Orbit of the Seventh Satellite of Jupiter: R. T. Crawford and A. J. Champreux.

This paper gives results of an application of Leuschner's 'Analytical Method of Determining the Orbits of New Satellites.' Three solutions were made which are designated in the tabulation given below by (Cr. & Ch.)$_1$, (Cr. & Ch.)$_2$ and (Cr. & Ch.)$_3$. The orbits are based on Perrine's positions of January 3, February 8 and March 6, 1905. The first set of elements was derived irrespective of any perturbations. The second represents the first approximation to elements osculating February 8 by taking immediate account of the attraction of the sun. The third set is the result of a close representation of the observations on the same basis. For comparison, the elements by Perrine (L. O. Bulletin No. 78) and by Ross (L. O. Bulletin No. 82) are also tabulated. The first set of elements does not represent an observation of August 9 any better than those by Ross. The third set, however, gives the residuals $(O - C)$:

$$\Delta p = +2°.7$$
$$\Delta s = +5'.2$$

the computed positions being derived directly from the third set of elements osculating for February 8 without applying the solar perturbations February 8 to August 9. The solar perturbations are being computed for all observations secured since the discovery observations, by an adaptation of Encke's method of special perturbations in rectangular coordinates. With these perturbations it is expected to represent recent observations closely. Outstanding differences will serve to correct the third set of elements. With each of the three sets, a second solution with retrograde motion was obtained.

Harold Jacoby.

ELEMENTS OF THE SEVENTH SATELLITE OF JUPITER (DIRECT MOTION) REFERRED TO THE EARTH'S EQUATOR.

Computer.	Ω	i	ω	e	Period.	a
					d	
(Cr. & Ch.)$_1$	279° 45′ 8″	26° 27′ 14″	006° 28′ 42″	0.12576	251.1415	49′ 48″
(Cr. & Ch.)$_2$	289 47 45	25 39 42	189 15 19	0.13195	255.5376	50 20
(Cr. & Ch.)$_3$	288 19 59	25 39 23	187 29 41	0.12152	258.9424	50 47
Perrine (prel.)	275 47	26 15	182 6	0.24	200	43 48
Ross (final)	281 7.8	26 12	331 16.8	0.0246	265.0	52.54

a (Cr. & Ch.) for log $(\rho) = 0.72124$; a (Ross) for log $(\rho) = 0.71624$.

THE AMERICAN ASSOCIATION FOR THE ADVANCEMENT OF SCIENCE. SECTION A—MATHEMATICS AND ASTRONOMY.

Vice-president—W. S. Eichelberger, United States Naval Observatory, Washington, D. C. In the absence of the vice-president, Professor Alexander Ziwet, the retiring vice-president, presided at the meetings of the section.

Secretary—Professor L. G. Weld, State University of Iowa, Iowa City, Iowa.

Member of the Council—President C. S. Howe, Case School of Applied Science, Cleveland, Ohio.

Sectional Committee—Dr. W. S. Eichelberger, vice-president, 1906; Professor Alexander Ziwet, vice-president, 1905; Professor L. G. Weld, secretary, 1904–1908; Professor J. R. Eastman, one year; Professor Ormond Stone, two years; Professor E. B. Frost, three years; Professor E. O. Lovett, four years; Professor Harris Hancock, five years.

Members of the General Committee—Professor G. B. Halsted.

Press Secretary—The secretary of the section.

Dr. Edward Kasner, of Columbia University, was elected vice-president for the year 1907.

The address of the retiring vice-president, Professor Ziwet, on 'The Relation of Mechanics to Physics,' was presented on the afternoon of December 29, in the general assembly room in Gibson Hall, Tulane

University. This address has already been published in SCIENCE for January 12 of the current year.

At the regular program meeting of the section, held on the morning of December 30, the following papers were presented:

On the groups of Order $p^{mn}q$ having Abelian Subgroups $H_{p^{mn}}$ of Type $(n, n, \cdots, n)$: Dr. O. F. GLENN, Drury College, Springfield, Mo.

This paper is supplementary to one presented by the author at the Philadelphia meeting of the association, in which the case $n=1$ was discussed. The defining relations of all groups described in the title are tabulated, and their properties discussed in relation to the properties of the Galois field $GF[p^{mn}]$ determined by the automorph of the subgroup H. It is found that all of the groups in question are members of a general family and have one general set of defining relations.

A New Straight in non-Euclidean Geometry: Professor G. B. HALSTED, Kenyon College, Gambier, O.

The paper sets forth the discovery that, in Riemannean non-Euclidean geometry, the six mid-points of the parts of the six rays from the vertices of any triangle obtained by prolonging the sides, are co-straight. This is a new and noteworthy straight associated with every triangle.

The theorem is demonstrated, and then interpreted in ordinary Euclidean space.

A Chapter in the Present State of Development of the Elliptic Functions: Professor HARRIS HANCOCK, University of Cincinnati, Cincinnati, O.

The paper is an attempt to show that practically all (American and European) writers on the elliptic functions have been giving too much emphasis to certain parts of Weierstrass's theory, while they have neglected many of the lines of thought which Weierstrass himself considered fundamental.

It is shown that the so-called Weierstrassean normal-form is not due to Weierstrass. The introduction of new functions gives a different aspect to the presentation of the elliptic functions, although little that is new has been added thereby to the theory itself. Weierstrass's great work lies in a somewhat different direction; with him the problem of determining all analytical functions which have algebraic addition-theorems is the leading idea.

The paper also brings into evidence several fundamental theorems of Hermite and shows some of the characteristics of Riemann's theory.

A Catalogue of 1,607 Zodiacal Stars for the Epochs 1900 and 1920, Reduced to an Absolute System: Mr. H. B. HEDRICK, U. S. Naval Observatory, Washington, D. C.

This catalogue was prepared by the writer, in the Nautical Almanac Office, in order to fill the widely felt need of more fundamental positions of stars within the zodiac.

In its construction fifty-two observational catalogues were used. Systematic corrections were applied to each catalogue to reduce it to the same absolute system as that of the 'Catalogue of Fundamental Stars,' by Professor Simon Newcomb, published in the 'Astronomical Papers of the American Ephemeris,' Volume 8, Part 2.

Definitive positions were obtained by least square solutions of all the observations of each star. Right ascensions, declinations, variations and proper motions are given for two epochs, 1900 and 1920. Other features are the reference numbers to well-known catalogues, the Besselian star constants for 1910 for each star, and an index of stars by letter and constellation or by a special name.

This paper will be printed in the 'Astronomical Papers of the American Ephemeris,' Vol. VIII., Pt. 3.

A Class of Central Forces: Dr. Edward Kasner, Columbia University, New York City.

There exists no field of force in which a particle started from an arbitrary position with arbitrary velocity will describe a circular path. In the case of a central force the only possible circular trajectories are, in general, those whose centers are at the origin of force. If, however, the force varies according to a function of the form $br(r^2 - a)^{-3}$, then a quadruple infinity of the trajectories are circular. In the simplest case, arising when a vanishes, the force varies inversely as the fifth power of the distance, and the circles all pass through the origin. In the general case they are orthogonal or diametral to a fixed sphere.

Solar Photographs: Professor F. H. Loud, Colorado College, Colorado Springs, Colo. Presented by title.

The Groups of Order p^m which contain Exactly p Cyclic Subgroups of Order p^α: Professor G. A. Miller, Stanford University, Cal.

The main theorems proved in this paper may be stated as follows: If a group of order p^m, p being any odd prime, contains exactly p cyclic subgroups of order p^α, $\alpha > 2$, it contains exactly p cyclic subgroups of every order which exceeds p and divides p^{m-1}. Hence it is one of the two non-cyclic groups of order p^m which contain operators of order p^{m-1}. When $\alpha = 2$ and $p > 3$ the theorem is still true. In fact, the only possible exception occurs when $\alpha = 2$, $p = 3$ and $m = 4$. In this special case there are three groups which contain exactly p cyclic subgroups of order p^α.

When $p = 2$ the preceding theorem is replaced by the following: If a group of order 2^m contains exactly two cyclic subgroups of order 2^α, $\alpha > 2$ it can not contain more than two cyclic subgroups of any higher order. If a group of order 2^m contains exactly two cyclic subgroups of order 2^β but does not contain any cyclic subgroup of order $2^{\beta+1}$, then m can not exceed $2^{\beta+2}$. These theorems involve the fundamental properties of all possible groups whose order is a power of any prime p and which involve exactly p cyclic subgroups of any given order p^α. From a well known theorem it follows that α is not unity, but it can have every other possible value less than m.

Inversion and Inversors: Professor J. J. Quinn, Warren, Pa.

In this paper are presented two new theorems relating to inversion, besides an explanation of the construction of certain linkages exhibiting the operation of inversion.

Observations of the Total Solar Eclipse of 1905, August 30, at Tripoli, Barbary: Professor David Todd, Amherst College, Amherst, Mass.

Observations were undertaken under six different heads, as follows:

1. Observations of the geometric contacts.

2. Coronal photography with a twelve-inch Clacey lens photographically corrected. Professor Todd's modified form of Burckhalter revolving occulter was employed. The corona was photographed to 30′. Bailey's 'beads' were also photographed before the second contact.

3. A duplex Clark lens of three inches was used for long exposures on the circumsolar stars and the outer coronal streamers. Neither these nor any intra-Mercurial planets were revealed, although 14 x 17 plates of the highest sensitiveness were used.

4. With a three-and-one-half-inch Goerz doublet of thirty-three and one half inches focus (used during the previous expeditions of 1896, 1900 and 1901), attached to one of the automatic movements, sixty-three fine pictures of the corona were secured during the 186 seconds of totality. Some of these show the coronal streamers to exceptional length.

5. Sketches of the corona were made with the usual results.

6. Observations of the shadow bands were begun at least ten minutes prior to totality. The bands were wavering and narrow, moving faster than one could walk and at right angles to the wind, their length with it. They were observed to wax and wane five times during the interval of observation preceding totality. These observations have been communicated in detail to Mr. Lawrence Rotch, of Blue Hill and embodied in his exhaustive study of this phenomenon.

Computed Traces and Totality-Durations of the Total Solar Eclipses of the Twentieth Century: Professor DAVID TODD and R. H. BAKER, Amherst College, Amherst, Mass. Read by title.

A Possible Extension of the Theory of Envelopes: Professor L. G. WELD, State University of Iowa, Iowa City, Ia.

(*a*) In the equation $f(x, y, \alpha) = 0$, representing a family of loci, by giving to α, first an increment and then a corresponding decrement, each of magnitude $\Delta\alpha$, solving the resulting equations for the coordinates of the point of intersection and, finally, letting $\Delta\alpha \doteq 0$, there will be obtained

$$x' = \phi(\alpha), \quad y' = \psi(\alpha).$$

These equations define a point of the envelope of the given family of loci and eliminating α between them gives

$$F(x', y') = 0,$$

the equation of the envelope.

The point (x', y'), determined as above, may be called the tracing point of the locus, that is, the point which, for the moment, is tracing the envelope. It was shown in the paper, by way of illustration, that the tracing point for the envelope of the family of ellipses,

$$\frac{x^2}{\alpha^2} + \frac{y^2}{\beta^2} = 1, \quad \alpha + \beta = c,$$

is the Fagnagni point.

(*b*) The inverse of the above notion was next developed with reference to the right line, viz.: A point on the line

$$\frac{x}{\alpha} + \frac{y}{\beta} = 1,$$

being assigned at will, to find the functional relation between the intercepts,

$$\Phi(\alpha, \beta) = 0$$

(*i. e.*, the law governing the motion of the line), in order that the given point may trace an envelope and, finally, to obtain the equation of the envelope. The required relation is given by either of the differential equations,

$$x' = \phi(\alpha, \beta) = \frac{\alpha^2}{\alpha - \beta\dfrac{d\alpha}{d\beta}}, \quad y' = \Psi(\alpha, \beta) = \frac{\beta^2}{\beta - \alpha\dfrac{d\beta}{d\alpha}}.$$

In general both equations will be needed in order to determine the constants of integration. Having thus obtained the function Φ, which is, in effect, the tangential equation of the envelope, the equation in rectangular coordinates readily follows.

Several examples applying the principles were presented and its application to other families of loci was suggested as a promising field of investigation for the amateur mathematician.

LAENAS GIFFORD WELD,
Secretary.

SCIENTIFIC BOOKS.

Die Schule der Chemie. Erste Einführung in die Chemie für Jedermann. Von WILHELM OSWALD. Zweiter Teil. Die Chemie der wichtigsten Elemente und Verbindungen. Braunschweig, Friedrich Vieweg und Sohn. 1904. Price, bound, 8 Marks.