, "Mechanics," p. 204; cf. Klein und Sommerfeld, "Theorie des Kreisels," p. 141, etc. These instances do not stand alone.

ing a more modern tendency, that sum, again recognized as force, is regarded as due to reactions of (m) upon bodies that transmit force to it. It is clear that neither view preserves the scheme of equation (1); the first uses the real equilibrium condition of equation (7) in order to exhibit the actual departure from that condition in equation (1), and the second includes forces acting, not upon (m) but upon surrounding bodies. Either view is of course tenable, both within the original scope of the principle and in the field of modern dynamics to which it has been extended. But it is only in this peculiar sense that d'Alembert made the criterion of equilibrium a basis for the measurement of unbalanced force.

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SOME APPLE LEAF-SPOT FUNGI[1]

SINCE 1892 leaf-spot disease has been frequently reported as doing considerable damage in apple orchards in various parts of the United States. Its occurrence has been noted in fifteen different states. Very little seems to be known about the etiology of the disease. That it is a fungous trouble is indicated by the ease with which it is controlled in most localities by spraying. *Coniothyrium pirina*[2] (Sacc.) Sheldon, *Phyllosticta limitata*,[3] *Phyllosticta prunicola*,[4] *Sphaeropsis Malorum*[5] and *Hendersonia Mali*[6] have been variously reported as causing, or being associated with, the disease.

The number of fungi found fruiting on the leaf-spots is the most confusing thing in determining the real cause of the disease. In an examination of apple leaf-spot specimens belonging to the West Virginia Agricultural Experiment Station, the following fungi were found: *Coryneum foliicolum, Coniothyrium pirina*, an undetermined species of the Tuberculariae (found by Sheldon in the spring of 1907), *Sphaeropsis Malorum, Monochaetia Mali, Pestalozzia breviseta, Phyllosticta limitata, Torula?* sp., *Macrosporium* sp., *Ascochyta* sp., *Phyllosticta? piriseda?, Phoma Mali, Septoria piricola?, Metasphaeria* sp., and an undetermined species of the Leptostromaceae. Of these fungi, only the first four were common enough to indicate any economic importance. *Coryneum foliicolum* is probably the fungus which has been reported by different writers as a *Hendersonia* on apple leaves. *Coniothyrium pirina* will be better recognized as *Phyllosticta pirina* Sacc., from which it was recently transferred by Sheldon.[7] *Coniothyrium tirolense* Bubàk, a portion of the original collection of which was examined by the writer, seems identical with *C. pirina*. *Phyllosticta Mali* Prill. & Dela. var. *comensis* Tray, was found to resemble *P. limitata* in all characters except the shape of the spot, which in the former is decidedly angular. A part of the type specimen of *P. tirolensis* Bubàk on pear leaves differed from *P. limitata* by the slightly shorter spores and more gregarious pycnidia.

It seems to have been generally taken for granted that *Coniothyrium pirina* and *Phyllosticta limitata* are the most important fungi causing apple leaf-spot, exceptions noticed being the reports of Clinton[5] and Sheldon.[8] *Coniothyrium pirina* has, on the other hand, been declared by Stewart and Eustace[9] to be a saprophyte. A more detailed study of the fungus therefore became desirable.

Pure cultures of it were obtained and grown on the ordinary culture media, with varying success; they were also grown very success-

[1] Read before Section G of the American Association for the Advancement of Science, January 2, 1908.

[2] Alwood, W. B., Va. Agr. Exp. Sta., Bull. 17:62 (1892).

[3] Stewart, F. C., N. Y. Agr. Exp. Sta., Ann. Rep. 14:545 (1895).

[4] Tubeuf, Karl Freiher von, and Smith, W. C., Diseases of Plants induced by Cryptogamic Parasites, 463 (1897).

[5] Clinton, G. P., Conn. Agr. Exp. Sta., Ann. Rep. 27:300 (1903).

[6] Alwood, W. B., Proc. Am. Acad. Adv. Sci., 47:415 (1898).

[7] Sheldon, J. L., Torreya 7:143 (July, 1907).

[8] Sheldon, J. L., W. Va. State Bd. of Agr., Ann. Rep. 1:57 (1906).

[9] Stewart, F. C., and Eustace, H. J., N. Y. Agr. Exp. Sta., Bull. 220:228–230 (1902).