venience of those who have to read their descriptions. This recommendation will be of great aid in identifying species and genera and will help towards that end when men will see there is honor in furnishing good diagnoses, but no honor in simply naming species.

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THE UNIVERSITY OF TEXAS,
June 4, 1907

ANOTHER WORD ON THE VULTUR CASE

MY brief allusion to Dr. Allen's inconsistency in his latest elimination of *Vultur* seems to have been clear to all with whom I have discussed the question except Dr. Allen, who fails entirely to see my point.

It seems necessary, therefore, to restate the matter. The case is as follows:

Sarcorhamphus 1806.
gryphus.
papa = type of Gypagus 1816.
auricularis = type of Torgos 1828.
Cathartes 1811.
papa = type of Gypagus 1816.
aura.
Gypagus 1816.
papa.
gryphus = type of Gryphus 1854.

Dr. Allen says that while *gryphus* is the type of *Sarcorhamphus* it was not the type in 1806 and only became so in 1828 by the removal of the other species. Therefore, he claims that in eliminating *Vultur* we have no right to remove *gryphus* at 1806 and can only remove it at the date at which *it became* the type of *Sarcorhamphus*.

This is absolutely contradictory to his own practise in all other cases, nor can I find a precedent in the "current usage" of other eliminators. For instance, *papa* is the type of *Gypagus* 1816, but it was not the type in 1816, and only became such in 1854; and yet Dr. Allen in all his eliminations removes *papa* at 1816, which any one can see is the date of establishment of the genus, not the date at which *papa* became its type. To be consistent *gryphus* must, of course, be removed at 1806, as I stated previously. Dr. Allen's recent note in which he repeats that *papa* must be removed at the date at which its genus was established, while *gryphus* must be removed at the date it became the type of its genus, only emphasizes his inconsistency—an inconsistency which is too self-evident to require the employment of any "imagination."

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May 24, 1907

SPECIAL ARTICLES

RELATION BETWEEN BIRTH RATES AND DEATH RATES

A SHORT notice appeared on page 641 of SCIENCE, 1907, of a paper read by C. E. Woodruff before the American Association for the Advancement of Science, on the relation between birth rates and death rates, etc.

In this connection, it may be of interest to note that a mathematical expression can be obtained for the relation between the birth rate per head b and the death rate per head d, for the case where the general conditions in the community are constant, and the influence of emigration and immigration is negligible.

Comparison with some figures taken from actual observation shows that these at times approach very nearly the relation deduced on the assumptions indicated above.

I give here the development of the formula, and some figures obtained by calculation by its aid, together with the observed values, for comparison.

Let $c(a)$ be such a coefficient that out of the total number N_t of individuals in the community at time t, the number whose age lies between the values a and $(a+da)$ is given by $N_t c(a)da$.

Now the $N_t c(a)da$ individuals whose age at time t lies between the values a and $(a+da)$, are the survivors of the individuals born in time da at time $(t-a)$.

If we denote by $B_{(t-a)}$ the total birth rate at time $(t-a)$, and by $p(a)$ the probability at its birth, that any individual will reach age a, then the number of the above-mentioned survivors is evidently $B_{(t-a)}p(a)da$.

Hence:

$$N_t c(a)da = B_{(t-a)}p(a)da$$

$$c(a) = \frac{B_{(t-a)}}{N_t} p(a)$$

Now if general conditions in the community are constant, $c(a)$ will tend to assume a fixed form. A little reflection shows that then both N and B will increase in geometric progression with time,[1] at the same rate $r = (b - d)$. We may, therefore, write:

$$B_{(t-a)} = B_t e^{-ra}$$

$$c(a) = \frac{B_t}{N_t} e^{-ra} p(a)$$

$$= be^{-ra} p(a) \qquad (1)$$

Now from the nature of the coefficient $c(a)$ it follows that

$$\int_0^\infty c(a)\, da = 1$$

Substituting this in (1) we have:

$$\frac{1}{b} = \int_0^\infty e^{-ra} p(a)\, da \qquad (2)$$

Equation (1) then gives the fixed age-distribution, while equation (2) (which may be expanded into a series if desired), gives the relation between b, the birth rate per head, and r, the rate of natural increase per head, and hence between b and d, since $r = b - d$.

Applying these formulæ to material furnished by the Reports of the Registrar-General of Births, etc., in England and Wales, the following results were obtained:

ENGLAND AND WALES 1871–80 (MEAN)

		Observed[2]	Calculated
Birth-rate per head	b	.03546	.0352
Death-rate per head	d	.02139	.0211
Excess ..... $(b - d) = r$		.01407	(.0141)

$p(a)$ from Supplement to 45th Ann. Rep. Reg. Gen. Births, etc., England and Wales, pp. vii and viii, assuming ratio:

$$\frac{\text{male births}}{\text{female births}} = 1.04.$$

[1] Compare M. Block, "Traité théorique et pratique de statistique," 1886, p. 209.

[2] Mean b and d from 46th Ann. Rep. Reg. Gen. Births, etc., England and Wales, p. xxxi.

Age Scale.—1,000 individuals, in age-groups of 5 and 10 years

$a_1 a_2$		$1000 \int_{a_1}^{a_2} c(a)\, da$	
0 – 5		136	138
5 – 10		120	116
10 – 15		107	106
15 – 20		97	97
20 – 25		89	87
25 – 35		147	148
35 – 45		113	116
45 – 55		86	87
55 – 65		59	59
65 – 75		33	33
75 – ∞		13	13

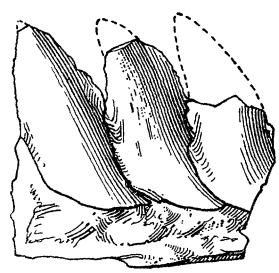
It will be seen that in the above example the values calculated for the age-scale and especially for b and d, show a good agreement with the observed values.[3]

The above development admits of further extension. But this, as well as further numerical tests, must be reserved for a future occasion. In view of the recent note of the work by Major Woodruff, it appeared desirable to the writer to publish this preliminary note.

ALFRED J. LOTKA

A NEW GENUS AND SPECIES OF FOSSIL SHARK RELATED TO EDESTUS LEIDY

THE specimen which serves as the type of the new genus and species, *Lissoprion ferrieri*, was secured in what are regarded as Permo-Carboniferous deposits near Montpelier, Bear



Lake County, Idaho. It was collected by Mr. W. F. Ferrier, of the town mentioned. The specific name is given in his honor. The

[3] The calculation is based on the observed value of $r = .0141$, as indicated by the brackets.