the same parents is due to any of the causes mentioned above.

But variation as between individuals of the same generation may be, to some extent, due to what has been aptly termed "place effect." This is especially true when a species is transported to new environment, a good deal of variation occurring which seems to be due to change in environment, such variation may partake partially of the nature of fluctuating variation, but this is a special case which we do not need to consider here.

This leaves two distinct types of variation due to wholly different causes which have hitherto been more or less confused. By some writers they have both been included under "fluctuating variations." The first of them is well exemplified in a field of corn, where ordinarily hardly any two individuals are alike. It is fully demonstrated by the work of Nilsson, in Europe, and Shull, in this country, which will be referred to below, that by far the greater part of the variation of our corn field is due to the fact that we have in the various individuals almost countless combinations of Mendelian characters and that these combinations change with each new generation. The same thing is true on a smaller scale with all species that cross-fertilize under any conditions, and even with close fertilization this condition exists to a greater or less degree. If we require a corn plant to close fertilize, by this process we permit the formation of a few individuals which are perfectly homozygote; *i. e.*, the inheritance of the individual from the two parents becomes exactly alike. Then if we take the pains to seek out these homozygotes and propagate them, allowing no cross-fertilization, we completely eliminate the type of variation here referred to and get forms that vary only in response to the immediate environment. Variation due to this re-combination of characters with each generation would naturally show correlation between parent and offspring, but when we have eliminated this type of variation such correlation would no longer exist.

Species that never cross-fertilize or that do so very rarely, and plants that are not allowed to do so, have been shown by careful study by Nilsson, Shull, Hopkins and others to consist of mixtures of strains which when separated show no variation except that due to environment, or rather the larger part of the species exhibits this condition, for even in such species there may be a small admixture of heterozygotes.

Nilsson in Europe and Shull in this country have obtained these perfectly homozygote individuals, Nilsson working with wheat and many other species, and Shull with corn. The individuals of a generation are as much alike as identical twins. These are the so-called elementary species, this term being applied through a misconception of their nature. (This subject will be fully discussed in another place.)

Finally we have variation due to the immediate environment of the individual. For instance, a variation in food supply may cause two individuals having identically the same inheritance to differ in size and in other characteristics.

Dr. Shull confines the term "fluctuating variations" wholly to variations of the last-mentioned type. He rigidly excludes variations due to the heterozygote nature of the parents. I fully agree with him in this use of the term. When we so limit it, we may then say that fluctuating variations are not transmitted.

W. J. Spillman
U. S. Department of Agriculture

SPECIAL ARTICLES

THE DERIVATION OF FECHNER'S LAW

About eighty years ago E. H. Weber observed that the least perceptible increment to a stimulus affecting several of the sense organs, under fixed subjective conditions of attention, expectation and fatigue, bore a definite relation to the amount of that stimulus.

G. T. Fechner, thirty years later, extended Weber's observations and formulated his results in mathematical terms. Calling the stimulus L and the least perceptible increment δL, then Fechner's statement of what he termed Weber's Law is that $\delta L/L = \text{constant}$.

This law appears to hold over a wide range of ordinary working intensities, breaking down only for very low and for excessively intense stimulation.

Fechner proceeded further by assuming that the above constant was proportional to the corresponding increment δB to the sensation. Hence $\delta L/L = c\delta B$ and by integration

$$B = c\ (\log L - \log L_0).$$

In this form or in similar forms differing only in the choice of integration constant, Fechner's law has been accepted by psychologists for half a century.

There are two very serious if not fatal defects in this deduction. In the first place, the increments δL and δB are finite quantities and by no means infinitesimal increments approaching zero as a limit, such as would be required for such an integration. The least perceptible increment to the stimulus (δL) is determined by the sensibility of the sensory organ concerned. At the threshold value it is as large as L itself, while at moderate intensities it bears a fixed ratio to L. The value of δB is entirely arbitrary, dependent upon the unit chosen in which to measure it. It may be greater than unity in special cases. In the second place, c is not a constant but a function of L. At low intensities approaching the threshold value it varies rapidly with L.

There appears to be no direct method for overcoming these defects. A method of avoiding them altogether has however occurred to the writer and been applied to the visual case in a way that may be perfectly satisfactory to psychologist and mathematician alike.

Consider any physical instrument—a galvanometer for instance, capable of indicating on a scale the amount of an external stimulus affecting it. The derivative of scale reading with respect to the stimulus will be a measure of the sensibility of the instrument at all parts of the scale. Conversely, the general integral of sensibility will give the scale reading as a function of the stimulus.

In the visual case we have sensibility to find scale reading. The best data on sensibility are those of König and Brodhun[1] covering about twenty different intensities for each of six different wave-lengths. The writer has elsewhere[2] shown that these data may be represented by the function

$$P = \delta L/L = P_m + (1 - P_m)(L_0/L)^n$$

where P_m is the minimum value of P, L_0 is the threshold value in light units and n a number varying from one third to two thirds with wave-length. The reciprocal of the least perceptible increment δL or $1/LP$ is a measure of the desired sensibility of the eye to differences of intensity. Hence we have for the scale reading or, in this case, the visual sensation of brightness,

$$B = \int K \frac{dL}{PL} = \frac{K}{P_m} \log [1 + P_m(L^n L_0^{-n} - 1)]^{1/n}$$

where K is a constant dependent upon the unit of sensibility chosen.

This general form includes Weber's law and Fechner's law as special cases for moderate intensities, but holds for low intensities down to the threshold of vision. Weber's law $\delta L/L =$ constant may be extended to cover low intensities by writing

$$\delta L/L = P_m + (1 - P_m) L^{-n} L_0^n.$$

P. G. NUTTING

BUREAU OF STANDARDS,
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ASTRONOMICAL NOTES

FLUCTUATIONS IN THE SUN'S THERMAL RADIATION[1]

Many scientists have attempted in the past to show that periodical fluctuations occur in meteorological phenomena, presumably dependent on changes in the solar radiations. The two most plausible periods of solar change are the sun-spot period, whose mean value is about eleven years, and the time of the synodic rotation. Professor Newcomb develops analytical methods for the investigation of fluctuations in a fixed period, and also when the

[1] *Berlin Sitz.*, 1888, 917–931.

[2] Bull. Bureau of Standards 3, 62.

[1] Simon Newcomb, "A Search for Fluctuations in the Sun's Thermal Radiation through their Influence on Terrestrial Temperature," Transactions of the American Philosophical Society, N. S., Vol. 21, V.