voted to their work; and the members of the scientific staff have carried out most faithfully their duties of preparing and preserving the collections thus far made.

We hoped to be docked at Callao, but owing to the prolonged occupation of the dock by a disabled steamer and the uncertainty of its becoming free within reasonable time, we decided to proceed without further delay to Easter Island and continue the expedition as we are.

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SCIENTIFIC BOOKS.

Rational Geometry. By GEORGE BRUCE HALSTED. New York and London, John Wiley and Sons. 1904. Pp. viii + 285.

For over two thousand years there has been only one authoritative text-book in geometry. 'No text-book,' says the British Association, 'that has yet been produced is fit to succeed Euclid in the position of authority!'. There is, in fact, little improvement to be made in Euclid's work along the lines which he adopted, and among the multitude of modern text-books, each has fallen under the weight of criticism in proportion to its essential deviation from that ancient author.

This does not mean that Euclid is without defect, but starting from the discussion of his famous parallel postulate, the modern development has been in the direction of the extension of geometrical science, with the place of that author so definitely fixed that the system which he developed is called *Euclidean geometry,* to distinguish it from new developments. The defects of Euclid arise out of a new view of rigorous logic whose objections seem finely spun to the average practical man, but which are based upon sound thought. The key to this modern criticism is the doubt which the mind casts upon the reliability of the intuitions of our senses, and the tendency to make pure reason the court of last resort. Thus, the sense of point between points, the perception of greater and less and many other tacit assumptions of the geometrical diagram, are the vitiating elements on which modern criticism concentrates its objections.

As an evidence of the ease with which the senses can be made to deceive, take a triangle ABC, in which AC is slightly greater than BC. Erect a perpendicular to AB at its middle point to meet the bisector of the angle C in the point D. From D draw perpendiculars to AC, BC, meeting them respectively in the points E, F. Let the senses admit, as they readily will in a free-hand diagram, that E is between A and C, and F between B and C; then from the equal right triangles $AED = BFD$, $DEC = DFC$, we find $AE = BF$, $EC = FC$, and, by adding, $AC = BC$, whereas AC is in fact greater than BC.

Are we to take our eyes as evidence that one point lies between two other points, or how are we to establish that fact? This query alone lets in a flood of criticism on all established demonstrations. The aim of modern rational geometry is to pass from premise to conclusion solely by the force of reason. Points, lines and planes are the names of things which need not be physically conceived. The object is to deduce the conclusions which follow from certain assumed relations between these things, so that if the relations hold the conclusions follow, whatever these things may be. Space is the totality of these things; its properties are solely logical, and varied in character according to the assumed fundamental relations. Those assumed relations which develop space concepts that are apparently in accord with vision constitute the modern foundations of Euclidean space.

Mr. Halsted is the first to write an elementary text-book which adopts the modern view, and in this respect, his 'Rational Geometry' is epoch-making. It is based upon foundations which have been proposed by the German mathematician, Hilbert. In point of fact, the book contains numerous diagrams, and is not to be distinguished in this respect from ordinary text-books, but these are simply gratuitous and not necessary accompaniments of the argument, designed especially for elementary students whose minds would be unequal to the task of reveling in the domain of pure reason. Also, in opening the book at random, one does not recognize any great difference from an ordinary geometry. In other words, those as-

sumed relations are adopted which lead to Euclidean geometry. In this respect the author is appealing to the attention of elementary schools, where no geometry other than the practical geometry of our world has a right to be taught.

The first chapter deals with the first group of assumptions, the assumptions of association. Thus, the first assumption is that *two distinct points determine a straight line.* This associates two things called points with a thing called a straight line, and is not a definition of the straight line. The definition of a straight line as the shortest distance between two points involves at once an unnamed assumption, the conception of distance, which is a product of our physical senses, whereas the rational development of geometry seeks the assumptions which underlie and are the foundations of our physical senses. In the higher court of pure reason, the testimony of our physical senses has been ruled out, not as utterly incompetent, but as not conforming to the legal requirements of the court. However, there is no objection to shortness in names, and a straight line is contracted into a *straight,* a segment of a straight line, to a *sect,* etc.

In the second chapter we find the second group of assumptions, the assumptions of betweenness, which develop this idea and the related idea of the arrangement of points. In the next chapter we have a third group, the assumptions of congruence. This chapter covers very nearly the ordinary ground, with respect to the congruence of angles and triangles, and all the theory of perpendiculars and parallels which does not depend upon Euclid's famous postulate. This postulate and its consequences are considered in chapter IV.

All the school propositions of both plane and solid geometry are eventually developed, although there is some displacement in the order of propositions, due to the method of development. Numerous exercises are appended at the end of chapters, which are numbered consecutively from 1 to 700.

Undoubtedly the enforcement upon logic of a blindness to all sense perceptions introduces some difficulties which the ordinary geometries seem to avoid, but as in the case of our conception of a blind justice, this has its compensation in the greater weight of her decisions. It seems as if the present text-book ought not to be above the heads of the average elementary students, and that it should serve to develop logical power as well as practical geometrical ideas. Doubtless, some progressive teachers will be found who will venture to give it a trial, and thus put it to the tests of experience. At least, the work will appear as a wholesome contrast to many elementary geometries which have been constructed on any fanciful plan of plausible logic, mainly with an eye to the chance of profit.

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A Treatise on the British Freshwater Algæ. By G. S. West, M.A., A.R.C.S., F.L.S. Cambridge, The University Press. 1904.

Certainly there is no book upon any phase of cryptogamic botany for which there has been so much need, and for which the demand, in recent years, has been so great, as one dealing comprehensively with the fresh-water algæ. It is nearly twenty years since any work of the kind has appeared in English, and whatever may have been said in favor of the works of Cooke and Wolle when they were published, there can be no question about their having been out of date for a long time. Indeed, the tremendous strides made in algology during the last ten years has made it difficult for any one but the specialists to keep informed regarding the physiology, phylogeny and morphology of this group, to say nothing of the new genera and species. Of the fresh-water algæ alone, approximately one fourth of the genera now recognized have been described since the appearance of Engler and Prantl's classification in 1890. Consequently, nearly all of the important literature upon the algæ has been in periodicals and separates, often difficult to obtain, the result being that the general student of botany has, of necessity, been many years behind in his ideas regarding this most important and interesting group of plants.