them, they are not convincing. Thus in the table of data on page 237 relating to oviposition, the maximum, minimum and average for any given species may be made up of more or less complete observations on two beetles or on a thousand; at all events, new observations will change the figures. Data on the oviposition of *L. decem-lineata* which Mr. A. A. Girault is about to publish will change the aspect of this table very materially.

It goes without saying that there is much excellent material in Professor's Tower's work. The observations on habits are most interesting. A point well worth the attention of experimental biologists is that tropical species, being less subject to fluctuating conditions than those of more northerly regions, respond more readily to change of environment.

The work, along with other Carnegie publications, suffers very materially through the absence of an index.

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SPECIAL ARTICLES

AGE OF A COOLING GLOBE IN WHICH THE INITIAL TEMPERATURE INCREASES DIRECTLY AS THE DISTANCE FROM THE SURFACE

KELVIN'S famous and epoch-making paper on the secular cooling of the earth was published in 1862.[1] His problem was to find the time which would elapse before a globe completely solid from center to surface and having throughout a certain uniform initial temperature would cool so far as to reduce the surface gradient of temperature to any given value. He assumed an initial temperature of 3,900° C., a diffusivity of 0.01178 in c.g.s. units and a final surface gradient of 1° C. in 27.76 m. or 1° F. in 50.6 feet. These data discussed by one of Fourier's theorems give for the age of the earth 98×10^6 years. Kelvin, however, expressly directed attention to the fact that the effect of temperature in modifying diffusivities is almost unknown, and that the original distribution of temperature is uncertain. He also referred to the great differences in the surface gradient of temperature, which varies with the locality, as he stated, from 1° F. in 15 feet to 1° F. in 110 feet. He, therefore, allowed very wide limits in his estimate and placed the age between 20 million and 400 million years.

In 1893 Clarence King made a very important contribution to the subject[2] by introducing the criterion of tidal stability. Mr. Barus determined for him the melting point of diabase in terms of depth. If in any hypothetical earth consisting solely of diabase the temperature in any couche were to exceed the melting point of diabase, then tidal instability would set in, the crust would break down and chaos would reign for the time being. In a real earth the same result would follow provided the couche were in a region where diabase or equally fusible rocks are to be expected. Excluding such cases, King found that the age of the earth could not exceed 24 million years when Kelvin's values for diffusivity and surface gradient are assumed. He also found that the corresponding initial temperature of such a globe would be 1,950° C.

Kelvin's last paper on a cooling earth[3] was read in 1897 and he there stated that after having worked out the problem of conduction of heat outwards from the earth by an elaborate method, he was not led to differ much from Clarence King's estimate. This he adopted as the most probable age and reduced his limits to between 20×10^6 and 40×10^6 years.

While King's earth is tidally stable, I confess that his solution of the problem seems to me to be fatally defective. He himself gives a temperature curve for the same earth at an age of 15 million years and this earth shows a couche at a temperature above the melting point of diabase, this layer extending from a depth of 34 miles below the surface to 66 miles. According to Laplace's law of densities these two levels correspond respectively to densities of 2.85 and 2.93, and it seems certain that the material must consist chiefly of basaltic rocks. Thus the 15-million-year

[1] *Trans. R. S. Edinburgh*, reprinted in Thomson & Tait, "Natural Philosophy," Pt. II., p. 468.

[2] *Am. Jour. Sci.*, Vol. 45, 1893, p. 1.

[3] *Trans. Victoria Institute*, Vol. 31, 1899, p. 11.

earth would be unstable and this instability would only just disappear at 24 million years. I am obliged to conclude that if an earth could cool in this way—if the crust could be prevented from breaking—the 24-million-year earth would only just have reached the "consistentior status" or the epoch of solidity.

The real earth, however, has been in a condition of tidal stability at least since the beginning of the Cambrian. For the strata are full of ripple marks, sands and pebbles rearranged by tidal currents, beach footprints and similar evidence of tides. Now oceanic tides would not exist upon a tidally unstable earth and therefore the consistentior status occurred long ago. It was the remoteness of this epoch which Kelvin attempted to calculate.

King gives data for only one earth which is satisfactory from this point of view. It had an initial temperature of 1,230° C. and reached a surface gradient of 1° F. in 50.6 feet in 10 million years. It was solid almost from the beginning. But apart from the excessive brevity of the age, it seems to me that this earth must likewise be rejected. The temperature was insufficient to melt even diabase a few miles below the surface, much less andesites and rhyolites, while there is a mass of well-known evidence that the earth has been fluid at least to depths of many miles from its growing surface. This is shown by the general dependence of gravity on latitude, the nearly spheroidal shape of the earth, the oblateness of the interior layers of equal density and the fact demonstrated by Kelvin,[4] Roche[5] and Wiechert[6] that a nucleus of constant high density (approximately the density of iron) surrounded by a shell of much smaller density (near 3) will satisfy the observations on precession, ratio of surface density to mean density and the ellipticity of sea level.

Considering the materials of which the earth is composed and the high pressures which must have existed at some distance from the surface at any stage of the earth's growth, it seems clear that very high temperatures must have prevailed within its mass, while for the reasons stated above tidal instability at any epoch since the ocean came into existence, is inadmissible. Hence the hypothesis of a constant initial temperature will not satisfy the conditions.

The question thus arises whether the initial temperature may be supposed to have been graduated in such a manner as to satisfy known conditions. I believe that this question may be answered affirmatively. Our great master in geophysics himself contemplated a very different distribution of temperature from the uniformity assumed in his equations. The earth, he said, "did in all probability become solid at its melting temperature all through or all through the outer layer"; "convective equilibrium of temperature must have been approximately fulfilled until solidification commenced" and "the temperature of solidification will, at great depths, because of the great pressure there, be higher than at the surface if the fluid contracts . . . in becoming solid."

If the initial temperature at the consistentior status increased with distance from the surface, it was probably according to some complex law, intimately related to that of convective equilibrium, but the thickness of the shell which has been sensibly affected by cooling is very small. At a distance of 80 miles below the surface the temperature is probably now very near 99 per cent. of what it was at the consistentior status. Hence if a layer double this thickness is considered, the conditions which prevailed in the remainder of the earth are of no consequence. The inner part, with a radius of say 3,840 miles, may have been originally at the temperature of ice or of the electric arc; it may conduct heat as well as silver or as ill as magnesia; in any case the influence on the outer surface would be insensible even after scores of millions of years. Now, though the temperature at the consistentior status did vary with distance from the surface according to a highly complex law, it is altogether probable that for so short a distance as 2 per cent.

[4] "Natural Philosophy," Pt. II., p. 420. This article also appeared in the first edition of the "Natural Philosophy," 1867.

[5] *Mém. Acad. Montpellier*, 1882.

[6] *Göttingen Nachrichten*, 1897, p. 221.

of the radius this law may be adequately represented by a straight line, the chord of an arc whose curvature is small. It would be comparable with, though not identical with, the superficial portion of Mr. Barus's nearly rectilinear curve representing the melting point of diabase as a function of depth. Hence it will be sufficient to assume that the initial temperature increased in simple proportion to distance from the surface.

It is easy to modify the Fourier equation employed by Kelvin to meet this condition. This equation is, strictly speaking, that of an infinite solid divided by a plane, on one side of which, at the initial instant, the temperature has one uniform value, while on the other side it has another uniform value. In other words, in Kelvin's problem the curvature of the earth is neglected because the phenomena are so superficial.

The equation used by Kelvin of course satisfies Fourier's law of the conduction of heat, viz.,

$$\frac{dv}{dt} = \kappa \frac{d^2v}{dx^2},$$

where v is temperature, t time, x distance from the dividing plane and κ diffusivity assumed to be constant. It follows that

$$\kappa \frac{d^2v}{dx^2} = -\frac{xV\varepsilon^{-x^2/4\kappa t}}{2t\sqrt{\pi\kappa t}},$$

and this integrated once gives

$$\frac{dv}{dx} = \frac{V}{\sqrt{\pi\kappa t}} \cdot \varepsilon^{-x^2/4\kappa t} + c. \qquad (1)$$

Here V is half the difference of the two initial temperatures at an infinitesimal distance from the dividing plane and c is a constant temperature gradient. In Kelvin's solution c is zero and the temperature on each side of the divisional plane is uniform. A second integration gives

$$v - v_0 = V \cdot \frac{2}{\sqrt{\pi}} \int_0^{x/2\sqrt{\kappa t}} \varepsilon^{-z^2} dz + cx. \qquad (2)$$

When $t = 0$, x being positive

$$v - v_0 = V + cx, \qquad (3)$$

while for negative x

$$v - v_0 = -V - cx.$$

This last equation represents the initial distribution of temperature in the hypothetical solid replacing outer space in the problem of a cooling earth. In these equations v_0 is the temperature in the dividing plane itself while V is the temperature at an infinitesimal distance from the plane at the initial instant. It is convenient to write $v - v_0 = E$ so that E is the excess of temperature of any point in the solid over the temperature in the limiting plane. For the present problem then

$$E = V + cx$$

represents the initial distribution of temperature in the earth.

If appropriate values of the constants can be found, equations (1) and (2) can be computed for any desired age and this computation is an easy task because the value of the definite integral in (2) has been tabulated by various mathematicians, the most complete table being by Mr. James Burgess and printed in 1900.[7]

Kelvin employed a diffusivity, κ, of 400, using the British foot and the year as units. In c.g.s. units this would be 0.01178. This value was obtained from experiments on the trap rock of Calton Hill, the sand of an experimental garden and the sandstone of Craigleith quarry, all at Edinburgh. Different weights were given to these observations, but how is not explained. Now, in considering the diffusivity of the earth it does not seem to me that the ragged pellicle of detrital matter on its surface need be considered. Over large areas it is absent and in most places the sedimentary rocks are saturated with water, so that their own intrinsic diffusivity is a minor feature of the flow of heat. The great bulk of the rocks in the shell affected by cooling are massive and at least comparable with the trap of Calton Hill, which is chiefly composed of Carboniferous basalt and andesite. The conductivity of this rock was observed by Forbes and Thomson (Kelvin) for no less than eighteen years, the thermal capacity was determined by Regnault, so that the value of the diffusivity, 0.00786, is undoubtedly very accurate. It does not stand alone. A com-

[7] *Trans. R. S. Edinburgh*, Vol. 39, 1900, p. 257.

mittee of the British Association,[8] Herschel and Lebour, reported for whin and traps $\kappa = .0067$, and for serpentine from .00594 to .0073, while Ayrton and Perry got for porphyritic trachyte .0103. I do not think a better choice can be made than the Calton Hill trap, and its diffusivity with the meter and year as units is the value which will be assumed here, *i. e.*, $\kappa = 24.8037$.

That κ varies with temperature and with pressure is probable. That in iron it decreases with increasing temperature is known and analogy would point to the conclusion that it should increase with pressure. Possibly diffusivity is simply related to density and for the same or similar rocks tends in the earth to a nearly constant value. At present it seems unavoidable in this problem to regard it as constant.

The outer portion of the earth is composed of various rocks which are believed to be arranged roughly in the order of density. If so the peridotites underlie the basaltic rocks, while the andesites and rhyolites overlie them. These latter are less fusible than diabase. How deep the level lies which would answer to the upper surface of the basaltic rocks can not be told with certainty. The best that can be done is to assume that Laplace's law of density is valid for a few score miles from the surface and to consider roughly the effects of heat and pressure. In this way I have reached the conclusion that at about 40 miles, or 0.01 times the radius, where the density should be 2.86, the temperature perhaps 1,300° C. and the pressure 17,400 atmospheres, basaltic rocks may begin to appear in place. A pressure of 13 or 14 atmospheres per degree centigrade is probably of the order of magnitude needful to preserve constancy of volume in a heated solid, while at atmospheric pressures the densities of basaltic rocks are from 2.85 to 3.10, with minor exceptions. I shall assume, therefore, that the outer crust to a depth of 40 miles is less fusible than basalt.

The line representing the melting point of diabase in terms of depth as determined by Mr. Barus may be taken as rectilinear for depths up to a hundred miles and is then represented by what I may call the diabase line,

$$y = 1170^\circ + \frac{430}{.01r}x,$$

where r is the radius of the earth, and according to the results of the last paragraph the original temperature distribution in the globe must be such that only the layer of rock within 40 miles of the surface was heated to a higher point than that at which diabase would melt. Thus V being the original surface temperature and u the original temperature at distance x,

$$u = V + \frac{1600 - V}{.01r}x,$$

and this line, intersecting the diabase line at $.01r$ or 63,710 meters, must be the asymptote of the temperature excess curve.

It is easy to perceive that whatever values of the constants and the age are chosen, the temperature curve will have one and only one tangent which is parallel to the diabase line. Of course the point of tangency is that at which the curve approaches the melting point of diabase most closely or at which the additional temperature which would be required to melt diabase is a minimum. It is at this level of tangency that any access of temperature due to the dissipation of mechanical energy or to other causes is most likely to produce fusion at depths where the rock is diabasic. If the constants are assumed at any value and the courses of the curves are considered for various periods of time, it is easily seen that the point of nearest approach to the diabase line sinks to greater depths as time elapses.

Now, strains must exist in the earth at all times. They may be and are partially relieved by rupture and by solid flow, but most completely by fusion. Thus in an earth the cooling of which is represented by (2) such strains as may be incident to upheaval and subsidence and to orogeny will probably be most completely relieved at the slowly sinking surface of easiest fusion.

Messrs. Tittmann and Hayford have recently discussed the whole body of geodetic data for the United States and have shown that the deflections of the vertical are best

[8] Brit. Assoc. Ad. Sci., 1881.

accounted for by the hypothesis that isostatic compensation is uniformly distributed and is complete at a depth of 140 kilometers or 71 miles from the surface.[9]

I, therefore, adopt the hypothesis that the tangent of the temperature curve, or equation (1), is parallel to the diabase line at 140 kilometers from the surface.

V is the value of the original temperature excess of the earth at its surface over the temperature of the atmosphere in contact with it. As was pointed out above, this must have been high enough to fuse rocks more refractory than diabase and was probably about equal to the temperature of the hottest eruptions which now reach the surface of the earth. It seems to me that 1,300° is a reasonable estimate. This is considerably below the melting point of pure iron and lower than the blast furnace, but above the melting point of copper (1,065°), which lavas are known to fuse, and of Barus's diabase (1,170°). So far as I know, no precise determinations have yet been made of the temperatures at which lavas issue from their vents, though the new optical method should make good observations possible.

To take advantage of the level of isostatic compensation x in equation (1) may be put at 140,000 meters, and dv/dx at the gradient of the diabase line, or $430°/.01r$. Then with $\kappa = .00786$ and

$$c = \frac{1600° - V}{.01r} = \frac{1600° - V}{63710}$$

it follows that

$$\frac{1}{V} = \frac{1}{1170°}\left\{1 - \frac{7217.2}{\sqrt{t}\,\varepsilon^{\frac{130.99\times10^6}{t}}}\right\}.$$

Although V should be about 1,300° and t might be computed as dependent variable, the form of this expression makes it easiest to assume values of t and then compute corresponding values of V and c. When these are known for any given age the corresponding value of the surface temperature gradient is

$$\left(\frac{dv}{dx}\right)_0 = \frac{V}{\sqrt{\pi\kappa t}} + c.$$

[9] Rep. to 15th general conference of the International Geodetic Assoc., Washington, 1906.

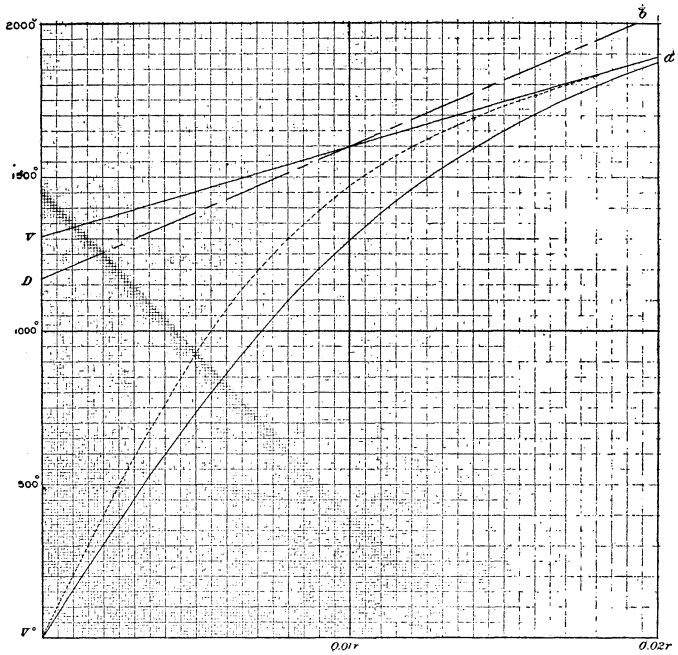
Carrying out this process I get the following table of related values:

A	30.	50.	55.	60.	65.	100.
V	1190.°	1264.	1286.	1307.	1329.	1453.
c	0.00644	0.00527	0.00493	0.00459	0.00426	0.00231
$1/c$	155.^{m}	190.	203.	218.	235.	433.
G°C.	32.^{m}2	39.2	40.7	42.2	43.6	53.3
G°F.	58 ′7	71.4	74.2	76.9	79.5	97.1

A is the age in millions of years; V is the initial surface temperature; c is the initial gradient of internal temperature and $1/c$ gives this gradient in terms of meters per degree centigrade. G° C. is the final surface gradient in terms of meters per degree centigrade and G° F. is the same gradient in terms of feet per degree Fahrenheit.

In all of these earths the upper surface of the diabase couche is supposed to be at one one-hundredth of the radius from the surface, or 63,710 meters. All of the excess of temperature curves have tangents parallel to the diabase line at a depth of 140,000 meters.

Of the six earths computed the one whose initial temperature comes nearest to 1,300° C. is that of the 60-million-year earth, and it is the one which appears to me most probable. The most evident objection to it is the low surface gradient of 1° F. in 77 feet, while Kelvin took 1° F. in 50.6 feet and King stated that in 1893 the last published value as reduced from all available data by the British Association committee is 64 feet per degree Fahrenheit. King himself considered 75 feet a maximum. To me, however, it does not seem that an average value is what is required. In discussing the cooling of the earth disturbing causes must be eliminated as far as possible. Now several causes must contribute more or less to raise the temperature of rocks near the surface; for example, thermal springs, volcanic heat, the dissipation of mechanical energy by faulting or solid flow, the liberation of heat in the decomposition of minerals and radioactivity. So far as I know, the only cause which can lead to a deceptively low gradient in rocks of a given type is abnormally high diffusivity. Furthermore, to include gradients observed in sedimentary rocks seems to me to complicate the problem unnecessarily. The gradients which should



serve as a guide are those in massive rocks, especially the nearly anhydrous basaltic, andesitic and rhyolitic massives. All cases where there are local evidences of heat due to thermal springs, etc., should be excluded, and when for a normal rock the gradient is unusually high, it should be considered as suspicious. In short, for the present problem the lower gradients in massive rocks are those most likely to give a correct value of the earth's age. So far as I can judge, the gradient of 1° F. in 77 feet is not much, if at all, too low from this point of view.

The accompanying diagram represents the temperature excess curve for the 60-million-year earth as a full line. It is asymptotic to the line *Va* and involves no tidal instability. A dotted curve in the diagram shows the temperature of the same earth when 30 million years of age. At that time the level of easiest fusion, or the eutectic level, was much nearer the surface than 140 kilometers in fact at about 86 kilometers, and the increment of temperature needful to produce fusion at the eutectic level was smaller, only some 80° instead of 140° C. At no stage of the life of this earth was there tidal instability. Only in the earliest stages did the curve cross the diabase line *Db*, and that only at less than 40 miles from the surface, where by hypothesis the rocks are andesites or rhyolites and less fusible than diabase. On the other hand, the temperature of the globe at great depths is high, 2,000° being reached within a hundred miles of the surface.

Perhaps the least satisfactory of the assumptions made in this discussion is that the layer of rocks less fusible than diabase is 40 miles in thickness. To obtain an idea of the importance of an error in this assumption I have computed the gradient, assuming the refractory layer to be only 30 miles thick in the 60×10^6 year earth, the other data remaining unchanged. This calculation gives 1° F. in 79.2 feet, so that the effect of even a very large error in estimating the thickness of the refractory layer is not great.

In the course of time it should be possible to obtain better values of the constants than I have employed. I urge a careful revision of surface temperature gradients in the sense of the remarks in a preceding paragraph, accurate determinations of the temperature of lava as it flows from the vents, and above all the study of the thermal diffusivity of massive rocks. Mr. Barus's investigation of diabase was most fruitful pioneer work and afforded the starting point for improvements which ought now to be applied to a revision of his results.

Notwithstanding the inadequacy of the data, I can not but believe that the 60-million-year earth here discussed is a fair approximation to the truth and that with better data this age will not be changed by more than perhaps 5 million years. It is in good accord with geological estimates from denudation and sedimentation, with the age of the ocean as inferred from the sodium content and with the age of the moon as computed by Sir Geo. Darwin. Finally, as I shall show elsewhere, it indicates that the part played by radioactivity in the heating of the earth is a subordinate one.

GEORGE F. BECKER
U. S. GEOLOGICAL SURVEY,
WASHINGTON, D. C.,
January, 1908

QUOTATIONS

THE GREAT BEQUEST TO TRINITY COLLEGE

BY the death without issue of Lady Pearce, who survived her husband, the late Sir William George Pearce, by less than two months, Trinity College, Cambridge, becomes immediately entitled, as we have recently recorded, to the large property in which she had a life interest. It does not often happen that the way is cleared for the owner of the remainder interest with such dramatic rapidity as in this instance. Nor does it often happen that so substantial a sum comes into the hands of any college or educational institution in this country. The total value of this bequest to Trinity College is probably considerably more than £400,000, but taken only at that figure the benefaction is an extremely handsome one. Trinity, as the most distinguished college in the two universities, is in every way worthy of this piece of good fortune, though there are doubtless many less prosperous colleges that may be pardoned for regarding it with somewhat envious eyes, and for quoting the hard saying, "to him that hath shall be given." Trinity will undoubtedly know how to make good use of the money for educational purposes; still, it may be regretted that, in view of the poverty of the university as distinguished from the colleges, some part at least of this large sum was not placed at its disposal. There are statutory provisions in force, both at Oxford and at Cambridge, whereby each college contributes a certain portion of its revenues either to some specific purpose, such as the payment of a professor's stipend, or to a common university fund to be applied to university purposes in general. Under these provisions, the university will, we presume, take its appointed toll of the Pearce benefaction to Trinity. But it is not otherwise a beneficiary. Yet in 1896 the total revenue of the university amounted to only £62,000 odd—only £844 of which was not specially appropriated—whereas in 1907 the gross annual revenue of Trinity was over £76,000.

A few comparative figures will serve to emphasize this contrast. Trinity already possesses the largest revenues of any college in the university, its gross income amounting, as stated above, to over £76,000 out of an aggregate total of £316,000 odd enjoyed by all the colleges. No doubt its outgoings and responsibilities are proportionate to this large income; but the new benefaction, probably amounting, as we have said, to considerably more than £400,000, may perhaps be taken as equivalent to an additional net income of £15,000 annually. There are no fewer than eleven out of the seventeen colleges at Cambridge of which the respective gross annual incomes amount to less than this, ranging from Magdalene with only £4,782 a year to Christ's with £14,371 a year; while a twelfth, Clare, only just exceeds it, having a gross annual income of £15,104. These figures are not cited invidiously. Their sole purpose is to show that Trinity is now about to enjoy an additional income, free of all charges, which is more than equivalent to the gross annual incomes respectively enjoyed by more than two thirds of the colleges at Cambridge. On the other hand, it is certain that at this juncture the needs and deserts of the university as