non is more extensively and clearly exhibited than in *Viola*. EZRA BRAINERD

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FORMULAS FOR THE COMPARISON OF ASTRONOMICAL PHOTOGRAPHS

THE present paper contains formulas suitable for the direct comparison of rectangular coordinates measured on different astronomical negatives. The problem here involved supplements what may be called the fundamental transformations in the reduction of celestial photographs; *viz.*, the calculation of right-ascensions and declinations from rectangular coordinates, and rectangular coordinates from right-ascensions and declinations. The writer has published formulas for all these transformations in 'Tables for the Reduction of Astronomical Photographs,' Contrib. Obs. Col. Univ., No. 23. In these formulas the problem is solved by expansion into series, taking advantage of the fact that the photographs under consideration cover but a very small part of the sky, so that measured coordinates may be regarded as small quantities.

It is sometimes desirable to compare rectangular coordinates of the same stars measured on two different overlapping photographs without computing right ascensions and declinations. For instance, Donner used this method for strengthening his determination of plate-constants in his reduction of the astrophotographic catalogue plates ('Sur le Rattachement des clichés astrophotographiques,' *Acta Soc. Sci. Fenn.*, Tom XXI., No. 8). Another important application will doubtless occur in the calculation of the solar parallax from Eros observations by the diurnal method.

For these reasons, the writer has thought it desirable to expand directly the x and y of a star on one plate in terms of its x and y on a second plate. The resulting series, though clumsy in appearance, are rapidly convergent, and in most practical cases, convenient in use. As here given, all terms to the fifth order, inclusive, have been retained; but a table is attached to the formulas showing the declination at which any term may be omitted in actual applications of the method. When this declination is greater than 75°, the table contains the number 75+. Inasmuch as we require a precision of 0''.01 up to 75° declination, the table has been arranged so as to exclude only terms less than 0''.005.

To obtain the desired expansions, we let:

$x_1, y_1,$ be the coordinates of a star on a correctly oriented plate whose center corresponds to the right-ascension α_1 and declination δ_1 on the sky.

$x_2, y_2,$ be the coordinates of the same star on a second correctly oriented plate whose center corresponds to the right-ascension α_2 and declination δ_2 on the sky.

$M_1, M_2, \ldots N_1, N_2, \ldots$ be certain auxiliary quantities, constant for all stars on a given pair of plates.

If we now put:

$$d\alpha = \alpha_1 - \alpha_2, \quad d\delta = \delta_1 - \delta_2, \quad \delta = \tfrac{1}{2}(\delta_1 + \delta_2),$$

we can express $x_2, y_2,$ in terms of $x_1, y_1,$ as follows:

$$(1)\ \begin{cases} x_2 = x_1 + M_1 + M_2x_1 + M_3y_1 + M_4x_1^2 + M_5x_1y_1 \\ \qquad + M_6y_1^2 + M_7x_1^3 + M_8x_1^2y_1 + M_9x_1y_1^2, \\ y_2 = y_1 + N_1 + N_2x_1 + N_3y_1 + N_4x_1^2 + N_5x_1y_1 \\ \qquad + N_6y_1^2 + N_7x_1^2y_1 + N_8x_1y_1^2 + N_9y_1^3. \end{cases}$$

Expressions for the M's and N's, with the table mentioned above, are given at the end of the present paper. The writer is under special obligations to Mr. G. W. Hartwell, assistant in mathematics, Columbia University, for help in this part of the work. Demonstrations are omitted here, because the formulas can be verified satisfactorily by means of a numerical example, such as the following particularly unfavorable one. Let us assume two plates and an imaginary star such that:

$$\alpha_1 = 0°\ 0'\ 0''.00, \qquad \alpha_2 = 2°\ 0'\ 0''.00,$$
$$\delta_1 = 74°\ 0'\ 0''.00, \qquad \delta_2 = 75°\ 0'\ 0''.00,$$
$$x_1 = +\ 3600'', \qquad \delta = 74°\ 30'\ 0''.00.$$
$$y_1 = +\ 3600'',$$

The right ascension and declination of the imaginary star, which we will call A and D, can then be computed readily from $x_1, y_1, \alpha_1, \delta_1,$ by means of our former series published in Contrib. Obs. Col. Univ., No. 23.

VALUES OF M'S AND N'S, WITH LIMITING DECLINATIONS.

Decl. at which Term can amount to 0″.005

	$x_1=30'$, $y_1=30'$			$x_1=1^\circ$, $y_1=1^\circ$		
	$da\cos\delta=10'$ $d\delta=10'$	30′ 30′	1° 1°	10′ 10′	30′ 30′	1° 1°
$M_1=+da\cos\delta$	0°.0	0°.0	0°.0	0°.0	0°.0	0°.0
$-1/2\,da\cos\delta\,d\delta\,\delta\tan\sin 1''$	0.3	0.0	0.0	0.3	0.0	0.0
$+3/8\,da\cos\delta\,d\delta^2\sin^2 1''$	75+	0.0	0.0	75+	0.0	0.0
$-1/6\,da^3\cos^3\delta\,(\tan^2\delta-2)\sin^2 1''$	70.4	0.0	0.0	70.4	0.0	0.0
$-11/48\,da\cos\delta\,d\delta^3\tan\delta\sin^3 1''$	75+	75+	48.7	75+	75+	48.7
$+1/12\,da^3\cos^3\delta\,d\delta\tan\delta\,(\tan^2\delta-2)\sin^3 1''$	75+	75+	62.3	75+	75+	62.3
$+19/128\,da\cos\delta\,d\delta^4\sin^4 1''$	75+	75+	75+	75+	75+	75+
$-1/16\,da^3\cos^3\delta\,d\delta^2\,(3\tan^2\delta-4)\sin^4 1''$	75+	75+	75+	75+	75+	75+
$+1/120\,da^5\cos^5\delta\,(16-13\tan^2\delta+\tan^4\delta)\sin^4 1''$	75+	75+	75+	75+	75+	75+
$M_2=+1/2\,d\delta^2\sin^2 1''$	0.0	0.0	0.0	0.0	0.0	0.0
$-1/2\,da^2\cos^2\delta\,(\tan^2\delta-2)\sin^2 1''$	0.0	0.0	0.0	0.0	0.0	0.0
$-1/8\,da^2\cos^2\delta\,d\delta^2\,(5\tan^2\delta-7)\sin^4 1''$	75+	75+	75+	75+	75+	75+
$+1/24\,da^4\cos^4\delta\,(\tan^4\delta-13\tan^2\delta+16)\sin^4 1''$	75+	75+	75+	75+	75+	75+
$+5/24\,d\delta^4\sin^4 1''$	75+	75+	75+	75+	75+	75+
$M_3=-da\cos\delta\tan\delta\sin 1''$	0.1	0.0	0.0	0.0	0.0	0.0
$+1/2\,da\cos\delta\,d\delta\sin^2 1''$	0.0	0.0	0.0	0.0	0.0	0.0
$-7/8\,da\cos\delta\,d\delta^2\tan\delta\sin^3 1''$	75+	75+	30.8	75+	67.3	16.6
$+1/6\,da^3\cos^3\delta\tan\delta\,(\tan^2\delta-5)\sin^3 1''$	75+	74.0	34.7	75+	71.7	17.7
$-1/12\,da^3\cos^3\delta\,d\delta\,(\tan^2\delta-5)\sin^4 1''$	75+	75+	75+	75+	75+	75+
$+23/48\,da\cos\delta\,d\delta^3\sin^4 1''$	75+	75+	75+	75+	75+	75+
$M_4=da\cos\delta\sin^2 1''$	0.0	0.0	0.0	0.0	0.0	0.0
$+1/2\,da\cos\delta\,d\delta\tan\delta\sin^3 1''$	75+	75+	64.4	75+	64.4	27.6
$+7/8\,da\cos\delta\,d\delta^2\sin^4 1''$	75+	75+	75+	75+	75+	75+
$-2/3\,da^3\cos^3\delta\,(\tan^2\delta-2)\sin^4 1''$	75+	75+	75+	75+	75+	75+
$M_5=d\delta\sin^2 1''$	0.0	0.0	0.0	0.0	0.0	0.0
$-3/2\,da^2\cos^2\delta\tan\delta\sin^3 1''$	75+	70.3	34.9	75+	34.9	9.9
$+5/6\,d\delta^3\sin^4 1''$	75+	75+	75+	75+	75+	75+
$-1/4\,da^2\cos^2\delta\,d\delta\,(5\tan^2\delta-7)\sin^4 1''$	75+	75+	75+	75+	75+	74.7
$M_6=-da\cos\delta\,d\delta\tan\delta\sin^3 1''$	75+	75+	46.3	75+	46.3	14.6
$+1/2\,da\cos\delta\,d\delta^2\sin^4 1''$	75+	75+	75+	75+	75+	75+
$+1/2\,da^3\cos^3\delta\tan^2\delta\sin^4 1''$	75+	75+	75+	75+	75+	75+
$M_7=1/6\,da^2\cos^2\delta\,(\sec^4\delta+6)\sin^4 1''$	75+	75+	75+	75+	75+	70.7
$M_8=2\,da\cos\delta\,d\delta\sin^4 1''$	75+	75+	75+	75+	75+	75+
$M_9=d\delta^2\sin^4 1''$	75+	75+	75+	75+	75	75+
$N_1=d\delta$	0.0	0.0	0.0	0.0	0.0	0.0
$+1/2\,da^2\cos^2\delta\tan\delta\sin 1''$	0.3	0.0	0.0	0.3	0.0	0.0
$-1/4\,da^2\cos^2\delta\,d\delta\,(\tan^2\delta-1)\sin^2 1''$	65.8	0.0	0.0	65.8	0.0	0.0
$+1/3\,d\delta^3\sin^2 1''$	75+	0.0	0.0	75+	0.0	0.0
$+1/4\,da^2\cos^2\delta\,d\delta^2\tan\delta\sin^3 1''$	75+	75+	46.3	75+	75+	46.3
$-1/24\,da^4\cos^4\delta\tan\delta\,(\tan^2\delta-5)\sin^3 1''$	75+	75+	69.7	75+	75+	69.7
$+1/48\,da^4\cos^4\delta\,d\delta\,(\tan^4\delta-6\tan^2\delta+5)\sin^4 1''$	75+	75+	75+	75+	75+	75+
$+2/15\,d\delta^5\sin^4 1''$	75+	75+	75+	75+	75+	75+
$N_2=da\cos\delta\tan\delta\sin 1''$	0.1	0.0	0.0	0.0	0.0	0.0
$+1/2\,da\cos\delta\,d\delta\sin^2 1''$	0.0	0.0	0.0	0.0	0.0	0.0
$+7/8\,da\cos\delta\,d\delta^2\tan\delta\sin^3 1''$	75+	75+	30.8	75+	67.3	16.6
$-1/6\,da^3\cos^3\delta\tan\delta\,(\tan^2\delta-5)\sin^3 1''$	75+	74.0	34.7	75+	71.8	17.9
$-1/12\,da^3\cos^3\delta\,d\delta\,(\tan^2\delta-5)\sin^4 1''$	75+	75+	75+	75+	75+	75+
$+23/48\,da\cos\delta\,d\delta^3\sin^4 1''$	75+	75+	75+	75+	75+	75+

VALUES OF M'S AND N'S, WITH LIMITING DECLINATIONS.

	Decl. at which Term can amount to 0″.005					
	$x_1=30'$, $y_1=30'$			$x_1=1°$, $y_1=1°$		
	$d\alpha\cos\delta=10'$, $d\delta=10'$	30′ 30′	1° 1°	10′ 10′	30′ 30′	1° 1°
$N_3=-1/2\, d\alpha^2\cos^2\delta\,(\tan^2\delta-1)\sin^2 1''$	0°.0	0°.0	0°.0	0°.0	0°.0	0°.0
$+d\delta^2\sin^2 1''$	0.0	0.0	0.0	0.0	0.0	0.0
$-3/4\, d\alpha^2\cos^2\delta\, d\delta^2\,(\tan^2\delta-1)\sin^4 1''$	75+	75+	75+	75+	75+	75+
$+1/24\, d\alpha^4\cos^4\delta\,(\tan^4\delta-12\tan^2\delta+5)\sin^4 1''$	75+	75+	75+	75+	75+	75+
$+2/3\, d\delta^4\sin^4 1''$	75+	75+	75+	75+	75+	75+
$N_4=d\alpha^2\cos^2\delta\tan\delta\sin^3 1''$	75+	75+	46.3	75+	46.3	14.6
$+1/2\, d\alpha^2\cos^2\delta\, d\delta\sec^2\delta\sin^4 1''$	75+	75+	75+	75+	75+	75+
$N_5=d\alpha\cos\delta\sin^2 1''$	0.0	0.0	0.0	0.0	0.0	0.0
$+3/2\, d\alpha\cos\delta\, d\delta\tan\delta\sin^3 1''$	75+	70.3	34.9	75+	34.9	9.9
$+15/8\, d\alpha\cos\delta\, d\delta^2\sin^4 1''$	75+	75+	75+	75+	75+	75+
$-1/6\, d\alpha^3\cos^3\delta\,(7\tan^2\delta-5)\sin^4 1''$	75+	75+	75+	75+	75+	74.8
$N_6=d\delta\sin^2 1''$	0.0	0.0	0.0	0.0	0.0	0.0
$-1/2\, d\alpha^2\cos^2\delta\tan\delta\sin^3 1''$	75+	75+	64.4	75+	64.4	27.6
$-3/4\, d\alpha^2\cos^2\delta\, d\delta\,(\tan^2\delta-1)\sin^4 1''$	75+	75+	75+	75+	75+	75+
$+4/3\, d\delta^3\sin^4 1''$	75+	75+	75+	75+	75+	75+
$N_7=1/8\, d\delta^2\,(3\tan^4\delta-2\tan^2\delta-1)\sin^4 1''$	75+	75+	75+	75+	74.4	68.8
$+d\alpha^2\cos^2\delta\sin^4 1''$	75+	75+	75+	75+	75+	75+
$N_8=2d\alpha\cos\delta\, d\delta\sin^4 1''$	75+	75+	75+	75+	75+	75+
$N_9=d\delta^2\sin^4 1''$	75+	75+	75+	75+	75+	75+

We find them to be:

Right ascension, $A_1 = 3° 51' 26''.08$,
Declination, $D_1 = 74° 58' 2''.52$.

From A, D, α_2, δ_2, we now compute x_2, y_2, also by our former series. These come out:

$$x_2 = +1733''.92, \qquad y_2 = -90''.28.$$

If we now apply equations (1) of the present paper to the data α_1, δ_1, α_2, δ_2, x_1, y_1, we should arrive at the same values of x_2, y_2. Actual calculation of the expressions appended below gives:

	″		″
x_1	$=+3600.000$	y_1	$=+3600.000$
M_1	$=-1984.573$	N_1	$=-3567.062$
M_2x_1	$=-1.176$	N_2x_1	$=-120.818$
M_3y_1	$=+121.404$	N_3y_1	$=-0.783$
$M_4x_1^2$	$=-0.568$	$N_4x_1^2$	$=+0.020$
$M_5x_1y_1$	$=-1.126$	$N_5x_1y_1$	$=-0.531$
$M_6y_1^2$	$=-0.037$	$N_6y_1^2$	$=-1.107$
$M_7x_1^3$	$=+0.003$	$N_7x_1^2y_1$	$=+0.020$
$M_8x_1^2y_1$	$=0.000$	$N_8x_1y_1^2$	$=0.000$
$M_9x_1y_1^2$	$=0.000$	$N_9y_1^3$	$=0.000$
x_2	$=+1733.927$	y_2	$=-90.261$

These numbers are in satisfactory accord with the values obtained in the previous calculation with the old series.

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CURRENT NOTES ON LAND FORMS

UPWARPED MOUNTAINS IN ALASKA

THE descriptions of certain ranges given by A. H. Brooks in 'The Geography and Geology of Alaska' (prof. paper no. 45, U. S. Geol. Survey, 1906) furnish additional examples of upwarped plateaus, carved into mountainous form by normal and glacial erosion, as already indicated in Gilbert's volume on 'Glaciers' in the reports of the Harriman Alaskan Expedition. The coast range, or southeastern part of the Pacific mountain system in Alaska, is said to be an irregular aggregate of mountain masses with little symmetry of arrangement except a rough alignment along a northwest-southeast axis. The whole aspect of the range is rugged and precipitous, from the needle peaks and knife-edge crests down to the sharply incised channels. This young