promoting local interest in pure and applied science.

Finally, and in the most comprehensive sense, to the local committee and especially to its presidents, Drs. Craighead and Beyer, its secretary, Mr. Mayo, and the chairman of its finance committee, Mr. Godchaux—in addition to the courtesies already mentioned—for providing ideal lunch arrangements, so convenient to the meeting places as to avoid a wasteful break in the day's work; for tendering a delightful reception—the peculiar charm of which was due in large part to the tactful management of Miss Minor and her associates in the ladies' reception committee; for a final ride, enabling us to carry away a coherent impression of New Orleans and its many points of historic interest; and for many acts of thoughtfulness—individual as well as collective—that will cause the past week to remain among the most pleasant memories that cluster about the many pleasant meetings of the association.

(Signed) WILLIAM TRELEASE, Chairman,
For the Committee,
Messrs. Trelease, Magie and Newcomb.

Response to these resolutions and farewell were given for the local committee by Professor Geo. E. Beyer, who extended a cordial invitation to the association to meet soon again in New Orleans. Response by President Woodward, who was also formally thanked by the association for his efficient and acceptable work as presiding officer. Adjourned.

GENERAL COMMITTEE.

At the meeting of the general committee on Monday evening, January 1, 1906, it was decided to hold a special summer meeting at Ithaca, New York, to close on or before July 3, 1906, and a regular winter meeting in New York City to begin on Thursday, December 27, 1906. The presidential and vice-presidential addresses will be omitted at the summer meeting and given at the winter meeting.

The officers elected at the New Orleans meeting will, therefore, hold over to the close of the New York meeting. Chicago was recommended as the place of the winter meeting of 1907.

The following officers were elected for the Ithaca and New York meetings:

President: Dr. W. H. Welch, Baltimore, Md.

Vice-Presidents:

Section A—Dr. Edward Kasner, New York City.

Section B—Professor W. C. Sabine, Cambridge, Mass.

Section C—Mr. Clifford Richardson, New York City.

Section D—Mr. W. R. Warner, Cleveland, O.

Section E—Professor A. C. Lane, Lansing, Mich.

Section F.—Professor E. G. Conklin, Philadelphia, Pa.

Section G—Dr. D. T. MacDougall, Washington, D. C.

Section H—Professor Hugo Münsterberg, Cambridge, Mass.

Section I—Mr. Chas. A. Conant, New York City.

Section K—Dr. Simon Flexner, New York City.

General Secretary: Mr. John F. Hayford, Washington, D. C.

Secretary of Council: President F. W. McNair, Houghton, Mich.

CLARENCE A. WALDO,
General Secretary.

THE RELATION OF MECHANICS TO PHYSICS.[1]

IN the historical development of mechanics the names of Galileo, Newton and Lagrange mark the principal epochs, each of the three periods, from Galileo to Newton, from Newton to Lagrange and from Lagrange to our time, covering roughly a century.

When Galileo in 1633, at the age of sixty-nine years, was forced by the prelates of Rome to abjure solemnly the truth of the Copernican system of the universe to the proof of which he had devoted the main efforts of a long and active life, he had still to write his most remarkable work, the 'Discorsi e dimostrazioni mate-

[1] Address of the vice-president and chairman of Section A, Mathematics and Astronomy, of American Association for the Advancement of Science, New Orleans, December 29, 1905.

matiche intorno à due nuove scienze attenenti alla mecanica et i movimenti locali' (1638).[2] He composed it while confined to a house at Arcetri, near Florence, under the close watch of the Inquisition, strictly forbidden to publish anything and struggling with ill-health and the infirmities of old age which were soon to deprive him completely of his eyesight. Considering these circumstances of its composition, the marvelous freshness and wealth of ideas of this work, which makes Galileo the first mathematical physicist, would be incomprehensible if we did not know from his correspondence that the materials for it had largely been in his mind ever since his early youth. If this be taken into account, the beginnings of both mechanics (apart from statics) and mathematical physics may be dated back to about the year 1600.

One of the two new sciences originated by Galileo in the 'Discorsi' is mechanics as the science of motion, especially in its application to falling bodies and projectiles. The genius of Newton, of Huygens, of Leibniz, was soon to prove the correctness of Galileo's prophetic insight in claiming for his speculations on motion the name of a new science. What Newton and his followers in the eighteenth century did for mechanics is too well known to be here rehearsed. By his careful formulation of the fundamental postulates and definitions and by his bold assumption of the law of universal gravitation, Newton laid the lasting foundations for astronomical mechanics; and his fluxional calculus opened up for this science a wide range of development.

The other of Galileo's two new sciences deals with the internal structure of matter and the so-called resistance of materials; it is the germ of the mechanics of deformable bodies. Progress along this line proved a far more difficult task. The seventeenth and eighteenth centuries contributed but little to the theory of elasticity. Indeed, a new mathematical tool, the theory of partial differential equations, had to be invented, and a physical phenomenon hitherto neglected, vibratory and wave motions, had to attract the attention of mathematicians, before the mechanics of deformable bodies could become a true science. Besides, the conception of mechanics itself had to be broadened; and this was accomplished by Lagrange in his 'Mécanique analytique' (first edition 1788, second edition 1811–15).

In view of the use made in the course of the nineteenth century of Lagrange's generalizations (it may suffice to mention the theory of the potential, the Lagrangian equations of motion with their generalized idea of force, the general 'principles' such as the principle of least action) it is, I believe, not too much to say that Lagrange's work is as great an advance on Newton's as Newton's was on that of Galileo.

By the contemporaries of Lagrange this advance was perhaps not fully appreciated. We find the physicists of the beginning of the nineteenth century still very strongly attached to the idea that all natural phenomena not only may, but must, be explained on the basis of Newton's laws[3] by central forces acting instantaneously at a distance. Newton's mechanics had done such admirable service in astronomy that

[2] It is to be regretted that there exists no good modern translation of this classical work. The German translation published in *Ostwald's Klassiker der exakten Wissenschaften* (Nos. 11, 24, 25), while it contains some helpful notes, is not always exact and trustworthy. The original has recently been edited with great care by A. Favaro in Vol. VIII. (1898) of the 'national edition' of Galileo's Works.

[3] See, however, Laplace, 'Mécanique Céleste,' livre I., Chap. VI. ('Oeuvres,' Vol. I., 1878, pp. 74–79), a passage to which E. and F. Cosserat have recently called attention.

it had come to be regarded as the only possible means of describing and discussing the actions of nature. The gradual abandonment of this position and the change to the modern view according to which all actions in nature are transmitted through a continuous medium and require time for their transmission was accomplished only after a long struggle that occupied the greater part of the nineteenth century.

The more or less conscious part taken in this struggle by *technical* mechanics, which in the same period developed into a science, has not always been insisted upon sufficiently. Technical mechanics has always been free of the idea of central forces. To the engineer the idea of forces acting at a distance is completely foreign, in spite of the curious fact that, until not so very long ago, the typical example of such a force, gravitation, was almost the only force with which he had to deal. The development of thermodynamics, which has given us the principle of the conservation of energy in its broadest aspect, was closely connected with the rise of technical mechanics, but proceeded rather independently of the development of the other branches of mathematical physics. Its fundamental principles are of a very general and abstract nature, and even where the molecular hypothesis is well worked out, as in the kinetic theory of gases, the idea of central forces is in no way essential.

Hydrodynamics, elasticity, optics, electricity and magnetism, though originally based on molecular hypotheses and the idea of central forces, in the course of their development found themselves more or less independent of these notions. In all of them the important common feature is the propagation of actions through a medium which can be regarded, at least in first approximation, as continuous. In hydrodynamics and in the theory of elasticity this medium is that unknown something which we call matter; in optics, and later in the theory of electricity and magnetism, it was found necessary to postulate the existence of another medium, the ether.

It is well known how the ideas of Faraday, of Maxwell, of Hertz, gradually gained ascendency over the older views and led to the abandonment of the idea of central forces acting instantaneously at a distance, in almost all branches of physics except in the theory of gravitation. It is also known that Maxwell, by a brilliant analysis, succeeded in establishing the connection between his electromagnetic theory and the analytical mechanics of Lagrange. Thus, at the end of the nineteenth century we find a general attitude toward physical phenomena essentially different from that prevailing at the end of the eighteenth century.

With the rise of the electron theory in the course of the last twenty-five years a new element has been introduced into this development, an element which seems destined to affect very radically not only our interpretation of physical phenomena, but also our general views about the principles of theoretical mechanics. The idea of the electron has grown out of the idea of ions as used in electrolysis. Each molecule of an electrolyte may break up into two ions, *i. e.*, two atoms, or groups of atoms, carrying equal and opposite charges. The current passing through the electrolyte then consists in the actual transfer of these ions to the cathode and anode to which they give up their charges. In his Faraday lecture, delivered in 1881, which marks an epoch in the ion theory, Helmholtz says: "If we accept the hypothesis that the elementary substances are composed of atoms, we can not avoid concluding that electricity also, positive as well as negative, is divided into definite elementary portions, which behave like atoms of electricity."

These 'atoms of electricity,' since encountered in a large number of more recondite phenomena, and often apparently free, *i. e.,* not attached to any matter in the ordinary sense, are the *electrons.* Thus physicists have been led to return in a certain sense to atomistic conceptions, without however, abandoning the idea of the propagation of electric, magnetic and optical disturbances through the ether in time. Lord Kelvin, in his Baltimore lectures in 1884, gave expression to this tendency so largely developed in the succeeding twenty years. The very first words of his first lecture are: "The most important branch of physics which at present makes demands upon molecular dynamics seems to me to be the wave theory of light."

Without discussing the experimental basis of the electron theory it must here suffice to say that on the one hand the dispersion and diffraction of light, on the other the phenomena exhibited by cathode and canal rays, Röntgen rays, the Becquerel rays emitted by radium, etc., all find their ready interpretation in this theory.[4] At the same time, the electron theory as developed by Lorentz, Wiechert, Drude and others seems to furnish an excellent basis for the whole theory of electricity, magnetism and light.[5] Indeed, attempts have already been made of interpreting matter itself as an electromagnetic phenomenon and of explaining gravitation by means of this electron theory of matter.

It should be observed that the electron theory does not upset that beautiful structure known as the electromagnetic theory of Maxwell and Hertz. It merely modifies it to a certain extent so as to give a more detailed account of electromagnetic phenomena in ordinary matter. It is related to the older theory somewhat as the kinetic theory of gases is related to the theory of heat and of ordinary matter in general. The kinetic theory assumes the laws of ordinary mechanics for the motion of the hypothetical molecule and then tries to determine the average effects arising from the motion of very large numbers of such molecules, these averages being the only thing actually observable. Similarly the electron theory must begin with postulating laws of motion for the single electron in the electromagnetic field and try to deduce the average effects due to swarms of electrons; the comparison of these calculated average effects with the results of observation and experiment must serve as verification of the postulated laws.

If, then, observation leads us to the assumption that electric charges may exist and move about without being attached to, or carried by, ordinary matter, what are the 'laws of motion' of such an electron? As the moving object is not ordinary matter we must not be astonished to find that Newton's laws of motion can not be applied blindly. The electron moves according to the laws of electrodynamics. We are thus confronted with the question as to the relation of the fundamental postulates of this science to those of ordinary mechanics.

An electric charge at rest manifests its presence only by the field which it excites in its vicinity, by the sheaf of lines of force issuing from it. To take a simple concrete example, a small charged sphere has lines of force radiating as if from its center in all directions, and the electric force, or intensity of the field, $\mathbf{E}$, at any point P, at

[4] See, for instance, W. Kaufmann, *Physikalische Zeitschrift*, 3 (1901), pp. 9 sq., translated in *The Electrician*, 48 (1901), pp. 95–97; O. Lodge, *Journal of the Institute of Electrical Engineers*, 32 (1902–3), pp. 45–115; P. Langevin, *Revue générale des sciences*, 16 (1905), pp. 257–276; H. A. Lorentz, 'Ergebnisse und Probleme der Elektronentheorie,' Berlin, Springer, 1905.

[5] It will be sufficient to mention Lorentz's articles in the *Encyklopädie der mathematischen Wissenschaften*, V., 13, 14, where full references are given, and to the systematic work of M. Abraham, 'Theorie der Elektrizität,' I. (1904), II. (1905), Leipzig, Teubner.

the distance r from the center of the sphere whose charge is e, has the direction of r and the magnitude e/r^2.

If the sphere is in motion it carries its field along almost unaltered, provided the velocity $\mathbf{v}$ of the sphere be small in comparison with the velocity of light. But it excites a magnetic field, the magnetic force, or intensity, being $\mathbf{H} = \mathbf{E} \times \mathbf{v}$; *i. e.*, the magnitude of the force at P is $= ev \sin (\mathbf{E}, \mathbf{v})/r^2$, its direction is at right angles to $\mathbf{E}$ and $\mathbf{v}$, and its sense is such that the three vectors $\mathbf{E}$, $\mathbf{v}$, $\mathbf{H}$ form a right-handed set. The lines of magnetic force are, therefore, coaxial circles about the direction of motion.

According to the electromagnetic theory, the energy of the magnetic field is distributed throughout the field, with volume density $(1/8\pi)\mu\mathbf{H}^2$, where μ is the magnetic permeability of the medium. The energy of the whole field is readily obtained by integrating over the space outside the sphere; it is found $= \frac{1}{3}\mu e^2 v^2/a$, where a is the radius of the sphere. This magnetic energy, being due to the motion of the charge, is analogous to kinetic energy.

If the charged sphere consists of an ordinary mass m carrying the charge e so that its ordinary kinetic energy is $\frac{1}{2}mv^2$, the total kinetic energy due to the motion of m and e with the velocity v is

$$T = \frac{1}{2}\left(m + \frac{2}{3}\mu\frac{e^2}{a}\right)v^2,$$

that is, the same as if the mass m of the sphere were increased by the amount $\frac{2}{3}\mu e^2/a$.

The result, then, is similar to that known in hydrodynamics for a sphere of mass m moving through a frictionless liquid. In moving, the sphere sets the surrounding liquid in motion; to move the sphere we have to set in motion not only the mass m, but also that of the liquid around it. Thus the sphere moves in the liquid just as a sphere of greater mass would move in vacuo. In the case of a sphere the mass is increased by one half of that of the liquid displaced. But in the case of a body whose mass is not distributed as symmetrically as in the case of the sphere the mass to be added depends on the direction of motion.

As the apparent mass of the charged sphere in motion, owing to the presence of the charge e, exceeds the ordinary mass m by $\frac{2}{3}\mu e^2/a$, the apparent momentum exceeds the ordinary momentum mv by $\frac{2}{3}\mu e^2 v/a$; and this additional momentum must be regarded as residing not in the sphere but in the surrounding field. This momentum possessed by the field is what Faraday and Maxwell used to call the electrotonic state.

In the case of the free electron we have $m = 0$; hence the total mass, momentum, kinetic energy, is magnetic and is distributed throughout the field. Moreover, if the velocity of the electron be comparable with the velocity of light, the apparent mass will depend not only on the direction, but also on the magnitude of this velocity.

Any variation in the velocity of the charged sphere, or of the electron, produces a variation in the momentum of the field, which is propagated as a pulse through the field with the velocity of light. If such a pulse strikes a charged body at rest, the body acquires velocity and momentum, the momentum acquired being equal to that lost by the pulse. As the pulse resides in the ether, the law of the equality of action and reaction would make it necessary to assume an action exerted on the ether itself. In the electron theory of Lorentz which does not admit such actions on the ether Newton's third law of motion is violated in as much as action and reaction take place neither at the same place nor at the same time.

These very brief and incomplete indications will perhaps suffice to call to mind some of the characteristic differences between the fundamental principles of ordinary mechanics and the modern electromagnetic theory. Is it necessary, then, to keep these two sciences distinct, or is it possible to build them up on a common foundation? Such a common foundation is certainly desirable; and it will ultimately amount to the same whether we try to generalize the principles of mechanics so as to embrace the electromagnetic theory, or whether we follow W. Wien[6] in deducing the principles of mechanics as a particular or rather limiting case from Maxwell's equations.

The question can be put in a somewhat different form. There seem to be two things underlying all the phenomena in the physical world: the ether and matter. To attain the unification of physical science, shall we consider the ether as a particular kind of matter? Or shall matter be interpreted electromagnetically? The older mechanics dealt exclusively with matter; and when it first became necessary to introduce the ether, this new medium was often endowed with properties very much like those of matter. The hydrodynamic analogy by which the apparent mass of the moving charge was interpreted above illustrates this tendency. The physics of the ether has, however, reached so full a development that the properties of the ether are now known far more definitely than those of matter. These properties are contained implicitly in the fundamental equations of Maxwell and Hertz which in their essential features are adopted in the electron theory of Lorentz.

In this theory the electromagnetic mass of the electron is nothing but the self-induction of the convection current produced by the moving electron. This mass depends on the velocity of the electron, or rather on the ratio of this velocity to that of light. Moreover, this mass, or inertia, may be of two kinds: longitudinal, as opposing acceleration in the direction of motion, and transverse, as opposing acceleration at right angles to the path. Any variation in the velocity is transmitted as a radiation through the ether with the velocity of light.

The electromagnetic energy does not reside in the moving electron, but is distributed through the whole field, with the volume density $(1/8\pi)(\mathbf{E}^2+\mathbf{H}^2)$, if $\mathbf{E}$ and $\mathbf{H}$ are the electric and magnetic vectors of the field. In determining the rate of work in any region we must take into account not only the time-rate of change of this energy in the region, but also the flux of energy through its boundary, which has the value $(c/4\pi)\ \mathbf{E}\times\mathbf{H}$, per surface element, c being the velocity of light.

M. Abraham[7] has shown that the fundamental equations of Lorentz's theory of electromagnetism can be given a form that bears a striking resemblance to the fundamental equations of ordinary mechanics. But he has pointed out at the same time that in spite of this analogy of mathematical form the real meaning of the equations is essentially different from their meaning in the older mechanics. The underlying invariant quantity is not ordinary mass, but the electric charge of the electron; mass, or inertia, is variable, depending on the velocity; momentum and energy are distributed through the field; the flux of energy, given by Poynting's radiation vector, is essential in determining the rate of working of a system. All these differences are ultimately due to the modern conception of the propagation of all actions, not instantaneously, but in time, through a medium. This idea, as seems to

[6] Ueber die Möglichkeit einer elektromagnetischen Begründung der Mechanik, *Archives néerlandaises* (2), 5 (Lorentz Festschrift), 1900, pp. 96–107.

[7] *Annalen der Physik*, Vol. 10 (1903), pp. 105–179.

have been foreseen long ago by Gauss[8] and Riemann,[9] requires a generalization of, or even a direct departure from, the ordinary laws of mechanics: the law of the relativity of motion, the conservation of linear and angular momentum and of energy in a closed system, the instantaneous equality of action and reaction.

It is now pretty generally recognized that Newton's 'laws of motion,' including his definition of 'force,' are not unalterable laws of thought, but merely arbitrary postulates assumed for the purpose of interpreting natural phenomena in the most simple and adequate manner. Unfortunately, nature is not very simple. "As the eye of the night-owl is to the light of the sun, so is our mind to the most common phenomena of nature," says Aristotle. And if since Newton's time we have made some progress in the knowledge of physics it is but reasonable to conclude that the postulates which appeared most simple and adequate two hundred years ago can not be regarded as such at the present time.

This does not mean, of course, that the mechanics of Newton has lost its value. The case is somewhat parallel to that of the postulates of geometry. Just as the abandonment of one or the other of the postulates of Euclidean geometry leads to a more general geometry which contains the old geometry as a particular, or limiting, case, so the abandonment or generalization of some of the postulates of the older mechanics must lead to a more general mechanics. The creation of such a generalized mechanics is a task for the immediate future. It is perhaps too early to say at present what form this new non-Newtonian mechanics will ultimately assume. Generalization is always possible in a variety of ways. In the present case, the object should be to arrive at a mechanics, on the one hand sufficiently general for the electron theory, on the other such as to include the Newtonian mechanics as a special case.

After the searching criticism to which Poincaré, especially in his St. Louis address,[10] in 1904, has subjected the foundations of mechanics and mathematical physics, almost the only one of the fundamental principles that appears to remain intact is the principle of least action. It seems, therefore, natural to take this principle as the starting point for a common foundation of mathematical physics and of a generalized mechanics, but with a broader definition of 'action,' or what amounts to the same, with a generalized conception of 'mass' so as to make the latter a function of the velocity.

A very notable attempt has recently been made in this direction by E. and F. Cosserat.[11] And although only a first instalment of their investigation has so far been published, the able way in which the difficult problem is here attacked seems full of promise for a solution as complete as the nature of the case may warrant.

It may, perhaps, be said that, in demanding a generalization of the foundations of mechanics on such broad lines, I have attached undue importance to the electron theory as developed by Lorentz and Abraham, a theory which is still in the formative stage. There exist electromagnetic theories that appear less radical in their departures from the older views

[8] Gauss, Werke, Vol. 5, p. 627. Compare *Encyklopädie der mathematischen Wissenschaften*, Vol. 12, pp. 45–46.

[9] Riemann, Werke, 2d edition, 1892, p. 288.

[10] *Bulletin des sciences mathématiques* (2), 28, pp. 302–324; English translation in the *Bulletin of the American Mathematical Society*, Vol. XIII., February, 1906.

[11] *Comptes rendus*, Vol. 140, pp. 932–934; for a more detailed development see the notes contributed by E. and F. Cosserat to the French translation of O. D. Chwolson's 'Traité de physique,' Paris, Hermann, 1905.

and not so much open to the objection of violating long established principles. But if I have insisted particularly on the theory of Lorentz, it was just for the purpose of bringing out as clearly and forcibly as possible the differences between the old and the new.

Besides, there is one minor feature in the form of presentation adopted by Lorentz and Abraham which appeals to me as worthy of attention: it is the consistent use of the vector analysis of Gibbs and Heaviside. And perhaps this is really somewhat more than a mere matter of form. Burkhardt[12] has shown that this vector analysis has a rational mathematical basis. And after the numerous and manifold applications that have been made of this method its usefulness can no longer be questioned. The diversity of notations used by different authors can hardly be regarded as a serious objection. Have we not a large variety of notations even in so old and well-established a branch of mathematics as the differential calculus? The important thing about vector analysis is that it teaches to think in vectors and fields. E. Picard,[13] in a lecture, has recently called attention to the importance of the field even in ordinary elementary mechanics. A. Föppl has led the way in using vector symbols in an elementary treatise on technical mechanics.

Vector addition is now more or less familiar even to the student of the most elementary mechanics, largely owing to the influence of graphical statics. Is it not time to introduce at least the scalar and vector products and the time-differentiation of vectors in the mechanics of the particle and the rigid body? The gain in clearness and conciseness in stating the more general propositions is certainly great. In the mechanics of deformable bodies and media (hydrodynamics, elasticity), the general theory of vector fields, with the fundamental notions of divergence and curl, flux and flow, lamellar and solenoidal fields, etc., should surely form the preliminary mathematical basis for all further study; and here the simple symbolism of vector analysis is particularly well adapted to the subject.

But whatever may be the form of presentation selected, the study of the fields of scalars, vectors and higher point functions, so intimately connected with the modern views of physical phenomena, might well claim more attention on the part of the pure mathematician than it has so far received.

ALEXANDER ZIWET.

[12] *Mathematische Annalen*, Vol. 43 (1893), pp. 197–215.

[13] 'Quelques réflexions sur la mécanique, suivies d'une première leçon de dynamique,' Paris, 1902.

THE SANITARY VALUE OF A WATER ANALYSIS.[1]

TWENTY years ago, the vice-president of this section, the late Professor William Ripley Nichols, took as the subject of his address, 'Chemistry in the Service of Public Health,' saying: "If any are inclined to criticize my choice of that branch of applied chemistry with which I am most familiar, I trust they will consider that, after all, few of us have the opportunity, or, let us confess it, the ability to carry research and speculation to the height to which chemistry is capable of rising." Agreeing fully in the sentiment of this last sentence, though not at all as applying to Professor Nichols, whose marked ability as an investigator was recognized by all, I feel that I can best fulfill the clause in our constitution which requires the several vice-presidents to give an address before their

[1] Address of the vice-president and chairman of Section C, Chemistry, American Association for the Advancement of Science, New Orleans, December, 1905.