SCIENTIFIC BOOKS

Lectures on the Theory of Functions of Real Variables. Volume I. By JAMES PIERPONT. Boston, Ginn and Company. Pp. xii + 560.

A considerable part of the present volume is in very close touch with problems which confront the students of elementary mathematics, dealing with such questions as the difference between rational and irrational numbers, the theory of limits and the concepts of continuity and discontinuity. Bright and thoughtful students frequently seek more light on these subjects than they can find in the elementary text-books and many teachers will doubtless rejoice to find that a large amount of most interesting information along these lines has been made accessible by a scholar in whom they can have the utmost confidence.

A little more than a hundred pages are devoted to the fundamental matters which are to serve as a basis for the notion of function in general. This notion is illustrated by means of the trigonometric functions with which the reader is supposed to be familiar and a very brief proof is given of the interesting fact that these functions are transcendental. The descriptive introduction to functions is followed by a similar introduction to point aggregates in which several fundamental theorems relating to limiting points are proved and a number of the common terms are defined and illustrated. The theory of point aggregates furnishes some of the most interesting instances of the distinction between finite and infinite multitudes, and the importance of this theory is partially illustrated by the fact that one of its terms (dense) is needed as early as page 20 to describe the system of rational numbers.

The greater part of the present volume deals with questions which the student approaches in the elementary calculus. The processes of differentiation and integration are treated with a completeness which seems impracticable in a first course, yet this completeness is essential for a thorough comprehension of the subject. A very helpful feature is furnished by the 'numerous examples of incorrect forms of reasoning currently found in standard works on calculus.' It has been "the author's experience that nothing stimulates the student's critical sense so powerfully as to ask him to detect the flaws in a piece of reasoning which at an earlier stage of his training he considered correct."

The vast extent of the applications of the processes of calculus have frequently led writers to overlook the regions where these processes do not always lead to correct results. Even some of the most useful formulas, such as

$$dy/dt = dy/dx \cdot dx/dt,$$

appear in nearly all, if not in all, of the other English texts with an incomplete demonstration. Arts. 378–80, which are devoted to a satisfactory demonstration of this formula, exhibit also the missing elements in the common demonstrations and suggest a method for an elementary demonstration in case a function has only a finite number of oscillations.

The last three chapters are devoted to improper integrals and to multiple proper integrals. These naturally contain much more original matter than those which precede. This is especially true of the last chapter, which is practically an original contribution. The definition of an integral is taken in the most general fashion and includes all the possible fields, whereas until then the most general was Jordan's and this is restricted to the inner points of a field. No other work contains such a complete treatment of the subject of uniform convergence as is found in these chapters.

The present volume, which is to be followed by another along the same lines, seems especially timely in view of the movement to employ the notion of function much more generally in the elementary courses in algebra and geometry. Teachers of secondary mathematics should have a clear understanding of the concept of function and we know of no other work where an accurate knowledge of this concept can be acquired as readily as from the earlier chapters and the criticisms of the present treatise. The fact that Ginn & Company should undertake the publication of such works as this and Goursat's 'Course in

Mathematical Analysis' is a very encouraging sign of the growing interest in higher mathematics and these works will doubtless do much towards increasing this interest. In following the pages of Professor Pierpont's work one feels that one is being led by a master of his subject and a sympathetic teacher, and these elements combined with the nature of the subject make the present work one of the most significant publications on pure mathematics that have ever appeared in this country.

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Electrical Nature of Matter and Radioactivity. By Harry C. Jones. New York, D. Van Nostrand Company. Pp. viii + 220. Price $2.

Another semi-popular book upon a well-worn subject, but a book which on the whole justifies its existence by the treatment, found in the last seventy-five pages, of the results of investigations and discussions so recent that they have not yet found place in other books on radioactivity. Thus the discussions of recent work on the origin and distribution of radium, of the properties of the α and β rays, as lately worked out by Rutherford and Bragg, of the 'radiobes' of Burke, of the decomposition products of actinium, and of radiothorium, are all new and all thoroughly commendable.

The book as a whole lacks somewhat in unity of treatment, the different sections differing considerably in value and in method of presentation. The treatment of radioactivity, which occupies all save the first third of the book, although it is non-mathematical, is on the whole thoroughly scientific, being characterized by an admirable moderation of statement, a scholarly collection of all the available experimental data, evidently from the original sources, and a judicious balancing of arguments for and against rival hypotheses. It will be read with interest and profit by physicists and chemists. It contains a commendably small amount of the sort of material which seems to be designed chiefly as food for the popular imagination.

The chapters dealing with the electrical nature of matter seem, on the other hand, to have been written largely for popular consumption and their faults are those most common to literature of this type, namely, incompleteness in the presentation of the facts and a rather immoderate haste in arriving at positive conclusions, the author's attitude being that of the ardent convert to the electrical-nature-of-matter *hypothesis* rather than that of the judicious disseminator of the present state of scientific knowledge in this field. Thus in discussing in the first chapter the value of e/m for the corpuscle, he slurs over the differences between the values found by different observers working with cathode rays, Lenard rays, photo-electric effects, the Zeeman effect, and radium rays, and says simply that the answer to the question as to the constancy of e/m for negative corpuscles is unmistakably given by the results which have been obtained. When it is remembered that these values vary for slow-moving corpuscles from 4×10^6 to more than four times that number, namely, 18.7×10^6, the statement appears rather too strong even for a popular article. Thus far these differences are certainly not to be explained by *probable* observational errors. It is to be hoped that further experimenting will soon reveal the causes of the discrepancies. The value of e/m which the author uses throughout the book is 7.7×10^6 instead of 18.7×10^6, the value given by the most reliable experiments, especially those of Seitz (*An. d. Phy.*, Vol. 8, p. 223), who succeeded in bringing the results obtainable by the three different methods used in the study of cathode rays into close accord. The value 7.7×10^6 is, of course, inconsistent with Kaufmann's measurements upon the variation of e/m with speed according to which this quantity changed from 6×10^6 to 13.1×10^6 as the speed varied from .94 to .7 that of light.

The feature of this part of the book, however, which is least commendable is the confusion either of ideas or of terms involved in such statements as the following: "Matter is then a pure 'hypothesis'—'there is not the least evidence for its existence.' Energy is the only reality." Now, of course, every trained reader knows that in the ultimate an-