Minneapolis; R. A. Harris, Washington; A. Heilprin, Philadelphia; R. T. Hill, Washington; E. Huntington, Milton, Mass.; M. S. W. Jefferson, Ypsilanti; Emory R. Johnson, Philadelphia; Wm. Libbey, Princeton; G. W. Littlehales, Washington; C. F. Marbut, Columbia, Mo.; F. E. Matthes, Washington (Camb.); W J McGee, Washington; R. Pumpelly, Newport, R. I.; H. F. Reid, Baltimore; W. W. Rockhill, Washington; R. D. Salisbury, Chicago; E. C. Semple, Louisville; G. H. Shattuck, Baltimore; L. Stejneger, Washington; R. S. Tarr, Ithaca; R. DeC. Ward, Cambridge; B. Willis, Washington.

Albert Perry Brigham,
Secretary.

SCIENTIFIC BOOKS.

Technical Mechanics. By Edward R. Maurer, professor of mechanics in the University of Wisconsin. New York, John Wiley & Sons. 1903.

Elements of Theoretical Mechanics. By Alexander Ziwet, junior professor of mathematics in the University of Michigan. Revised edition of 'An Elementary Treatise on Theoretical Mechanics,' especially designed for students of engineering. New York, The Macmillan Company. 1904.

Die Technische Mechanik. Elementares Lehrbuch für Mittlere Maschinentechnische Fachschulen und Hilfsbuch für Studierende Höherer Technischer Lehranstalten. Von P. Stephan, Regierungsbaumeister, Lehrer an der Kgl. Höheren Maschinenbauschule in Posen. Erster Teil: Mechanik Starrer Körper. Leipzig und Berlin, B. G. Teubner. 1904.

The teacher of mechanics who undertakes to write a text-book for students of engineering is confronted with a difficult problem. He is compelled to recognize the justice of the demand that the course shall be practical, while resisting the tendency to interpret the practical too narrowly. While, a rather extensive course seems to be demanded by the manifold applications of mechanics in engineering, his experience in the class-room emphasizes strongly the limitations imposed by restricted time and lack of maturity of students. It will scarcely be questioned that the matter of first importance to the student is a clear understanding of principles rather than an assortment of special rules for solving particular problems. The presentation of principles in a sound and intelligible manner should, therefore, be the chief aim of a text-book, and methods of presentation and illustrative examples should be chosen primarily with reference to this aim.

The success with which this requirement is met by the three books under review will be differently estimated by different teachers. Each possesses merit of a high order, and there is little room for adverse criticism except such as implies a fair difference of opinion as to what methods of treatment are to be regarded as best. It will here be attempted only to indicate the character and scope of each of the books, and to make some general observations regarding methods of presenting the principles of mechanics in an elementary text-book.

As a sound and practical text-book for the use of students of engineering Professor Maurer's book possesses high merit. The exposition is nearly always concise—indeed, this is perhaps often carried to a fault—but the soundness of the logic is rarely open to question. The author shows close sympathy with the point of view of the beginner, and appreciation of the fact that at certain points the conventional treatment of fundamental principles fails to meet the needs of the ordinary student.

Professor Ziwet's book is an excellent introduction to the science of analytical mechanics. His exposition is in general sound and logical, and the book will be read with pleasure and profit by a student of mathematical tastes and ability who has the requisite mathematical training. The maturity and mathematical equipment required for reading it at all easily appear to be greater than are possessed by most of those who take up the subject in the second or third year of the ordinary four-year course in engineering, but the book will doubtless prove effective in the hands of a teacher who is in sympathy with the methods and point of view of its author.

Stephan's book is admirable for the simplicity with which elementary principles and methods are presented. If written in English it would probably find favor with many teachers in America who desire a text-book not presupposing calculus. It should be said, however, that while calculus notation is not employed by Stephan, he does employ the conceptions of both differential and integral calculus. The fundamental conceptions of the calculus are, of course, necessarily employed in any sound presentation of the principles of mechanics, and it may be doubted whether real simplicity is gained by avoiding its notation.

The three books are all designed for students of engineering, and each aims to be practical by including many numerical exercises and illustrative examples of the kind met in engineering practise, but each is a text-book of theoretical, rather than applied, mechanics. All have much the same scope, covering the statics, kinematics and kinetics of particles and of rigid bodies. Two of the books—those of Maurer and Stephan—agree somewhat closely in order of treatment, beginning with statics and following with kinematics and kinetics. Ziwet, on the other hand, begins with geometry of motion and kinematics, follows with an introduction to dynamics (statics being treated as a special case) and concludes with kinetics. In all the treatment is mainly restricted to the simpler force systems and the simpler cases of motion. Of the three books that of Stephan is the most elementary in treatment, while that of Ziwet would probably be the most difficult reading for the average student beginning the subject in its usual place in a course in engineering.

As features of Maurer's book may be mentioned the emphasis everywhere given to the vector nature of the quantities dealt with, the parallel treatment of graphical and analytical methods in statics, the admirable chapter on work and energy, and the satisfactory treatment of the subject of units. Professor Ziwet also gives prominence to vector notions, and also includes graphical methods in statics, though less fully than Maurer. His book contains no systematic presentation of the theory of energy, though the main features of the theory may be gathered from detached passages. His treatment of kinematics and kinetics is throughout more elaborate on the theoretical side than that of Maurer or of Stephan, and more use is made of general analytical methods. Stephan does not use the language of vectors. In statics he makes free use of graphical methods, but does not give the student the aid which comes from the use of Bow's notation for the designation of forces. His treatment of kinematics and of kinetics is relatively brief, and only the merest introduction to the theory of energy is given, potential energy not being mentioned.

Dimensional equations and the theory of units are explained by both Maurer and Ziwet, the former devoting to this subject an appendix of six pages. In Professor Ziwet's book (Art. 58) occurs an erroneous illustration which is likely to confuse the student: '* * * we have of course the proportion: 30 miles an hour is to one mile an hour as 44 feet per second is to one foot per second.' Both gravitational and kinetic systems of units are explained in each of the three books. The simple treatment of the engineers' kinetic system adopted by Maurer and Stephan should effectually clear away the traditional haziness surrounding the equation $m = W/g$. The unit mass is taken as a derived unit, and defined as the mass to which the gravitation unit force (the pound-force or kilogram-force) gives unit acceleration; this unit mass is thus equal to g pounds or g kilograms, and the equation expresses the reduction from one unit mass to another. The usefulness of a name for the unit thus defined will be agreed to even by those who hesitate to adopt the names *geepound* and *geekilogram* suggested by Maurer.

In considering the general question as to the best method of presenting the fundamental principles of mechanics in an elementary text-book, two requirements must be kept in view, soundness and intelligibility. Critics are by no means agreed as to what constitutes a sound formulation of the laws of motion.

Newton's laws have long held their place in the majority of English and American books, and in spite of the fact that the philosophical validity of the Newtonian system has been seriously questioned by able critics, this system, properly understood, still appears to furnish substantially the best foundation. It does not follow, however, that a literal translation of Newton's words is the best formulation of the laws of motion for the purpose of elementary instruction. That Newton's formulation is not easily understood by the beginner is tacitly recognized by most writers, much space being ordinarily devoted to explanations of the meaning of Newton's language. Without here attempting a full analysis, it may be profitable to suggest certain points in regard to which students may be aided by a departure from the usual method of stating and explaining the fundamental laws.

(1) Recognizing force as a fundamental quantity whose nature is known roughly, at least, from ordinary experience, its definition should be so stated as to include the fact that a force is exerted *by* a body. This should also be embodied in the statement of the first law, which might take the form 'a body uninfluenced by other bodies would move uniformly in a straight line or remain at rest.' It should also be embodied in the statement of the law of action and reaction: 'When one particle exerts a force upon another the latter exerts one upon the former, and the two forces are equal, collinear and opposite.'

(2) The full explanation of the second law should be preceded by a clear explanation of the meaning of acceleration as a vector quantity. The law itself might be stated as follows: 'A force acting alone upon a particle gives it an acceleration whose direction is that of the force and whose magnitude is proportional directly to that of the force and inversely to the mass of the particle.'

(3) The parallelogram law should receive explicit statement: 'Two forces acting simultaneously upon a particle give it an acceleration which is the vector sum of the accelerations which would be due to the forces acting separately.' 'Two forces acting simultaneously upon a particle are equivalent to a single force equal to their vector sum.' These statements are seen to be equivalent by virtue of the second law. An experimental statical proof of the parallelogram law is instructive, but its acceptance as an exact law rests on the same basis as that of the rest of the laws of motion—the apparent exact agreement of these laws with all experience.

Without entering into a detailed account of the treatment of the laws of motion in each of the three books under review, it is of interest to notice the different methods of defining and explaining force. Maurer's treatment is in close agreement with that here suggested; the point emphasized in (1) is explicitly stated at the outset, and the above statement of the law of action and reaction is in Maurer's words. Stephan gives the common but vague definition of force as the cause of a change of motion, the elementary but important fact that a force is always exerted by a body being explicitly stated only at the end of the three pages devoted to the preliminary explanation of force and of the law of action and reaction. In Ziwet's book the treatment of force oscillates between two different points of view. Force is defined mathematically as the product of the mass of a particle into its acceleration, and the author evidently agrees with those who regard force as a fiction, while he does not find it easy or advisable to discard the conception of force as a cause of motion in explanations addressed to beginners. The definition of force as the product of mass into acceleration, and the denial of force as a physical reality, are in harmony with what is, perhaps, the prevailing view among philosophical critics. Such a view is, however, wholly meaningless to the beginner, and it must be insisted that the treatment of force in an elementary text-book should build upon common notions and everyday experience.

Although, in an elementary text-book, logical rigor is not to be too strictly insisted upon, it is important to avoid false logic, and especially the appearance of logically proving what is really assumed. At certain points many current expositions of the principles of mechanics appear to be open to criticism on the ground of false or defective logic.

Consider, for example, Stephan's treatment of the law of composition of forces, which is substantially identical with that found in many text-books. In Art. 69 is the statement: 'If several forces act simultaneously upon a particle, the acceleration which each force imparts to the particle is independent of the existing velocity and of the action of the other forces.' For the explanation of simultaneous accelerations and of the method of combining them reference is made to Art. 66. But this explanation relates to a particle having a certain motion with respect to one base of reference, while this base is itself in motion with respect to a second base, so that the two 'simultaneous accelerations' refer to different bases or axes of reference. This throws no light on what is meant by simultaneous accelerations of a particle when only a single base of reference is in question. In the composition of forces we are not concerned with moving axes,* and in the analysis of motion with respect to any single base it is only by an arbitrary use of language that a particle can be said to have at the same time two different accelerations. Its actual acceleration may, of course, be expressed as the vector sum of components, but this may be done arbitrarily and in any number of ways; in choosing a particular set of components and associating each with a force we are merely *assuming* the parallelogram of forces.

From a logical standpoint the treatment of the theory of energy is an unsatisfactory feature of many text-books. Commonly energy is defined as the 'capacity of a body to do work,' or as the 'quantity of work a body can do,' while the meaning of work as done by a body is nowhere explained, work being defined only as done by a force. Another logical defect is to make $\frac{1}{2}mv^2$ the definition of kinetic energy instead of proving from a general definition of energy that a particle possesses by virtue of its motion the quantity of energy $\frac{1}{2}mv^2$. Of the three books under review, that of Professor Maurer is the only one that includes a logical and systematic presentation of the theory of energy.

Although the discussion of questions of terminology often seems fruitless, it may be worth while to refer to certain of these because of their importance as affecting the acquirement of sound notions by the beginner. That there has been little progress toward general agreement in the use of such terms as stress, centrifugal force, inertia force, is unfortunately due in part at least to the fact that discussions over them have involved more than mere questions of terminology.

The word stress is too often used vaguely, without attempt at exact definition. Among writers whose usage is clear, two definitions are current, which were formulated by Rankine and by Maxwell respectively. Rankine defined as stresses the forces which the particles of a body exert upon one another to resist strain (*i. e.,* departure from the 'natural' configuration). By Maxwell the action and reaction between any two portions of matter was called stress.* The usage of engineers, so far as it is definite, usually conforms more or less closely to the former definition, while the latter has been adopted in a number of works on both theoretical and applied mechanics. There are reasons in favor of each of these definitions, but it is to be regretted that the writer of a text-book should depart from both. Professor Ziwet apparently uses stress to designate any pair of equal and opposite forces in the same line, whether constituting an action and reaction or not. This sacrifices the chief value of Maxwell's definition, which is that it keeps clearly before the mind the fact that every force has its reaction and that action and reaction act upon different

* It is worth while to emphasize the argument by remarking that the accelerations of a particle with respect to two different sets of axes are not related by a simple parallelogram law unless the relative motion of the two bases is a translation.

A full logical analysis of the laws of motion, including the parallelogram law, must include a consideration of the meaning of absolute and relative motion—a question which may well be omitted from an elementary book and which will not be entered into here.

* See Rankine's 'Miscellaneous Scientific Papers,' p. 120; Maxwell's 'Matter and Motion,' Chapter III. It should be said that neither author used the word in a uniform sense throughout his writings.

portions of matter. One of the most common and vicious errors is that action and reaction are counterbalancing forces. This error will inevitably be made if stress is defined as action and reaction, and then used to designate a pair of counterbalancing forces. Professor Maurer's usage, while departing from both the above definitions, is clear and consistent. He defines stress as any force whose place of application is a surface.

Most present-day text-books, including the three before us, define centrifugal force as the reaction which a particle constrained to describe a curved path, or a rigid body constrained to rotate about a fixed axis, exerts upon the constraining body. This definition is clear, and would be satisfactory if it were not inconsistent with general usage in the only class of problems in which the term is really needed—*i. e.,* problems in which motion is referred to rotating axes. It is convenient in such cases to give the equation of motion of a particle the same form as if the axes were fixed, introducing such fictitious forces as would produce the accelerations actually due to the motion of the axes. One component of the fictitious force for each particle is the centrifugal force, which is thus not a reaction exerted by the particle but a force assumed to act upon it. This must be regarded as the legitimate use of the term centrifugal force. Inconsistency in the use of this term in elementary text-books is responsible for much confusion in the mind of the student. An example of this inconsistency occurs in Stephan's book, pp. 279, 281. Centrifugal force is defined as the reaction exerted by a particle upon the body which deflects it from a straight path. But in the discussion of the belt and pulley an element of the belt is said to 'experience' a centrifugal force.

So much confusion of thought has been shown in discussions of 'inertia-force' that it seems desirable to drop the term entirely. Those who use it often appeal to the authority of Newton; but it is well known that Newton did not restrict the word force to its present specialized meaning, and that which he meant by force of inertia is not force at all in the present meaning of the word. Professor Ziwet defines force of inertia of a particle as the reversed effective force, *i. e.,* a force $-mj$, m being the mass of the particle and j its acceleration; and he explains that this force is exerted not on the particle but by it, being the reaction to the force which acts upon the particle to produce its acceleration. A student who compares this statement with the following (p. 160) is likely to be somewhat bewildered: "The fact that any change of motion in a physical body is affected by its mass is sometimes ascribed to the so-called 'inertia,' or 'force of inertia,' of matter, which means, however, nothing else but the property of possessing mass." This latter statement is practically Newton's explanation of force of inertia.

The preceding definition (also given by Stephan) is sanctioned by various writers of high authority. It may, however, be doubted whether there is any real need of a term to designate the reversed effective force $-mj$; at all events the term inertia-force used in this sense seems inappropriate and misleading. The nature of the action to which we give the name force does not depend upon whether the body exerting it has or has not acceleration. Suppose, for example, that a particle is acted upon by two bodies only, A exerting a force P upon it and B a force Q, and let R be the vector sum of P and Q. The particle reacts upon A with a force $-P$ and upon B with a force $-Q$; there is no body upon which it exerts a force $-mj = -R$. The 'inertia-force' is thus merely the vector sum of two forces exerted by the particle upon different bodies. There is nothing peculiar about these forces, and no reason why either of them should be attributed to the 'inertia' of the body. If P and Q become equal and opposite, the so-called inertia-force becomes zero, but the nature of the forces P and Q and of the reactions to them is unchanged. Neither is the nature of P or of its reaction changed if Q ceases to act; there is no more reason in this case than in the preceding for attributing the force exerted upon A to the inertia of the particle.

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